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J.C.Brunelli

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 - São Paulo, S.P.
Brazil

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Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 288-5643
Telefax: 55 (11) 36-3449
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J. C. Brunelli

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405 - São Paulo - SP
BRAZIL

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Abstract

Using the Langevin approach for stochastic processes we study the renormalizability of the massive Thirring model. At finite fictitious time, we prove the absence of induced quadrilinear counterterms by verifying the cancellation of the divergencies of graphs with four external lines. This implies that the vanishing of the renormalization group beta function already occurs at finite times.

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1. Introduction

The stochastic quantization method of Parisi–Wu[1] (for a review see Ref. 2) when applied to fermionic theories usually requires the use of a Langevin system modified by the introduction of a kernel[3]

$$\frac{\partial\psi(x,t)}{\partial t} = -\int d^n y\, K(x,y)\frac{\delta S}{\delta\overline{\psi}(y,t)} + \eta(x,t) \tag{1.1a}$$

$$\frac{\partial\overline{\psi}(x,t)}{\partial t} = \int d^n y\, \frac{\delta S}{\delta\psi(y,t)}\overline{K}(x,y) + \overline{\eta}(x,t) \tag{1.1b}$$

where

$$\langle \eta_a(x,t)\overline{\eta}_b(x',t')\rangle = 2K_{ab}(x,x')\delta(t-t')\,. \tag{1.2}$$

Here ψ, $\overline{\psi}$ and the Gaussian noises η, $\overline{\eta}$ are independent Grassmann variables. $K(x,y)$ is the aforementioned kernel which ensures the proper equilibrium limit configuration for massless theories. The specific form of the kernel is quite arbitrary but in what follows, we use

$$\begin{aligned} K(x,y) &= (i\overrightarrow{\partial\!\!\!/}_x + m)\,\delta^n(x-y) \\ \overline{K}(x,y) &= \delta^n(x-y)\,(-i\overleftarrow{\partial\!\!\!/}_x + m)\,. \end{aligned} \tag{1.3}$$

In a number of cases, it has been verified that the stochastic quantization procedure does not bring new anomalies and that the equilibrium limit correctly reproduces the basic properties of the models considered[4]. It should be remarked that, for interacting fermionic models, a rigorously general proof of the equivalence between the stochastic method and the usual quantization procedures is lacking in contrast with the bosonic case[2]. It is still necessary to investigate specific examples to gain further insights on the peculiarities of the method. In this sense two dimensional models offer a good laboratory, providing a rich structure to test the general grounds of the theory.

In this paper we will focus our attention on the stochastic quantization of the massive Thirring model. One interesting aspect of the field quantized version of the model is the absence of infinite coupling constant renormalization[5]. As it is well known, this implies in

the vanishing of the renormalization group beta function to every finite order of perturbation theory. As we shall see, this happens already at finite fictitious time in the stochastic quantization approach. In Sec. 2 we establish the perturbation framework for the massive Thirring model in the stochastic quantization of Parisi-Wu in 2 space-time dimensions. In Sec. 3 the renormalizability of the model is discussed and we derive the renormalization group equation. In Sec. 4 we show by explicit calculations, at finite fictitious time and at one loop order, the vanishing of the beta function of the model. Finally, Sec. 5 is devoted to the conclusion.

2. Langevin Equation and Perturbative Expansion

The massive Thirring model, in Euclidian space-time, is defined by the Lagrangian density

$$\mathcal{L} = -i\overline{\psi}(\not{\partial} + im)\psi - \frac{1}{2}g\left(\overline{\psi}\psi\right)^2 \tag{2.1}$$

where the relation $\left(\overline{\psi}\gamma^\mu\psi\right)^2 = \left(\overline{\psi}\psi\right)^2$, valid in two dimensions, was used to rewrite the *current-current* interaction in a more convenient form. Throughout this paper we will use an antihermitian representation for the γ_μ matrices which obey $\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$ and we adopt $\gamma_0 = i\sigma_1$, $\gamma_1 = i\sigma_3$ and $\gamma_5 = \gamma_0\gamma_1 = i\sigma_2$ (σ_i, $i = 1,2,3$ are the Pauli matrices).

