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A SQUEEZED STATE HOLOMORPHIC PHASE SPACE REPRESENTATION OF EQUATIONS OF MOTION

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A squeezed state holomorphic phase space representation of equations of motion

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Abstract

A mapping scheme is presented which takes quantum operators associated to bosonic degrees of freedom into complex phase space integral kernel representatives; the procedure consists in using the Schrödinger squeezed state as the starting point for the construction of the integral mapping kernel which, due to its inherent structure, is suited for the description of second quantized operators. Products and commutators of operators have their representatives explicitely written which reveal new details when compared to the usual $q-p$ phase space description. The classical limit of the equations of motion for the canonical pair $q-p$ is discussed in connection with the effect of squeezing the quantum phase space celular structure.

Keywords
Quantum Phase Spaces
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1 Introduction

The representation of abstract operators (acting on proper Hilbert spaces) in terms of classical variables has attracted special interest since the pioneering works by Weyl [1] and Wigner [2]. Those approaches were later put on solid grounds [3] and constitute nowadays in a well defined tool to treat quantum mechanical problems in a phase space - like way. Besides being a particular manner of implementing semiclassical descriptions of quantum systems, the Weyl - Wigner scheme has interesting aspects of its own. In this connection the mapping procedure which generates the operator representative in the quantum phase space deserves a particular attention since it envolves the concept of decomposition of an operator in an operator basis

$$\hat{O} \equiv \int O_w(q,p)\hat{\Delta}(q,p)\, dq\, dp\,, \tag{1}$$

where $O_w(q,p)$ is the well known Weyl - Wigner transform of operator $\hat{O}$, and $\hat{\Delta}(q,p)$ is the suited operator basis for the degree of freedom we are treating.

This point of view has been exploited in previous papers [4] in which the construction of operator basis has been studied for degrees of freedom that do not have a classical counterpart, mainly those manifested by a finite number of states. There it was shown how to generalize the starting definition, eq.(1), by using the Schwinger unitary operator bases [5], i.e.,

$$\hat{O} \equiv \sum_{m,n} O(m,n)\hat{S}(m,n)\ , \tag{2}$$

where now $\hat{S}(m,n)$ is the Schwinger conveniently constructed basis for the particular degree of freedom. The main result in that formalism was that $O(m,n)$, although discrete, still is the Weyl - Wigner transform of operator $\hat{O}$, which then represents it in a new discrete phase space. Besides drawing attention to general geometrical aspects of this discrete phase space [6], this formalism also points to the second quantization structure manifested in such quantum phase spaces.

It is the aim of this paper to show how to implement a mapping scheme in such a way to produce representatives of operators in a holomorphic

phase space that may also describe second quantized operators. Although extensively used in studies on quantum optics [7], the holomorphic representation of quantum operators is also of great importance in general cases of low lying vibrational energy states in many - body physics [8] because of their inherent harmonic oscillator coherent state character, besides being used in path integration techniques in the context of gauge theories [9]; here we have adapted our mapping scheme so as to describe the quantum mechanical observables associated to spatial motion while emphasizing the second quantization character of the corresponding phase space. It was then developed a mapping kernel that simultaneously embodies all those characteristics through the use of the Weyl - Wigner transformation of the Bargmann's kernel [10]. Moreover, in order to allow for a possible deformation of the phase space cell, we explicitely kept the harmonic oscillator parameter, present in the Bargmann's kernel, as a free parameter. This in fact corresponds to using a Schrödinger squeezed state [11] mapping kernel. With this new mapping scheme we are able to obtain the representatives of second quantized operators as well as the usual canonical pair q - p, in a complex variable representation. The inclusion of squeezing in the phase space cell, on the other hand, introduce new elements in this description; to study these new effects we obtain the mapped expressions of products and commutators of operators with which we can write the von Neumann - Liouville equation of motion for the mapped density operator. It is then direct to obtain the Ehrenfest equations for the expectation values of operators in which case the squeezing manifests itself through genuine quantum effects in the corresponding equations of motion, which is to be compared to the classical ones [12].

