IFT-P.033/92
Fermilab Library
0 1160 0011923 4

IFT

Instituto de Física Teórica
Universidade Estadual Paulista

June/92

IFT–P.033/92

Unexpected strong attraction in the presence of continuum bound state

A. DELFINO
Department of Physics and Astronomy, University of Maryland
College Park, Md 20742
and
Departamento de Física, Universidade Federal Fluminense
24210 Niterói, RJ, Brazil

SADHAN K. ADHIKARI, LAURO TOMIO[+]
Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, SP, Brazil
and
[+]*School of Physical Sciences*
The Flinders University of South Australia
Bedford Park, SA 5042 Australia

TOBIAS FREDERICO
Instituto de Estudos Avançados, Centro Técnico Aeroespacial
12231 São José dos Campos, SP, Brazil

FERMILAB
OCT 1 1992
LIBRARY

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 288-5643
Telefax: 55 (11) 36-3449
Telex: 55 (11) 31870 UJMFBR
Electronic Address: LIBRARY@IFT.UESP.ANSP.BR
47553::LIBRARY

Unexpected strong attraction in the presence of continuum bound state

A. Delfino

Department of Physics and Astronomy, University of Maryland, College Park, Md 20742

and

Departamento de Física, Universidade Federal Fluminense, 24210 Niterói, RJ, Brasil

Sadhan K. Adhikari, Lauro Tomio†

Instituto de Física Teórica, Universidade Estadual Paulista, 01405 São Paulo, SP, Brasil

and

†School of Physical Sciences, The Flinders University of South Australia

Bedford Park, SA 5042, Australia

T. Frederico

Instituto de Estudos Avançados, Centro Técnico Aeroespacial

12231 São José dos Campos, SP, Brasil

The result of few-particle ground-state calculation employing a two-particle non-local potential supporting a continuum bound state in addition to a negative-energy bound state has occasionally revealed unexpected large attraction in producing a very strongly bound ground state. In the presence of the continuum bound state the difference of phase shift between zero and infinite energies has an extra jump of π as in the presence of an additional bound state. The wave function of the continuum bound state is identical with that of a strongly bound negative-energy state, which leads us to postulate a pseudo bound state in the two-particle system in order to explain the unexpected attraction. The role of the Pauli forbidden states is expected to be similar to these pseudo states.

PACS NUMBERS: 21.45.+v, 03.65.Nk

I. INTRODUCTION

Usually, in quantum mechanical problems with short-range local potentials the binding energy of the two-particle system reflects the strength of the underlying potential.[1] Qualitatively, the stronger the potential, the larger the two-particle binding. For an S-wave square-well potential the quantity V_0R could be considered a measure of the strength of the potential, where V_0 is the depth of the square-well and R is its range. [1] Whenever V_0 and/or R increase the potential becomes stronger and the two-particle binding increases. Though, for a realistic local potential the depth and the range parameters, V_0 and R, cannot always be unambiguously defined, the two-particle ground state binding serves as an useful parameter to indicate the strength of the potential. One also has the very useful Levinson's theorem in this case. Levinson has shown that for a short-range local potential the phase-shift at zero energy $\delta(0)$ and the phase-shift at infinite energy $\delta(\infty)$ are related by the condition[1]

$$\delta(0) - \delta(\infty) = N\pi, \tag{1}$$

where the quantity $N \equiv [(\delta(0)-\delta(\infty))/\pi]$ is the number of bound states and should be directly related to the strength of the potential.

The above simple argumentation does not necessarily work in the case of a general partly attractive and partly repulsive non-local potential. Such non-local potentials are common in various areas of physics and usually they appear when an effective interaction, suppressing certain internal degrees of freedom, is derived. This happens, for example, for an intercluster potential in nuclear physics, where the nucleonic degrees of freedom are suppressed. Such a potential between clusters appears naturally in the resonating group method.[2] Non-locality of the effective interaction also appears naturally in the Feshbach unified theory of nuclear reactions.[3] The non-local intercluster interaction in the resonating group method seems to be, in general, stronger than an equivalent local counterpart. For example, the resonating group calculation of the relative motion between two alpha particles often requires the wave function to have additional node(s) at small distances indicating the presence of excited states in the system.[2,4] Also, the expression $[\delta(0) - \delta(\infty)]$ may have an extra jump of π (or several π's) not dictated by the Levinson's theorem. This is true for the S wave $\alpha - \alpha$ scattering in the resonating group method[2]

and for the S wave spin quartet neutron-deuteron scattering[5] using the Faddeev equations.[6, 7] These facts suggest the existence of excited state(s) not present in the system. They are the so called Pauli forbidden states[4, 5] which contribute to N on the right hand side of Eq. (1).[5]

All the above mentioned features of a forbidden state can be simulated in a simple non-local rank-one potential model possessing a continuum bound state (CBS)[4, 8] in addition to the usual negative-energy bound state(s). We shall occasionally refer to a usual negative-energy bound state simply by bound state. In order to possess a CBS the potential should, however, be partly attractive and partly repulsive.[9] The difference $[\delta(0) - \delta(\infty)]$ for such a potential has the required extra jump of π.[4, 5, 9] Also, the wave function for relative motion has a node at short distances. Somehow, the behavior of the phase-shift or the wave function suggest the presence of an excited state of the system. In the presence of a CBS the binding energy of the usual bound state does not necessarily reflect the strength of the potential. Whenever this potential is used in a multi-particle system it produces stronger binding compared to that produced by an 'equivalent' local potential which produces the same negative-energy bound state.[10, 11] However, we recall that a local potential cannot reproduce all the peculiarities of the non-local potential.[4]

We study this potential model yielding a CBS in some detail with a view to understand the origin of this unexpectedly large attraction. Also, as this potential model simulates the essential features of a non-local intercluster potential which excludes the Pauli forbidden state(s) our study will shed light on the unexpected behavior[4] of such a potential.