The Langevin equations for this system were considered previously by Damgaard and Tsokos[3] and, in momentum space[a], they read

$$\frac{\partial\psi(k,t)}{\partial t} = -(k^2+m^2)\psi(k,t) + \frac{2g}{(2\pi)^4}\int d^2p\, d^2q\, \overline{\psi}(p,t)\,\psi(q,t) \times(-\not{k}+m)\,\psi(k+p-q,t) + \eta(k,t) \tag{2.2a}$$

[a] we consider that ψ and $\overline{\psi}$ have opposite Fourier transforms, i.e.:

$$\psi(x,t) = \int \frac{d^nk}{(2\pi)^n}\, e^{ikx}\,\psi(k,t) \quad \text{and} \quad \overline{\psi}(x,t) = \int \frac{d^nk}{(2\pi)^n}\, e^{-ikx}\,\overline{\psi}(k,t)$$

$$\frac{\partial\overline{\psi}(k,t)}{\partial t} = -(k^2+m^2)\overline{\psi}(k,t) + \frac{2g}{(2\pi)^4}\int d^2p\, d^2q\, \overline{\psi}(p,t)\,\psi(q,t) \times\overline{\psi}(k-p+q,t)\,(-\not{k}+m) + \overline{\eta}(k,t) \tag{2.2b}$$

where

$$\langle\eta(k,t)\overline{\eta}(k',t')\rangle = 2(2\pi)^2(-\not{k}+m)\delta^2(k-k')\delta(t-t'). \tag{2.3}$$

Using $\psi(k,0) = \overline{\psi}(k,0) = 0$ as initial conditions the solutions of (2.2) are

$$\psi_a(k,t) = \int_0^t d\tau\; e^{-(k^2+m^2)(t-\tau)}\,\eta_a(k,\tau) + 2g\int_0^t d\tau \int \frac{d^2p\, d^2q}{(2\pi)^4} \times\Gamma_{ab}(k;t-\tau)\psi_b(k+p-q,\tau)\,\overline{\psi}(p,\tau)\psi(q,\tau) \tag{2.4a}$$

$$\overline{\psi}_a(k,t) = \int_0^t d\tau\; e^{-(k^2+m^2)(t-\tau)}\,\overline{\eta}_a(k,\tau) + 2g\int_0^t d\tau \int \frac{d^2p\, d^2q}{(2\pi)^4} \times\overline{\psi}(p,\tau)\psi(q,\tau)\,\overline{\psi}_b(k-p+q,\tau)\Gamma_{ba}(k;t-\tau). \tag{2.4b}$$

We have introduced the notation

$$\Gamma(k,t) \equiv G(k,t)(-\not{k}+m) = \theta(t)\,e^{-(k^2+m^2)t}(-\not{k}+m) \tag{2.5}$$

where $G(k,t)$ is the Green function of the linearized part of (2.2).

Setting $g = 0$ we can calculate the two point function obtaining

$$\langle\psi_a(k,t)\overline{\psi}_b(k',t')\rangle = (2\pi)^2\delta^2(k-k')D_{ab}(k;t,t')$$

with

$$D_{ab}(k;t,t') = |t-t'| \int_1^{\frac{t+t'}{|t-t'|}} d\beta\,(-\not{k}+m)_{ab}\, e^{-\beta|t-t'|(k^2+m^2)} \tag{2.6}$$

which in the equilibrium limit (taking $t = t' \to \infty$ after the integration) reduces to the usual Euclidian fermion propagator.

For $g \neq 0$ the integral equations (2.4) can be solved by iteration treating g as a small parameter. In figure 1 we illustrate the solution obtained in this way. The Green functions

$G(k,t)$ are represented by solid lines while the $\Gamma(k,t)$ are represented by broken lines. Noise fields $\eta(k,t)$ and $\overline{\eta}(k,t)$ are represented respectively by crosses and boxes. The wavy lines help in visualizing the charge flow by separating the term $\Gamma\psi$ or $\overline{\psi}\Gamma$ from $\overline{\psi}\psi$. In this representation the propagator associated to the wavy line is just the identity.