Our paper is organized as follows. In section 2 we present the basic ideas and the holomorphic mapping scheme through which we obtain the operators representatives. The mapping of operators and commutators is described and the Wigner function associated to the unidimensional harmonic oscillator is discussed; the squeezing character of the mapping is clearly presented in this context. The von Neumann - Liouville equation is presented in section 3 where the Ehrenfest as well as the Hamilton - like equations of motion are explicitely written and the squeezing effects manifested in the classical limit of these equations are discussed. Finally section 4 is devoted to a summary and the conclusions.

2 The holomorphic mapping

The Bargmann treatment of the Fock space states as well as of the creation and annihilation operators in terms of entire complex analytic functions and their derivatives [10] strongly influenced the study of the harmonic oscillator coherent states of wide use in quantum optics and low lying vibrational states of many - body systems [8]. If it is known that the operators are mapped as $a^\dagger \longrightarrow z$ and $a \longrightarrow \partial/\partial z$, on the other hand the Fock states are led to

$$|\, n\rangle \longrightarrow \frac{z^n}{\sqrt{n!}}\,, \tag{3}$$

this new set constituting a Hilbert space of entire functions with a measure governed by the weight function

$$\rho_r(z) = exp(-zz^*)/\pi^r \tag{4}$$

for a r - dimensional space. Furthermore, the mapping kernel connecting the square integrable space $\mathcal{L}_r$ to the complex analytic functions Hilbert space $\mathcal{F}_r$ is given by

$$A_r(z,q) = \frac{1}{\pi^{r/4}b^{r/2}} exp[-\frac{1}{2}(z^2 + q^2/b^2) + \sqrt{2}zq/b] \tag{5}$$

in such a way that

$$\int A_r(z,q)A_r^*(w,q)\, d^rq \;=\; e^{zw^*}\,, \tag{6}$$

where e^{zw^*} is the Bargmann's reproducing kernel, which plays the role of a Dirac's delta in this new space and b is the usual harmonic oscillator parameter (here considered as a free parameter). Now, in order to connect this complex variable description of the harmonic oscillator with a two real variables picture describing the quantum phase space, we must establish a relationship between the Bargmann mapping kernel and that of Weyl - Wigner's [3]. In this way it is direct to see that (hereafter r = 1 for simplicity only)

$$O_w(q,p) = \int dy d\mu(z) d\mu(w^*) e^{ipy/\hbar} A^*(z, q - y/2) O_h(z, w^*) A(w^*, q + y/2)\,, \tag{7}$$

where $O_h(z,w^*)$ is the Bargmann representative of an operator $\hat{O}$ and $d\mu(z) = \rho(z)d^2z$. Using (5) it is direct to see that

$$O_w(q,p) = 2e^{-|\alpha|^2} \int e^{\sqrt{2}(z^*\alpha^* + w\alpha) - z^*w} O_h(z,w^*) d\mu(z) d\mu(w^*)\,, \tag{8}$$

where we have called

$$\alpha = \frac{q}{b} + i\frac{bp}{\hbar}. \tag{9}$$

The inverse mapping is obtained in precisely the same way as we did before and is given by

$$O_h(z,w^*) = \frac{2}{\hbar} e^{-zw^*} \int dq\, dp\, e^{-|\alpha|^2} e^{\sqrt{2}(z\alpha + w^*\alpha^*)} O_w(q,p)\,. \tag{10}$$

These two expressions allow us to pass from the Bargmann description to the Weyl - Wigner one and vice - versa.

It is important to remark that, since we are representing operators by integral kernels, the present procedure makes use of two complex variables instead of only one. This number of variables may seem excessive, but it is to be observed that, although all functions $F(z,w^*)$ of interest are required to be entire in z and w^* so that they can be uniquely determined by its restriction to the diagonal [13], here we must perform the two complex integrations so as to take care of the quantum ordering effects embodied in the mapping process.