A CBS is an S-matrix pole on the real positive energy axis.[4, 9] The association with the bound state lies in the fact that the corresponding wave function, ϕ, has no outgoing propagation. It is in fact a localized function of momentum $\phi(p)$ and its Fourier transform $\phi(r)$ decreases exponentially like a bound-state wave function. The number N in the Levinson's theorem (1) should include all exponentially decaying states – bound state(s) and CBS. A virtual state is exponentially growing and does not contribute to the Levinson's theorem, although it implies attraction.

Tabakin[12] constructed a rank-one separable NN potential with a two-term form factor, capable of reproducing S wave NN phase shifts upto moderately high energies where these phase shifts become negative in agreement with experiment.

This potential supports a CBS in addition to a bound deuteron. A class of similar potentials has been identified later. The use of these potentials in (three-nucleon) $3N$ and four-nucleon bound-state calculations[10] leads to ground states of unusually large binding of several hundred MeV's in addition to a weakly bound excited state. This puzzling result has eversince been studied by several group of workers.[10, 11, 13, 14]

One may ask if the Tabakin potential reproduces the experimental NN phase-shifts reasonably well why does it fail drastically in the $3N$ bound-state problem? Strictly speaking, the experimental NN phase-shifts become negative at higher energies and eventually goes to zero at infinite energy in accordance with the Levinson's theorem reflecting the presence of a single bound deuteron. On the other hand, the Tabakin phase-shift becomes negative and goes to -π at infinite energy reflecting the presence of a CBS in addition to the bound deuteron in violation of the usual Levinson's theorem (1); for a Tabakin-type potential $[\delta(0) - \delta(\infty)] = 2\pi$. It is this CBS and the associated behavior of the phase-shift which is responsible for the collapse. The collapse becomes more drastic, contrary to expectations, as the CBS moves to higher energies. [10, 11, 14]

This problem is best studied using the momentum-space connected-kernel three-particle dynamical equations.[6] The input to such calculations are the negative-energy two-particle t matrix elements. The presence of the CBS, or the unusual behavior of the positive-energy phase-shifts has a dramatic effect on the positive-energy t matrix elements but does not have a direct consequence on the negative-energy t matrix elements. It is interesting to investigate how does the presence of a CBS manifests at negative energies leading to the collapse.

We shall see that because of an unexpected pole cancellation the momentum-space CBS wave function is found to be independent of the energy of the CBS and be identical with that of a negative-energy bound state of a one-term separable Yamaguchi potential.[15] Both the binding-energy and the potential parameters of this negative-energy bound state, which we shall call a pseudo bound state, are determined by the parameters of the original potential. There are no free parameters in defining the pseudo bound state. We shall see that whenever the energy of this pseudo bound state is large the three- and four-particle systems experience bound state collapse (BSC). In other words, as in the case of the local potentials, if we

would like to use a single energy to reflect upon the strength of these non-local potentials in situations of collapse, it is not the binding energy of the bound state but the binding energy of the pseudo state which should reflect the real strength of the potential while calculating the binding energy of the ground state of the few-particle system.

First, we illustrate the idea of the pseudo state in the case of the S wave spin triplet NN channel. We use a partly attractive and partly repulsive non-local separable potential, possessing a CBS but no real bound state, to simulate the bound state and scattering properties in this channel. This non-local potential simulates a pseudo-deuteron at the correct energy. Consequently, the phase-shifts and the Fredholm determinant for the model containing the CBS simulates the actual state of affairs reasonably well except near the deuteron pole. If this non-local potential containing the CBS is used in a three-boson bound-state calculation it will produce an energy comparable to that obtained in a three-boson calculation employing a Yamaguchi potential with the range parameter $1.4fm^{-1}$, supporting a deuteron of $2.225MeV$.

We suggest that the BSC could be understood in terms of the pseudo bound state. Whenever the energy of the CBS increases, more dramatic is the collapse. As we would like to understand the strength of a potential in terms of its discrete spectrum, we hope that a simple model which includes the negative-energy bound state of the original non-local potential and replaces the CBS by the pseudo bound state should suffice. Usually, the collapse is most dramatic when the $B_s >> B_b$, where B_s is the binding energy of the pseudo state and B_b is the binding energy of the original bound state.[10, 11] As one of the objectives is to understand how a CBS leads to BSC, we shall be limited to the simple case where $B_s >> B_b$, and in this qualitative study just to simplify the model we neglect the physical bound state in the presence of the very strongly bound pseudo state. We find that the presence of this pseudo state alone in a simple model with no free parameters is able to explain the essential features of the BSC. The parameters of this simple model are determined by the parameters of the original non-local potential.