Figure 1

3. Renormalization

The $2N$-point correlation functions are obtained by substituting the N fields ψ and N fields $\overline{\psi}$ for its diagrammatical expansion given in figure 1 generating, in this way, a series of stochastic diagrams after the averages over the η's and $\overline{\eta}$'s are taken. The analytical form of these diagrams[6], out of the equilibrium, is given by

$$\int \prod_{i=1}^{L} \frac{d^2 k_i}{(2\pi)^2} \int \prod_{i=1}^{V} d\tau_i \prod_{\ell=1}^{N_{IB}} \Gamma(Q_\ell; \tau_{\ell_1} - \tau_{\ell_2}) \prod_{\ell'=1}^{N_{I\times}} D(Q_{\ell'}; \tau_{\ell'_1}, \tau_{\ell'_2}) \tag{3.1}$$

where L is the number of loops, V is the number of vertices, N_{IB} is the number of internal broken lines representing $\Gamma(k; t-t')$ (given by (2.5)), $N_{I\times}$ is the number of internal crossed lines representing the propagator $D(k;t,t')$ (given by (2.6)) obtained by contracting the lines with noises attached to its ends. In the above expression we have omitted matricial indices and the factors corresponding to external lines. In the $\ell(\ell')^{\text{th}}$ broken(crossed) line, joining the vertices with times $\tau_{\ell_1}(\tau_{\ell'_1})$ and $\tau_{\ell_2}(\tau_{\ell'_2})$, flows momentum $Q_\ell(Q_{\ell'})$ which is a linear combination of external and loop momentum k_i.

Using (2.5) and (2.6) the expression (3.1) assumes the form

$$\int \prod_{i=1}^{L} \frac{d^2 k_i}{(2\pi)^2} \int \prod_{i=1}^{V} d\tau_i \prod_{\ell=1}^{N_{IB}} \theta(\tau_{\ell_1} - \tau_{\ell_2}) \exp\left[-(\tau_{\ell_1} - \tau_{\ell_2})(Q_\ell^2 + m^2)\right] (-\not{Q}_\ell + m)$$
$$\times \prod_{\ell'=1}^{N_{I\times}} |\tau_{\ell'_1} - \tau_{\ell'_2}| \int_1^{\frac{\tau_{\ell'_1}+\tau_{\ell'_2}}{|\tau_{\ell'_1}-\tau_{\ell'_2}|}} d\beta_{\ell'} (-\not{Q}_{\ell'} + m) \exp\left[-\beta_{\ell'} |\tau_{\ell'_1} - \tau_{\ell'_2}| (Q_{\ell'}^2 + m^2)\right] . \tag{3.2}$$

Replacing k_i by λk_i in (3.2) and allowing $\lambda \to \infty$ we get for the superficial degree of divergence d, for the stochastic diagrams, the expression

$$d = 2L - 2(V-1) + N_{IB} - N_{I\times}$$

where the contributions $-2(V-1)$ and $-N_{I\times}$ come from the observation that the replacement of the k_i's in the exponentials (inside the terms Q_ℓ and $Q_{\ell'}$) are equivalent to the replacement of the temporal differences $\tau_i - \tau_j$ by $\lambda^{-2}(\tau_i - \tau_j)$ in the integrand and in the integration measure. By construction $V = N_B$, where N_B is the total number of internal and external broken lines. This relation plus the topological one $4V = N_E + 2N_I$, with N_E and N_I standing for the total number of external and internal lines respectively, yields

$$d = 4 - \frac{5}{2} N_{EB} - \frac{1}{2} N_{E\times} \tag{3.3}$$

with N_{EB} and $N_{E\times}$ as the total number of external broken and crossed lines respectively. One may observe that for every diagram with loops there is at least one external broken line (that is $N_{EB} \neq 0$) what leads to the fact that linear divergent diagrams appear only if $N_{EB} = N_{E\times} = 1$ and logarithmic divergent ones for $N_{EB} = 1$ and $N_{E\times} = 3$.