Now, the Weyl transform of the operator $\hat{\Delta}(\hat{q},\hat{p})$, that is an element of the $(\hat{q} - \hat{p})$ operator basis, is of fundamental importance and is given by [3]

$$\Delta_w(q,p) = h\delta(q - q')\delta(p - p')\,, \tag{11}$$

which is then mapped onto

$$\begin{aligned} B(z,w^*;q,p) &= 2e^{-zw^*} e^{-|\alpha|^2 + \sqrt{2}(z\alpha + w^*\alpha^*)} \\ &= h\Delta(z,w^*;q,p). \end{aligned} \tag{12}$$

This result is nothing but the mapping kernel already presented in (8) and (10) in such a form that we verify the correspondence between the element of the operator basis in the quantum phase space and the new mapping kernel $\Delta(z,w^*;q,p)$; those equations are then rewritten as

$$O_w(q,p) = \int \Delta^*(z,w^*;q,p) O_h(z,w^*) d\mu(z) d\mu(w^*) \,, \tag{13}$$

and

$$O_h(z,w^*) = \int \Delta(z,w^*;q,p) O_w(q,p) dq\, dp \tag{14}$$

respectively.

The simplest example for which we can apply this mapping is $O_w(q,p) = 1$, which gives

$$\begin{aligned} O_h(z,w^*) &= \int \Delta(z,w^*;q,p)\, dq\, dp \\ &= e^{zw^*}; \end{aligned} \tag{15}$$

this result shows that the unity operator, (here represented by its Weyl - Wigner expression $O_w(q,p) = 1$) is directly mapped into the Bargmann reproducing kernel.

With the help of equations (13) and (1), it is a simple task to obtain the mapped expression of any function $f(\hat{q},\hat{p})$ through the use of its Weyl - Wigner transform $f_w(q,p)$. In this connection the harmonic oscillator Hamiltonian (without the zero - point energy) is written as

$$\begin{aligned} H_h(z,w^*) &= \tfrac{\hbar\Omega}{4} e^{zw^*}[(z+w^*)^2 - (z-w^*)^2] \\ &= \hbar\Omega z w^* e^{zw^*} \,, \end{aligned} \tag{16}$$

that together with the operators

$$(|\, n\rangle\langle m\, |)(z,w^*) = \frac{1}{h}\frac{z^n w^{*m}}{\sqrt{n!m!}} \,, \tag{17}$$

where $(o)(z,w)$ stands for the holomorphic representation of (o), constitute the basic operators necessary to describe that system. The contact with the creation and annihilation operators description of the harmonic oscillator

is explicit here. On the other hand, it is interesting to note that the phase space usual operators mapped as

$$(\hat{q})(z,w^*) = \frac{b}{\sqrt{2}} e^{zw^*}(z+w^*) \tag{18}$$

and

$$(\hat{p})(z,w^*) = i\frac{\hbar}{\sqrt{2}b} e^{zw^*}(z-w^*) \,, \tag{19}$$

give the usual construction of the a and $a^\dagger$ operators

$$(a)(z,w^*) = ([\frac{1}{\sqrt{2}}(\frac{\hat{q}}{b} + i\frac{b\hat{p}}{\hbar})])(z,w^*) = w^* e^{zw^*} \tag{20}$$

and

$$(a^\dagger)(z,w^*) = ([\frac{1}{\sqrt{2}}(\frac{\hat{q}}{b} - i\frac{b\hat{p}}{\hbar})])(z,w^*) = z e^{zw^*} \tag{21}$$

respectively.

The scalar product of two functions in the holomorphic phase space is defined in the standard way, i.e.,

$$(f,g) = \int d\mu(z) d\mu(w^*) f^*(z,w^*) g(z,w^*) \,, \tag{22}$$

such that the expectation value of an operator is then given by

$$\langle \hat{O} \rangle = h \int d\mu(z) d\mu(w^*) O(z,w^*) \rho_h^*(z,w^*) \,, \tag{23}$$

where $\rho_h(z,w^*)$ is now the equivalent to the Wigner function [2] in the Weyl - Wigner representation of quantum phase space. Using (16) and (17) it is direct to see that for the harmonic oscillator the expectation value relation

$$\langle \hat{H} \rangle_n = h \int d\mu(z) d\mu(w^*) H_{HO}(z,w^*) \rho_{h,n}^*(z,w^*) = n\hbar\Omega \tag{24}$$

holds.