Recently, the general phenomena of BSC in the presence or absence of CBS has been interpreted to be a short-range mechanism.[14] Here we make a special model in the presence of the CBS in order to understand the BSC in terms of a

pseudo bound state. It is to be noted that the present model is consistent with the above interpretation in possessing a short-range in situations of BSC in addition to the pseudo bound state. For a fixed pseudo state energy the increase of the three-particle ground-state binding with a decrease of the range of potential is shown to be an approximation to the Thomas effect[16] as in Ref. [14]

In Sec. II we present the separable potential model that we employ. We also consider in some detail the problem of the CBS in this case and provide arguments for replacing the CBS by the pseudo bound state. In Sec. III we illustrate our idea in the case of the NN problem in the S wave spin-triplet channel. In Sec. IV we present the alternative model incorporating the pseudo bound state. We also compare the results for the original non-local separable potential model and the alternative model both for the two- and the three-particle systems and find that the alternative model provides a realistic description of the actual state of affairs in situations of collapse. Finally, in Sec. V we present some concluding remarks.

II. THE MODEL

The phenomenon of BSC has been observed for many separable two-particle potentials of the Tabakin-type.[12] A Tabakin-type separable potential is mainly identified by a momentum space form-factor $g(p)$ which may change sign as a function of p^2. Consequently, the two-particle phase shift changes sign at higher energies. The phenomenon of BSC is common to all Tabakin-type potentials. The BSC is not only observed for rank one potentials, it has also been observed for higher rank potentials.[10]

We consider the following Tabakin-type S-wave two-particle potential in the momentum space[8]

$$V(p,q) = \lambda g(p) g(q), \tag{2}$$

with

$$g(p) = \frac{\alpha_1}{(p^2+\beta_1^2)} - \frac{\alpha_2}{(p^2+\beta_2^2)}, \tag{3}$$

$$= \frac{\kappa(p^2-p_c^2)}{(p^2+\beta_1^2)(p^2+\beta_2^2)}, \tag{4}$$

where $\kappa \equiv (\alpha_1 - \alpha_2)$ and $p_c^2 \equiv (\alpha_2\beta_1^2 - \alpha_1\beta_2^2)/\kappa$ is the position of the CBS. We consider this potential with attractive and repulsive parts because of its simplicity

which allows us to understand the origin of the additional strength in the presence of the CBS. All the equations of this study are written in units $\hbar = 2\mu = 1$, where μ is the reduced mass. For the NN system the calculation was performed with $\hbar^2/2\mu = 41.47 MeV fm^2$.

The two-particle t-matrix for this potential, at energy $E = k^2$, is given by

$$t(p,q,k^2) = g(p)\frac{\lambda}{\tau(k^2)}g(q), \tag{5}$$

where

$$\tau(k^2) = 1 - (2/\pi)\lambda\int_0^\infty g^2(p)p^2dp/(k^2 - p^2 + i0) \tag{6}$$

$$= 1 - \frac{\lambda\alpha_1^2}{2\beta_1(k+i\beta_1)^2} - \frac{\lambda\alpha_2^2}{2\beta_2(k+i\beta_2)^2} + \frac{2\lambda\alpha_1\alpha_2}{(k+i\beta_1)(k+i\beta_2)(\beta_1+\beta_2)}. \tag{7}$$

The phase-shift δ is defined in this case by

$$t(k,k,k^2) = -\frac{e^{i\delta}sin\delta}{k}. \tag{8}$$

For potential (2) the momentum space Schrödinger equation for a bound state or CBS at energy E_b, described by the wave function ϕ, is given by

$$\phi(p) = \frac{1}{(E_b - p^2 + i0)}g(p)\lambda\frac{2}{\pi}\int_0^\infty q^2dqg(q)\phi(q), \tag{9}$$

$$= C\frac{g(p)}{(E_b - p^2 + i0)}. \tag{10}$$

The normalization constant C is determined by the condition

$$\frac{2}{\pi}\int_0^\infty \phi^2(p)p^2dp = 1. \tag{11}$$

If the system has a bound state at $E_b = -\gamma^2$ its wave function is given by

$$\phi(p) = -C\frac{g(p)}{(\gamma^2 + p^2)}. \tag{12}$$

In the present study the binding energy γ^2 of the NN system is always fixed at the value (2.225/41.47) fm^{-2}, when we vary the parameters of potential (2). This corresponds to a deuteron binding of 2.225 MeV.

If the potential (2) possess a CBS at energy p_c^2, in addition to $g(p_c) = 0$ we also have $\tau(p_c^2) = 0$, and its wave function $\phi_c(p)$ is given by

$$\phi_c(p) = -C\frac{g(p)}{(p^2 - p_c^2)}, \tag{13}$$

$$= -C\frac{\kappa}{(p^2 + \beta_1^2)(p^2 + \beta_2^2)}, \tag{14}$$

where we have substituted for $g(p)$ from Eq. (4). Unlike the bound-state wave function (12) which depends on binding energy γ^2, the CBS wave function (14) is independent of energy p_c^2. We note that at large distances the CBS wave function decays exponentially as exp(-$\beta_i r$) and not as exp(-$p_c r$). A comparison between Eqs. (12) and (14) reveals that the CBS wave function (14) is identical with the bound-state wave function (12) provided that we identify the binding energy γ^2 with β_i^2 and the form-factor $g(p)$ with $(p^2+\beta_j^2)^{-1}$, where β_i (β_j) is the smaller (greater) of β_1 and β_2. This association is made because in the configuration space the bound-state wave function of binding γ^2 should behave as $\exp(-\gamma r)$ asymptotically.