These ultraviolet divergencies, once the theory is regularized, can be reabsorbed by a renormalized Langevin equation obtained from (2.2) and (2.3) through the following redefinition of parameters and field normalization[b]

$$\begin{aligned} \psi(k,t) &= Z_\psi^{1/2}\, \psi_R(k,t_R) \\ g &= Z_g\, Z_\psi^{-2}\, g_R \\ m^2 &= m_R^2 + \frac{\delta m^2}{Z_\psi} \\ \eta(k,t) &= Z_\psi^{-1/2}\, \eta_R(k,t_R) \\ t &= Z_t^{-1}\, Z_\psi\, t_R \end{aligned} \tag{3.4}$$

[b] In fact we could employ a renormalization constant Z_η in the redefinition of the noise, so that $\eta(k,t) = Z_\psi^{-1/2} Z_\eta^{-1} Z_t^{1/2} \eta_R(k,t_R)$ (of course Z_η and Z_t should enter in the redefinition of the others parameters). But as show by Zinn-Justin[7] we have the Ward identity $Z_t = Z_\eta^2$, for Markovian process, which allow us the use of (3.4).

where $\psi(k,t)$, g, m, $\eta(k,t)$ and t are the bare parameters of the theory. The counterterms, and therefore the renormalization constants $Z = \{Z_\psi, Z_g, \delta m, Z_t\}$ are calculated, order by order in perturbation theory, through the renormalization conditions which fix the mass, the physical coupling constant, the normalizations of the field ψ and the noise η. The renormalization constants are dimensionless functions of the coupling constant g_R, of the dimensional parameters t_R, m_R and the renormalization point μ. However, in what follows, we use a minimal subtraction scheme[8] where the finite parts of the counterterms are set to zero so that the renormalization constants do not depend on the dimensional parameters.

In our calculations we apply an analytic regularisation[9] (for a description of this regularization in the stochastic approach see Ref. 10) which implies the introduction of an arbitrary constant μ with the dimension of mass. Now g_R is dimensionless and in (3.4) we have to make the substitution $g_R \to \mu^\lambda g_R$, where λ is an analytic regulator, and we emphasize that the Z's do not depend on m_R and t_R because the finite parts of the counterterms are set to zero.

Using (3.4) the $2N$-point bare and renormalized correlation functions are related by

$$G^{(N)}(\tilde{p}, g, m, \tilde{t}, \lambda) = Z_\psi^{N/2} G_R^{(N)}(\tilde{p}, g_R, m_R, \tilde{t}_R, \lambda, \mu) \tag{3.5}$$

where the bare correlation functions are regularized in a properly manner. The $\tilde{p}$ and $\tilde{t}$ refer to the set of external momenta and fictitious time.

The renormalization group equation is derived by the observation that the bare correlation functions are independent of the renormalization scale μ while the renormalized ones depend on μ explicitly and implicitly through g_R, m_R and t_R. Taking the derivative $\frac{\partial}{\partial\mu}$ of (3.5) we come up with

$$\left(\frac{N}{2}\mu\frac{\partial \ln Z_\psi}{\partial\mu} + \mu\frac{\partial g_R}{\partial\mu}\frac{\partial}{\partial g_R} + \mu\frac{\partial m_R}{\partial\mu}\frac{\partial}{\partial m_R} + \mu\frac{\partial \tilde{t}_R}{\partial\mu}\frac{\partial}{\partial \tilde{t}_R} + \mu\frac{\partial}{\partial\mu}\right) G_R^{(N)} = 0 \tag{3.6}$$

as the generalization of the renormalization group equation to the stochastic case. By dimensional analysis we know that

$$G_R^{(N)}(\rho\tilde{p}, g_R, \rho m_R, \rho^{-2}\tilde{t}_R, \lambda, \rho\mu) = \rho^{2-\frac{3}{2}N} G_R^{(N)}(\tilde{p}, g_R, m_R, \tilde{t}_R, \lambda, \mu)$$

which, by the Euler's theorem for homogeneous functions, satisfies

$$\left(s\frac{\partial}{\partial s} + m_R\frac{\partial}{\partial m_R} - 2\tilde{t}_R\frac{\partial}{\partial \tilde{t}_R} + \mu\frac{\partial}{\partial\mu} - D\right) G_R^{(N)}(s\tilde{p}, g_R, m_R, \tilde{t}_R, \lambda, \mu) = 0$$

where s is a scale for the momenta. We can use this expression to eliminate $\mu\frac{\partial}{\partial\mu}$ in (3.6) and we are left with