In general, the mapping of products of two operators is of fundamental importance; in this phase space description, using the standard technique, it is straightforward to obtain

$$([\hat{f}_1(\hat{q},\hat{p})\hat{f}_2(\hat{q},\hat{p})])(z,w^*) = e^{zw^*}e^{\Psi_{1,2}}f_1(z,w^*)f_2(z,w^*) \ , \tag{25}$$

where

$$\Psi_{1,2} = (\frac{\partial^1}{\partial s} - \frac{\partial^1}{\partial d})(\frac{\partial^2}{\partial s} - \frac{\partial^2}{\partial d}) \ . \tag{26}$$

Here

$$s = z + w^* \ , \tag{27}$$

$$d = z - w^* \tag{28}$$

and the labels 1 and 2 refer to the first or second function respectively. With these results we are led to the description of the commutator of two operators. Similarly to what has been presented in other description [7], the commutator has the form

$$([\hat{O}_1,\hat{O}_2])(z,w^*) = 2e^{zw^*}e^{\Theta_{1,2}}\sinh\Lambda_{1,2}O_1^R(z,w^*)O_2^R(z,w^*) \ , \tag{29}$$

where

$$\Theta_{1,2} = \frac{\partial^1}{\partial s}\frac{\partial^2}{\partial s} - \frac{\partial^1}{\partial d}\frac{\partial^2}{\partial d} \tag{30}$$

and

$$\Lambda_{1,2} = \frac{\partial^1}{\partial s}\frac{\partial^2}{\partial d} - \frac{\partial^1}{\partial d}\frac{\partial^2}{\partial s} \ . \tag{31}$$

The functions $O_i^R(z,w^*)$ represent the reduced mapped holomorphic expressions for $\hat{O}_i(\hat{q},\hat{p})$, and are defined as

$$O_i^R(z,w^*) = e^{-zw^*}O_i(z,w^*) \ . \tag{32}$$

Alternatively we could have written the new operators appearing in the commutator in the form

$$\Theta_{1,2} = \frac{1}{2}(\frac{\partial^1}{\partial w^*}\frac{\partial^2}{\partial z} + \frac{\partial^1}{\partial z}\frac{\partial^2}{\partial w^*}) \tag{33}$$

and

$$\Lambda_{1,2} = \frac{1}{2}(\frac{\partial^1}{\partial w^*}\frac{\partial^2}{\partial z} - \frac{\partial^1}{\partial z}\frac{\partial^2}{\partial w^*}) \tag{34}$$

respectively, thus stressing the fundamental role of the creation and annihilation operators in the construction of the commutator also, while (30) and (31) highlight the $q-p$ description.

It is a trivial matter to extend this procedure to the mapping of the anticommutator, which is then expressed as

$$(\{\hat{O}_1,\hat{O}_2\})(z,w^*) = 2e^{zw^*}e^{\Theta_{1,2}}\cosh\Lambda_{1,2}O_1^R(z,w^*)O_2^R(z,w^*) \ . \tag{35}$$

In both cases, eqs. (29) and (35), we have similar structures as those as the Weyl - Wigner $q-p$ mapped expressions, since both have the sine and cosine series of derivatives of the fundamental operator heralding the symplectic structure embodied in that formalism. It is important to note, however, that here we have additionally the series associated to the Θ operator. If the Λ_{12} operator takes into account the ordering of the $\hat{q}$ and $\hat{p}$ operators, the Θ_{12} is responsible for the $\hat{a}$ and $\hat{a}^\dagger$ ordering effects on the commutator. In this sense, if we consider the second quantization as the fundamental one, our description is wider than that of Weyl - Wigner.

Of great interest, the holomorphic mapping of the Wigner function, associated to the minimum wave packet of width b_0, is directly obtained through the use of equation (10)

$$\rho(z,w^*;b_0,b_1) = \frac{2}{h}e^{-zw^*}\int e^{-|\alpha|^2}e^{\sqrt{2}(\alpha z+\alpha^* w^*)}\rho_w(q,p;b_0)dq\,dp \ , \tag{36}$$

where the $\alpha's$ associated to the mapping kernel, now embodies an oscillator parameter b_1, which must not be equal to b_0. This fact is of significance since now we have an holomorphic mapping kernel constructed out of a Schrödinger squeezed state and a Wigner function associated to yet another squeezed state. Since the Wigner function for a minimum wave packet is known to be of the form [12]