The original potential which supports the CBS has an attractive and a repulsive parts. However, because of pole cancellation the CBS wave function (14) has the behavior of a normal negative-energy bound-state wave function with a purely attractive separable potential. It is this negative-energy bound state which we call a pseudo bound state which will be seen to play a crucial role in predicting the collapse. The parameters of this pseudo state is given completely by the original non-local potential.

The phase-shift difference $[\delta(0) - \delta(\infty)]$ of potential (2-4) has an additional jump of π not accounted for by the normal bound state, or its wave function has a node[4] at short distances reflecting the presence of a second bound state. This is the pseudo bound state predicted by the CBS.

Rupp, Streit, and Tjon[11] considered a local potential equivalent to a potential of type (2-4) and calculated the two-particle bound-state wave function with this potential. The wave function showed a node typical of an excited state. This feature is consistent with the introduction of the pseudo state in addition to the original bound state.

III. ILLUSTRATION IN A SIMPLE MODEL

We would like to illustrate in a simple model that the presence of a CBS could simulate a pseudo bound state. We consider the NN problem in the S wave spin-

triplet channel employing a purely attractive Yamaguchi potential[15]. This is the special case of potential (2) with $\alpha_2 = 0$. The parameters of this potential are determined by fitting to a deuteron binding of $2.225MeV$ and a NN spin triplet scattering length of $5.4fm$. This results in $\lambda\alpha_1^2 = -7.45fm^{-3}$ and $\beta_1 = 1.4fm^{-1}$.

We would like the full non-local potential (2-4), possessing only a CBS and no true bound state, to simulate the results of the above Yamaguchi potential via a pseudo deuteron of $2.225MeV$ binding. For this purpose the parameters of the Tabakin-type potential (2-4) have to be appropriately chosen. The discussion related to Eq. (14) suggests that we choose $\beta_1 = 0.23fm^{-1}$, and $\beta_2 = 1.4fm^{-1}$; β_1^2 is the deuteron binding. The parameter α_2 is taken to be 1. We would like this potential to have a CBS. The position of this CBS, p_c^2, is still arbitrary. Once this position is given the other parameters of the potential, λ and α_1, are determined using the conditions $g(p_c) = 0$ and $\tau(p_c^2) = 0$. We varied the position of the CBS a little in order to have a good agreement between the calculations of the two potentials. Finally, we used $p_c = 0.1fm^{-1}$, $\alpha_1 = 0.032$ and $\lambda = -9.57fm^{-3}$. The result is very weakly sensitive to the value of p_c provided that for p_c^2 we choose an energy smaller than deuteron binding.

It is interesting to comment at this stage that once a separable non-local potential possessing a CBS is given, the passage to the separable potential possessing the pseudo bound state is unique as we have commented in the last section. In this section we would, however, like to predict a CBS which simulates a given (pseudo) bound state. In this situation there is some arbitrariness in defining the position of the CBS.

In Fig. 1 we plot the phase-shifts of the original Yamaguchi potential possessing a deuteron and those simulated via the Tabakin-type potential possessing a CBS. In Fig. 2 we plot the function $\tau(k^2)$ of Eq. (7) for the two potentials. It should be noted that this function is the usual Fredholm determinant. The agreement between the two calculations is good except near zero energy. In both cases $[\delta(0) - \delta(\infty)] = \pi$. For the Yamaguchi potential the bound state contributes to N and for the Tabakin-type potential the CBS contributes to N in the Levinson's theorem (1). For the Yamaguchi potential, $\tau(k^2)$ has a zero at the deuteron energy; for the Tabakin-type potential, $\tau(k^2)$ does not have a zero at negative energies. As the low-energy t matrix is dominated by the presence of the deuteron pole, it is natural

that the two results be different near the deuteron pole. If we exclude a small domain, -$10MeV$ to $10MeV$, the results for the two calculations are surprisingly similar. For a multi-particle ground-state calculation the two potentials should produce similar results. We performed a three-boson calculation with this Tabakin potential and find a three-boson binding of $38.8MeV$ to be compared with $24.7MeV$ for the Yamaguchi potential.[17] These two numbers are in qualitative agreement with each other. The Tabakin-type potential supports no two-particle bound state; yet it produces a reasonable result for three-boson binding energy.

The Tabakin-type potential in this section has a CBS and no real bound state and simulates a pseudo bound state at the deuteron energy. The original Tabakin potential has a bound deuteron and also a CBS. The CBS of the original Tabakin potential simulates a strongly bound pseudo state, as a consequence the triton has a binding of several hundred MeV's as we see in the next section.