$$\left[-s\frac{\partial}{\partial s} + (N\gamma_\psi + D) + \beta(g_R)\frac{\partial}{\partial g_R} + (\gamma_m - 1)m_R\frac{\partial}{\partial m_R} + 2(\gamma_t - \gamma_\psi + 1)\tilde{t}_R\frac{\partial}{\partial \tilde{t}_R}\right] G_R^{(N)}(s\tilde{p}, g_R, m_R, \tilde{t}_R, \lambda, \mu) = 0 \tag{3.7}$$

where

$$\begin{aligned} \beta(g_R,\lambda) &= \mu\frac{\partial g_R}{\partial\mu} & \gamma_\psi(g_R,\lambda) &= \frac{1}{2}\mu\frac{\partial \ln Z_\psi}{\partial\mu} \\ \gamma_t(g_R,\lambda) &= \frac{1}{2}\mu\frac{\partial \ln Z_t}{\partial\mu} & \gamma_m(g_R,\lambda) &= \frac{1}{2}\mu\frac{\partial \ln m_R^2}{\partial\mu} \end{aligned} \tag{3.8}$$

Equations (3.6) and (3.7) are the generalization of renormalization group equations to the stochastic approach and show the behaviour of the correlation functions by a shift in the renormalization point or a momentum reescaling respectively. The study of these equations as well as the derivation of the Callan-Symanzik equation, even for non-Markovian stochastic process, can be found in Ref. 11.

4. Beta Function Calculation

In the equilibrium situation the $\beta(g_R)$ function for the massive Thirring model vanishes to all orders in perturbation theory[5]. Now, in the SQM, we show that the same result is valid out of the equilibrium at least to the one loop order.

The β function, in a minimal subtraction scheme, is given by

$$\beta(g_R) = \lim_{\lambda\to 0}\beta(g_R,\lambda) = \lim_{\lambda\to 0}\mu\frac{\partial g_R}{\partial\mu}\Big|_{g,\lambda} \tag{4.1}$$

where we have used (3.8). The relation between the bare and renormalized coupling constant (3.4) can be rewritten perturbativaly as

$$g = \mu^\lambda Z_g Z_\psi^{-2} g_R = \mu^\lambda\left(1 + \sum_{n=1}^{\infty}\frac{a_n(g_R)}{\lambda^n}\right) g_R \tag{4.2}$$

where the coefficients a_n are power series in the renormalized coupling constant g_R. Taking the derivative of $g_R = \mu^{-\lambda}(Z_g Z_\psi^{-2})^{-1} g$ on μ we get

$$\beta(g_R,\lambda) + \lambda g_R + g_R \beta(g_R,\lambda)\frac{\partial \ln(Z_g Z_\psi^{-2})}{\partial g_R} = 0\,.$$

Using the series (4.2) we look for a power series solution for $\beta(g_R,\lambda)$ and we finally arrive at[8]

$$\beta(g_R) = g_R^2 \frac{da_1}{dg_R} \tag{4.3}$$

showing that $\beta(g_R)$ is totally determined by the residue of the simple poles in λ of $Z_g Z_\psi^{-2}$.

To calculate[c] β in order g^2 we must determine Z_ψ and Z_g in order g. However, as shown in figure 2, the graphs (a) and (b) do not contribute for the wave function renormalization because their loops do not have external momenta dependence. So we can write

$$Z_\psi = 1 + \mathcal{O}(g^2)\,. \tag{4.4}$$

Figure 2

The stochastic Feynman graphs for the correlation functions with four external lines can be generated at order g^2 by substituting ψ and $\overline{\psi}$ given in figure 1 in

$$\langle \psi_{a_1}(p_1,t_1)\psi_{a_2}(p_2,t_2)\overline{\psi}_{a_3}(p_3,t_3)\overline{\psi}_{a_4}(p_4,t_4)\rangle\,.$$

These graphs can be separated in classes, as the one shown in figure 3, which differ only by the type of lines distribution.