IV. RESULTS AND DISCUSSIONS

In order to demonstrate the plausibility of our hypothesis we performed numerical calculations in a simple separable potential model including this pseudo state both for the two- and the three-particle systems and compared them with the original potential model supporting the CBS. On the one hand we have potential (2-4) supporting a regular bound state and a CBS, on the other hand we consider the following form-factor to be used in potential (2):

$$g(p) = \frac{1}{p^2 + \beta_j^2}, \tag{15}$$

where β_j is the greater of β_1 and β_2. By adjusting the strength parameter λ in Eq. (2) the alternative potential given by Eqs. (2) and (15) is made to reproduce a (pseudo) bound state at energy β_i^2, where β_i is the smaller of β_1 and β_2. The pseudo state and the potential (15) follows uniquely from the original non-local potential, there are no free parameters.

As the energy of the pseudo state is much larger than the energy of the normal state in situations of collapse, in order to illustrate our idea of the dominance of the pseudo state we have neglected the normal state and considered only the pseudo state in the alternate potential model.

In the two-particle case we compare in Fig. 3 the function $\tau(k^2)$ at negative energies for potentials (4) and (15) with $\beta_1 = 1.4fm^{-1}$, $\beta_2 = 5.73fm^{-1}$, $\alpha_1 = 0.1$, $\alpha_2 = 1$, and $\lambda = -1096fm^{-3}$. In this case the original potential (4) produces a two-particle bound state at energy 2.225 MeV in addition to a CBS at $p_c = 1.213fm^{-1}$ or at $p_c^2 \equiv 61.02MeV$. The alternate potential (15) has a pseudo bound state at $\beta_1^2 \equiv 81.28MeV$ and a range parameter of 5.73 fm^{-1}. We also show in Fig. 3 $\tau(k^2)$ for a normal Yamaguchi potential[15] with range parameter 1.4 fm^{-1} fitted to the deuteron binding of 2.225 MeV. For energies smaller than about -100MeV the results for the original potential (4) and the alternate potential (15) are very similar. The result for the normal Yamaguchi potential fitted to deuteron binding is very different from the above two.

In order to reproduce the three-particle ground state using the Faddeev equations one needs the two-particle t matrix elements between energies $-\infty$ to $-B_3$, where B_3 is the binding energy of the three-particle ground state. We find from Fig. 3 that in this energy domain the function $\tau(k^2)$ for the original potential and the alternate potentials are quite similar and hence it seems plausible that the alternate potential will be able to simulate the collapsed state of the original potential.

With the separable potentials given by (4) and (15) we calculated the three-boson binding energies by solving the usual Faddeev equations in a routine fashion.[6]

We consider the full potential (4) with $\alpha_2 = 1$ and $\beta_1 = 1.4fm^{-1}$, which allows for the possibility of a CBS to appear. The constants α_1 and β_2 were varied arbitrarily, and then λ is adjusted to yield a fixed two-particle binding of 2.225 MeV and a CBS. In the calculation we have maintained the parameter β_1 constant (=1.4 fm^{-1}) and the parameter β_2 is always larger than β_1. The pseudo state always has a binding of β_1^2, or 81.28MeV produced by the alternate potential (15). The three-boson binding in this model should be typically about several times the two-particle binding. This puts a threshold for the three-boson ground state binding of about couple of hundred MeV's. Qualitatively, this explains a large binding for the three-particle ground state via a pseudo state.

In Table 1 we exhibit the potential parameters α_1, β_2, the position of the CBS, p_c and the corresponding three-particle binding B_3 for the potential (2-4) and calculation on B_3, from Ref. [18], given by

$$(B_3)_{LB} = 2B_2(\frac{3}{2}\lambda) \tag{16}$$

for the two potentials. Here B_2 stands for the two-particle binding energy, calculated when the potential strength is $3\lambda/2$. The LB calculations are shown in the parenthesis in the last two columns. The exact results for the three-particle ground state for both these potentials are qualitatively similar; the LB calculations provide good approximations to the exact calculations. Hence the presence of the pseudo state explains the principal features of the collapse of the three-particle system.

In a previous study[14] the increase of the three-particle binding for the original non-local potential (2-4), as the position of the CBS moves to higher energies, has been claimed to be a short-range effect similar to the Thomas effect.[16] The Thomas effect causes the three-particle ground state to collapse when the range of the potential is reduced while maintaining the two-particle binding fixed.

In Table 1 the increase in the three-particle binding in the case of the original non-local potential is due to the Thomas effect. In order to demonstrate this point we needed to consider in a previous study[14] the average kinetic energy of the two-particle bound state in the case of the original potential (2-4). This average kinetic energy was used to indicate the range of the potential. For a partly attractive and partly repulsive non-local potential (2-4) neither β_1 nor β_2 provides a precise measure of the range.[14] It is interesting to note that the Thomas effect also manifests in the case of the alternate potential (15), where there is only one range parameter β_2 and this parameter should reflect the range of the potential. An increase of β_2 denotes a passage to a smaller range while maintaining the two-particle (pseudo) bound-state energy fixed at 81.28MeV; consequently, the three-particle system tends to collapse due to a manifestation of the Thomas effect.[16] Hence the present interpretation of the collapse via the pseudo bound state is consistent with that of Ref. [14].

More quantitatively, there is an interesting scaling property indicating an approximation to the Thomas effect. It was demonstrated in Ref. [11] analytically through a consideration of the momentum space Faddeev equations that in the pres-

$$B_3 \sim \beta_2^2. \tag{17}$$

This was also confirmed in their numerical calculations. It has also been numerically demonstrated[14] that for a simple Yamaguchi potential of range parameter β_2, scaling (17) is accurately obeyed. The analytic scaling arguments of Ref. [11] can be trivially extrapolated to the case of a simple Yamaguchi potential, so that Eq. (17) holds good, in accordance with the numerical study of Ref. [14]. In the presence of a pseudo bound state the alternate potential (15) has the simple Yamaguchi form and Eq. (17) holds true in this case. This scaling has been shown to be a manifestation of the Thomas effect in Ref. [14]. Hence the increase of the three-particle binding in Table 1 with the increase of p_c and β_2 is due to an approximation to the Thomas effect.

V. SUMMARY

Here we have investigated the origin of the unusual strong attraction, known as BSC, in the problem of multi-particle ground state while using two-particle non-local potential that supports a CBS. The presence of the CBS requires that the non-local potential be partly attractive and partly repulsive.[9] In this situation the Levinson's theorem (1) requires modification in that the number N should be modified to include the CBS.[4, 5]

We analyse the wave function of the CBS in a simple separable non-local potential model (2-4) and find that it is identical to the wave function of a negative-energy bound state which is called a pseudo bound state. The energy and the separable potential that defines this wave function are given by the parameters of the original potential. The inclusion of this pseudo bound state in a simple potential model explains the unusual attraction in the presence of the CBS.

Delfino et al.[14] claimed the BSC to be a short range phenomena as in the Thomas effect.[16] The present interpretation of the collapse in terms of the pseudo bound state is also a short-range phenomena as in the Thomas effect as the model that incorporates the pseudo state is found to possess a short range in the case of collapse.

The pseudo bound state provides excess binding for the multi-particle bound state but it cannot really explain the whole multi-particle spectrum. For example,

the presence of the strongly bound pseudo state eliminates in the simple model the appearance of the weakly bound excited states of the multi-particle system present in the original potential. The original non-local potential provides this excess binding for the multi-particle bound state in a nontrivial fashion via the CBS. The purpose of the present study is not to simulate all the subtle intricacies of a non-local potential with attractive and repulsive parts via a simple potential model but just to understand how to visualize some of the consequences of a CBS. It is also to be noted that the mere presence of a CBS or an associated pseudo state, although, imply some additional binding for the multi-particle ground state, but does not obviously imply a dramatic effect as in the BSC. One has to consider the parameters of the pseudo state to see if it leads to a dramatic effect.

We studied the simple separable potential model containing a CBS because such a potential model simulates the essential features of a non-local intercluster potential which excludes the Pauli forbidden states. Though we can not provide a general proof, it seems quite plausible that many of our conclusions should carry over to the case of Pauli forbidden states. Such a proof, or even a rigorous demonstration, will be an welcome addition to the literature.

LT thanks I. R. Afnan for the kind hospitality at the Flinders University of South Australia and AD thanks E. F. Redish for the kind hospitality at the University of Maryland. The work is supported in part by the Conselho Nacional de Desenvolvimento - Científico e Tecnológico (CNPq) of Brasil.

REFERENCES

[1] L. I. Schiff, Quantum Mechanics, McGraw-Hill, New York, 1968; R. G. Newton, Scattering Theory of Waves and Particles, Second Ed., Springer-Verlag, New York, 1982.

[2] Y. C. Tang, M. LeMere, and D. R. Thompson, Phys. Rep. 47C ,167 (1978); K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. 7, 150 (1958); 9, 449 (1958).

[3] H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958).

[4] B. Mulligan, L. G. Arnold, B. Bagchi, and T. O. Krause, Phys. Rev. C 13, 2131 (1976); B. Bagchi, T. O. Krause, and B. Mulligan, ibid. 15, 1623 (1977).

[5] I. H. Sloan, Phys. Lett. 34B, 243 (1971).

[6] See, for example, S. K. Adhikari and K. L. Kowalski, Dynamical Collision Theory and Its Applications, Academic Press, Boston, 1991; W. Glöckle, The Quantum Mechanical Few-Body Problem, Springer-Verlag, Berlin, 1983.

[7] L. D. Faddeev, Mathematical Aspects of the Three-Body Problem in Quantum Scattering Theory, Davey, New York, 1965.

[8] P. Beregi, Nucl. Phys. A206, 217 (1973).

[9] S. K. Adhikari, Phys. Rev. A 42, 6 (1990).

[10] G. Pantis, H. Fiedeldey, and D. W. L. Sprung, Z. Phys. A291, 367 (1979); A294, 101 (1980).

[11] G. Rupp, L. Streit, and J. A. Tjon, Phys. Rev. C 31, 2285 (1985).

[12] F. Tabakin, Phys. Rev. 174, 1208 (1968).

[13] J. E. Beam, Phys. Lett. 30B, 76 (1969), V. A. Alessandrini and C. A. Garcia Canal, Nucl. Phys. A133, 950 (1969).

[14] A. Delfino, S. K. Adhikari, L. Tomio, and T. Frederico, Phys. Rev. C 46, xxxx (1992).

[15] Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[16] L. H. Thomas, Phys. Rev. 47, 903 (1935).