Figure 3

The graphs of figure 3 have logarithmic divergencies but we can show a cancellation among them at finite fictitious time. First let us write the analytic expression associated

[c] from now on we shall omit completely the subscript R from the renormalized parameters.

to each one of them

$$\text{(a)} = -4g^2 \int_0^{t_1} d\tau \int_0^{\tau} d\tau' \int d^2k \left[\Gamma(p_1;t_1-\tau)\Gamma(p_1+p_2-k;\tau-\tau')D(p_4;\tau',t_4)\right]_{a_1a_4} \times \left[D(p_2;t_2,\tau)\,D(k;\tau,\tau')\,D(p_3;\tau',t_3)\right]_{a_2a_3} \tag{4.5a}$$

$$\text{(b)} = -4g^2 \int_0^{t_1} d\tau \int_0^{\tau} d\tau' \int d^2k \left[\Gamma(p_1;t_1-\tau)\Gamma(p_1-p_3+k;\tau-\tau')D(p_4;\tau',t_4)\right]_{a_1a_4} \times \left[D(p_2;t_2,\tau')\,D(k;\tau',\tau)\,D(p_3;\tau,t_3)\right]_{a_2a_3} \tag{4.5b}$$

$$\text{(c)} = -4g^2 \int_0^{t_1} d\tau \int_0^{\tau} d\tau' \int d^2k \left[\Gamma(p_1;t_1-\tau)\,D(p_4;\tau,t_4)\right]_{a_1a_4} \times \mathrm{Tr}\Big(\Gamma(k;\tau-\tau')D(k-p_2+p_3;\tau,\tau')\Big)\left[D(p_2;t_2,\tau')\,D(p_3;\tau',t_3)\right]_{a_2a_3} \tag{4.5c}$$

$$\text{(d)} = 4g^2 \int_0^{t_1} d\tau \int_0^{\tau} d\tau' \int d^2k \left[\Gamma(p_1;t_1-\tau)\,D(k-p_2+p_3;\tau,\tau') \times\Gamma(k;\tau-\tau')\,D(p_4;\tau,t_4)\right]_{a_1a_4}\left[D(p_2;t_2,\tau')\,D(p_3;\tau',t_3)\right]_{a_2a_3} \tag{4.5d}$$

$$\text{(e)} = 4g^2 \int_0^{t_1} d\tau \int_0^{\tau} d\tau' \int d^2k \left[\Gamma(p_1;t_1-\tau)\,D(p_4;\tau,t_4)\right]_{a_1a_4} \times\left[D(p_2;t_2,\tau')\Gamma(k+p_2-p_3;\tau-\tau')\,D(k;\tau,\tau')\,D(p_3;\tau',t_3)\right]_{a_2a_3} \tag{4.5e}$$

For the graph (a), given by (4.5a), we can use (2.5) and (2.6) to obtain

$$-4g^2 \int_0^{\infty} d\tau \int_0^{\infty} d\tau'\, \Gamma_{a_1b}(p_1;t_1-\tau)\,D_{ca_4}(p_4;\tau',t_4)\,D_{a_2d}(p_2;t_2,\tau)\,D_{ca_3}(p_3;\tau',t_3) \times \int d^2k\,\theta(\tau-\tau')\left[-(\not p_1+\not p_2-\not k)+m\right]_{bc} e^{-[(p_1+p_2-k)^2+m^2]\,|\tau-\tau'|} \times \int_{|\tau-\tau'|}^{\tau+\tau'} d\alpha\,(-\not k+m)_{de}\, e^{-\alpha(k^2+m^2)} \tag{4.6}$$

Performing the change of variables $a = \tau - \tau'$, $b = \frac{1}{2}(\tau + \tau')$ and $\alpha = a\beta$ the expression (4.6) assumes the form

$$-4g^2 \int_{-\infty}^{+\infty} da \int_0^{+\infty} db\, \Gamma_{a_1 b}(p_1; t_1 - a/2 - b)\, D_{ca_4}(p_4; -a/2 + b, t_4)$$

$$\times D_{a_2 d}(p_2; t_2, a/2 + b)\, D_{ca_3}(p_3; -a/2 + b, t_3)\, \theta(a)\, a \int_1^{2b/a} d\beta\, e^{-a(p^2 + m^2 + \beta m^2)}$$

$$\times \int d^2k\, [-(\not p - \not k) + m]_{bc} (-\not k + m)_{de}\, e^{-a[k^2(1+\beta) - 2pk]} \tag{4.7}$$