[17] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. 136, B650 (1964).

[18] J. W. Humberston, R. L. Hall, and T. A. Osborn, Phys. Lett. 27B, 195 (1968).

TABLES

TABLE I. The three-boson binding B of the original Tabakin-type potential (2-4) and that calculated in a simple Yamaguchi model (15) incorporating a pseudo state dictated by the Tabakin-type potential with $\alpha_2 = 1$ and $\beta_1 = 1.4 fm^{-1}$. The simple Yamaguchi model has the range parameter given by β_2 and is fitted to a bound state at energy $1.4 fm^{-2}$. The numbers in parenthesis in the last two columns refer to LB calculation employing Eq. (16)

α_1	$\beta_2(fm^{-1})$	$p_c(fm^{-1})$	$B(MeV)(Tabakin)$	$B(MeV)(Yamaguchi)$
0.05	8.47	1.307	930.5 (1173)	902.3 (1086)
0.1	5.73	1.213	545.3 (698)	644.8 (748)
0.13	4.92	1.167	446.1 (576)	578.5 (660)
0.15	4.53	1.149	396.3 (518)	539.4 (619)
0.2	3.82	1.097	317.3 (420)	486.5 (549)

Figure Caption

1. Phase-shifts of the Yamaguchi potential (dashed line) possessing a deuteron of $2.225 MeV$ binding, and the Tabakin-type potential (full line) possessing a CBS, which simulates this deuteron via a CBS.

2. The Fredholm determinant $\tau(k^2)$ for the above Yamaguchi (dashed line) and Tabakin-type (full line) potentials at negative energies.

3. The Fredholm determinant $\tau(k^2)$ for the Tabakin-type potential (full line) possessing a CBS and a deuteron of $2.225 MeV$ binding, for the Yamaguchi potential (dotteded line) possessing the pseudo bound state of $81.28 MeV$ binding, and for the Yamaguchi potential (dashed line) possessing a deuteron of $2.225 MeV$ binding. See text for details of potential parameters.