where we have introduced the notation $p \equiv p_1 + p_2$. A divergence in the integrand appears, as a pole in $a = 0$, after the integration of the term $\not k \not k$. Isolating this term we obtain

$$4\pi g^2 \int_{-\infty}^{+\infty} da \int_0^{+\infty} db\, \Gamma_{a_1 b}(p_1; t_1 - a/2 - b)\, D_{ca_4}(p_4; -a/2 + b, t_4)\, D_{a_2 d}(p_2; t_2, a/2 + b)$$

$$\times D_{ca_3}(p_3; -a/2 + b, t_3)\, [(\gamma_\mu)_{bc} (\gamma_\mu)_{de}]\, \theta(a)\, a^{\lambda - 1} \int_1^{2b/a} d\beta\, \frac{e^{-am^2(1+\beta) - \frac{ap^2\beta}{1+\beta}}}{2(1+\beta)^2} \tag{4.8}$$

where we have regularized analytically the integral in the variable a by the introduction of a complex parameter λ, the regulator. By an analytic continuation of the above expression for $\lambda = 0$, the divergence appears as a pole at $\lambda = 0$[9].

The pole of expression (4.8) yields[12]

$$4\pi g^2 \int_{-\infty}^{+\infty} da \int_0^{+\infty} db\, \Gamma_{a_1 b}(p_1; t_1 - a/2 - b)\, D_{ca_4}(p_4; -a/2 + b, t_4)\, D_{a_2 d}(p_2; t_2, a/2 + b)$$

$$\times D_{ca_3}(p_3; -a/2 + b, t_3)\, [(\gamma_\mu)_{bc} (\gamma_\mu)_{de}]\, \frac{1}{\lambda} \delta(a) \int_1^{2b/a} d\beta\, \frac{e^{-am^2(1+\beta) - \frac{ap^2\beta}{1+\beta}}}{2(1+\beta)^2}$$

where, after performing the integration in a, the integration in β is trivial. So, the final result for the divergent piece of graph (a) of figure 3 is given by

$$-\pi g^2 \frac{1}{\lambda} \int_0^{\infty} db\, \Big[\Gamma(p_1; t_1 - b) \gamma_\mu D(p_4; b, t_4)\Big]_{a_1 a_4} \Big[D(p_2; t_2, b) \gamma_\mu D(p_3; b, t_3)\Big]_{a_2 a_3} \tag{4.9}$$

Following the same procedure method for the graph (b) we find that the divergent part cancels exactly with the contribution coming from graph (a) (given by (4.9)). Also, by similar calculations we obtain for the graphs (c), (d) and (e) of figure 3 the following divergent terms

$$-4\pi g^2 \frac{1}{\lambda} \int_0^{\infty} db\, \Big[\Gamma(p_1; t_1 - b)\, D(p_4; b, t_4)\Big]_{a_1 a_4} \Big[D(p_2; t_2, b)\, D(p_3; b, t_3)\Big]_{a_2 a_3}$$

$$2\pi g^2 \frac{1}{\lambda} \int_0^{\infty} db\, \Big[\Gamma(p_1; t_1 - b)\, D(p_4; b, t_4)\Big]_{a_1 a_4} \Big[D(p_2; t_2, b)\, D(p_3; b, t_3)\Big]_{a_2 a_3}$$

$$2\pi g^2 \frac{1}{\lambda} \int_0^{\infty} db\, \Big[\Gamma(p_1; t_1 - b)\, D(p_4; b, t_4)\Big]_{a_1 a_4} \Big[D(p_2; t_2, b)\, D(p_3; b, t_3)\Big]_{a_2 a_3}$$

which obviously cancel each other when added. The same occurs for the graphs with four external lines of the other classes.

Then, at order g^2, all divergent terms for the four point correlation functions cancel at finite fictitious time (note that this result independs of the minimal subtraction scheme we have used) and the renormalization constant Z_g gives

$$Z_g = 1 + \mathcal{O}(g^2). \tag{4.10}$$

Finally from (4.3)

$$\beta(g) = \mathcal{O}(g^3). \tag{4.11}$$

5. Conclusions

We have shown explicitly, at order of one loop, the cancellation of the divergencies in the stochastic four point function for the massive Thirring model at finite fictitious time. This result implies in the vanishing of the renormalization group beta function at finite times. The generalization of the result (4.11), for all orders in g, depends on an analysis which shows explicitly the cancellation mechanism of divergencies among the graphs in

each order, as happens with the ones in figure 3. However, the asymptotic conservation of vectorial and chiral currents in the massless limit are the key elements responsible for the vanishing of the beta function in the equilibrium situation[5]. The generalization of these proprieties for the stochastic approach can be used, on all order perturbation calculation, to demonstrate the maintenance of the asymptotic scale invariance of the massive Thirring model. These results will be published elsewhere.