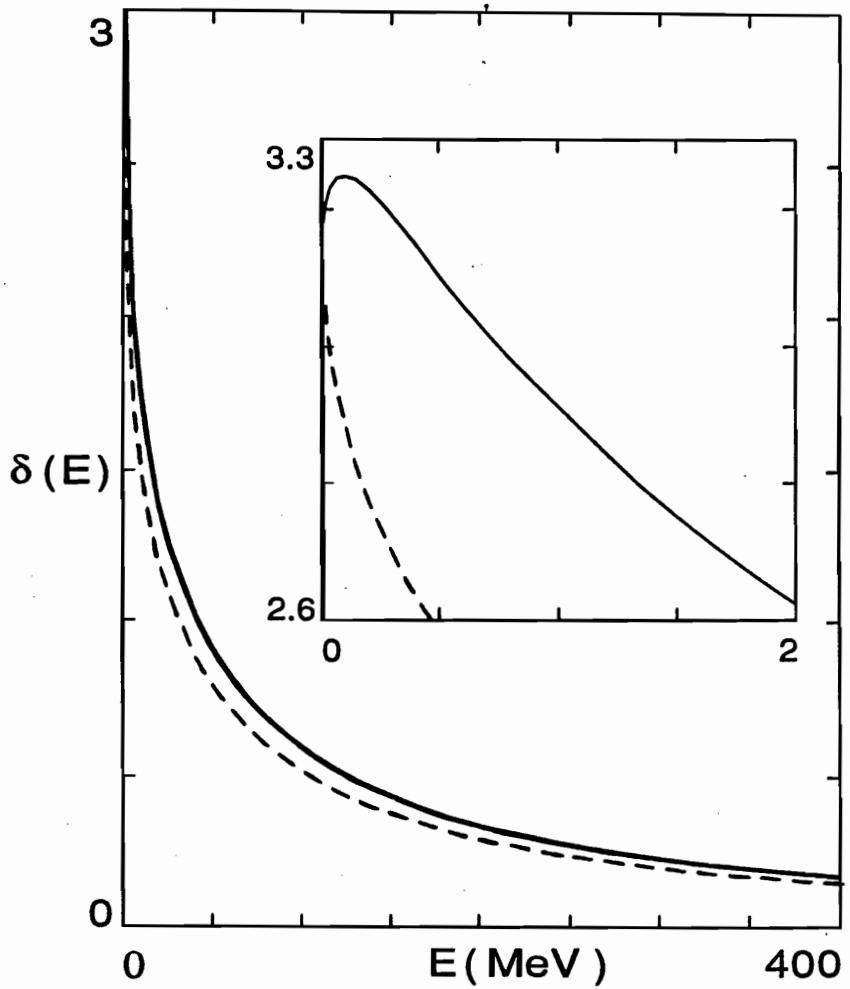

Fig. 1 - Delfino, Adhikari, Tomio and Frederico

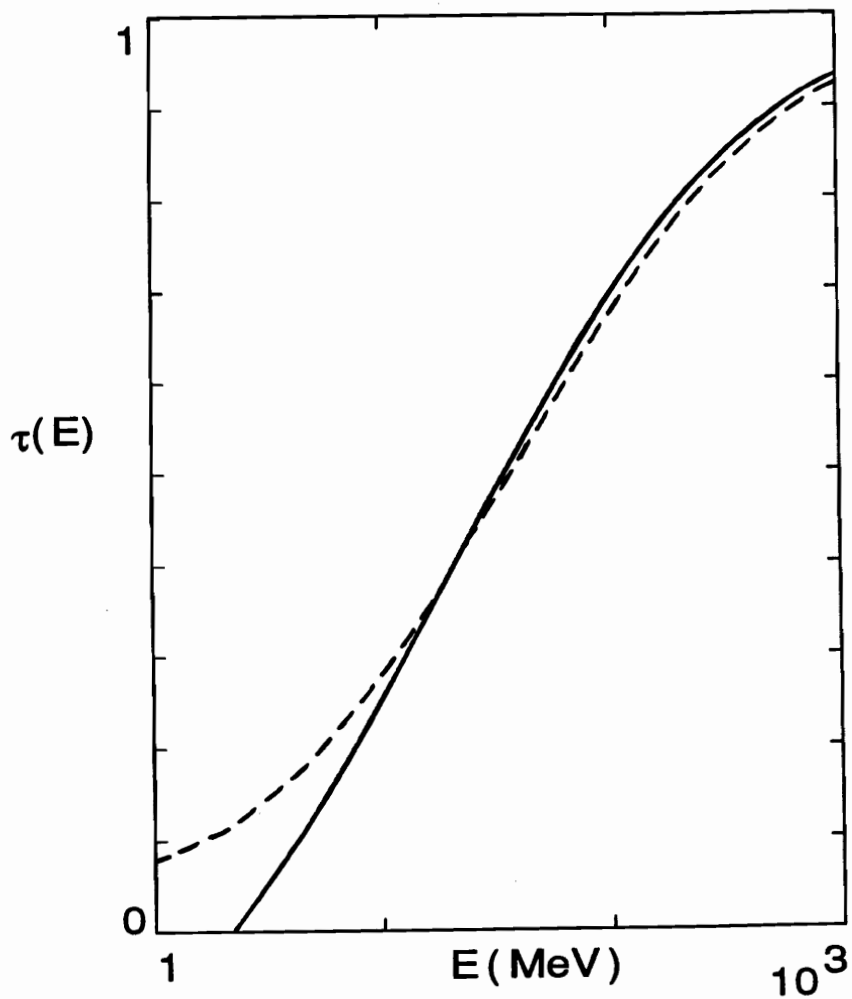

Fig. 2 - Delfino, Adhikari, Tomio and Frederico

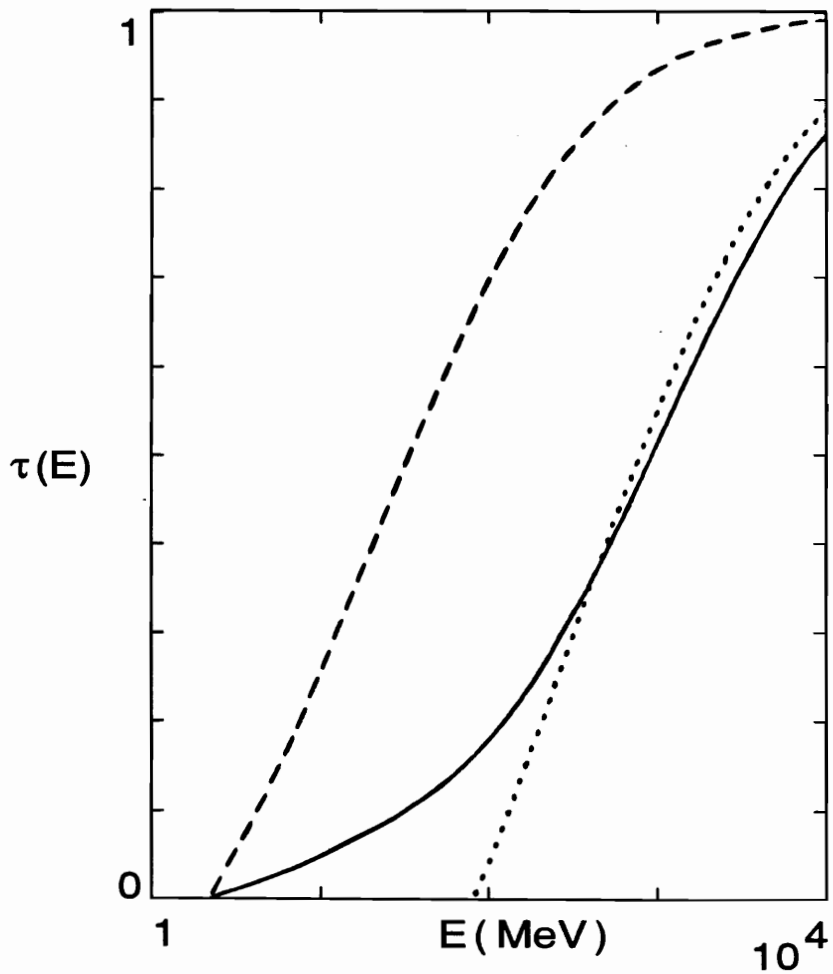

Fig. 3 - Delfino, Adhikari, Tomio and Frederico.