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Figure Captions

Figure 1 – Perturbative expansion of the fermion fields. Broken lines represent Γ, solid lines G and wavy lines emphasize the separation of the terms $\Gamma\psi$ or $\overline{\psi}\Gamma$ from $\psi\overline{\psi}$. The noises η and $\overline{\eta}$ are represented by crosses and boxes, respectively.

Figure 2 – The 2-point function to one-loop order.

Figure 3 – A class of stochastic diagrams contribuing to 4-point function to one-loop order.

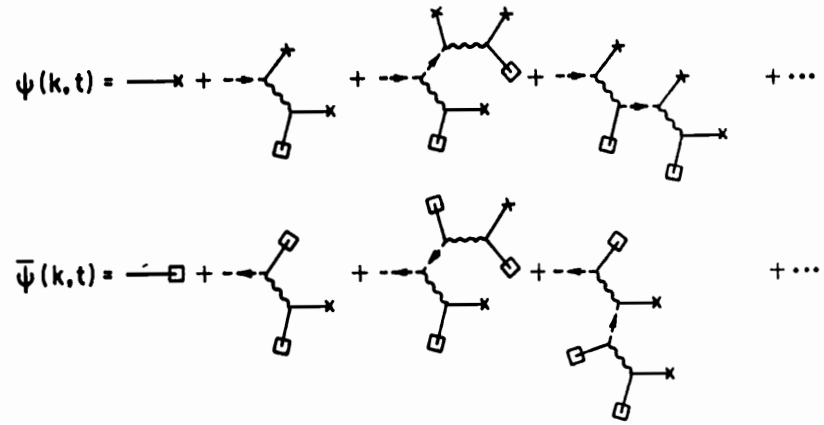

Figure 1

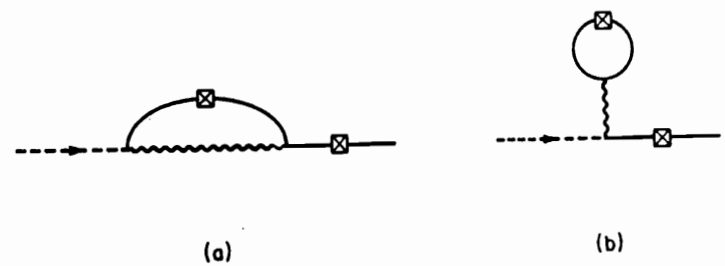

Figure 2

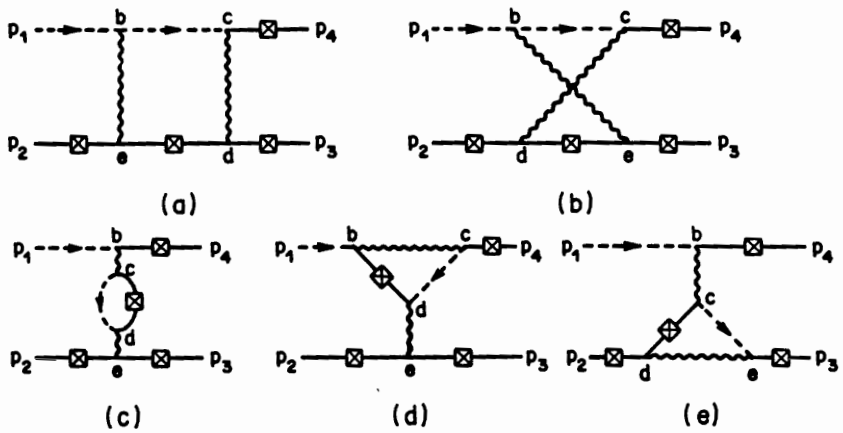

Figure 3