$$\rho_w(q,p;b_0) = \mathcal{N}(b_0)e^{-(q-q_0)^2/b_0^2}e^{-b_0^2(p-p_0)^2/\hbar^2} \ , \tag{37}$$

where $\mathcal{N}(b_0)$ is a normalization constant, we get in the holomorphic representation

$$\rho(z,w^*;b_0,\beta) = N(b_0,\beta)exp[-\frac{1}{(1+\beta^2)}(\frac{q_0^2}{b_0^2}+\beta^2\frac{b_0^2p_0^2}{\hbar^2})]$$
$$exp[\frac{1}{2}\frac{1-\beta^2}{1+\beta^2}(z^2+w^{*2})]exp[\frac{\sqrt{2}\beta}{1+\beta^2}(\alpha z+\alpha^* w^*)] \quad (38)$$

Here we made $\beta = b_1/b_0$, and $N(b_0,\beta)$ is another normalization constant. It is interesting to note that the holomorphic density depends on two parameters, namely the harmonic oscillator width associated to the squeezed mapping kernel, b_1, and the width of the starting wave packet; this fact gives rise to some interesting effects which are associated to the squeezed character of the mapping. This has been studied in the Weyl - Wigner representation of the density operator of a many - body nuclear system, which has shown some semiclassical features when β is conveniently chosen [14]. In particular, if we consider $\beta = 1$, we get

$$\rho(z,w^*;b_0) = \frac{N(b_0)}{2}e^{-\frac{|\alpha|^2}{2}}e^{\frac{(\alpha z+\alpha^* w^*)}{\sqrt{2}}}, \quad (39)$$

which is an expression akin to the holomorphic mapping kernel, as expected. On the other hand, when $b_1 > b_0$, i.e., $\beta > 1$, we note that the presence of terms in z^2 and w^{*2} in the exponential is an indication of the squeezed character of our particular holomorphic representation of the minimum wave packet.

3 The von Neumann - Liouville equation and the classical equations of motion

Lets consider now the case of the time evolution equation for the density operator in the holomorphic representation; it is completely clear that it corresponds to the mapped version of the von Neumann - Liouville equation and is expressed as

$$i\hbar\dot{\rho}(z,w^*) = 2[e^{\Theta_{1,2}}\sinh\Lambda_{1,2}]H_1^R(z,w^*)\rho_2(z,w^*) \ . \quad (40)$$

It is important to notice here that the density operator representative does not carry the exponential factor $exp(zw^*)$ in such a way that the equation above is slightly different from the one expected from the previous commutator calculations. Since in the present case $\rho(z,w)$ plays the role of a density in this holomorphic phase space, it is the associated Wigner function [2] in this space.

One interesting point to be discussed here concerns the time evolution of a density function representing an eigenstate of the Hamiltonian. In this case it is expected that the time evolution of the density labels will describe a particular trajectory in the phase space so as to keep the density constant; each one of these curves will define a section of equal density known as an equi - Wigner [15]. For example in the harmonic oscillator case, we have

$$(\hat{H})(z,w^*) = \hbar\Omega z w^* e^{zw^*} \quad (41)$$

and

$$(\hat{\rho})(z,w^*) = \frac{(zw^*)^n}{hn!} \quad (42)$$

for the Hamiltonian and density operators respectively. If we put these results in eq. (40), we get

$$\frac{\dot{z}}{z} = -\frac{\dot{w}^*}{w^*} \quad (43)$$

which clearly admits the solutions

$$z(t) = z(t_0)e^{i\Omega(t-t_0)} \quad (44)$$

and

$$w^*(t) = w^*(t_0)e^{-i\Omega(t-t_0)} \tag{45}$$

respectively. In this way, when eqs. (44) and (45) hold, the curves are in an equi - Wigner, i.e., the form of the distribution function does not change with time. This means that these equations for z and w^* are the equivalents to the classical description of the points q and p in the Lee and Scully's picture for the associated problem in Weyl - Wigner scheme [15].

In a general form, we define the Liouville operator

$$\mathcal{L}(z,w^*) = \frac{-2i}{\hbar}[e^{\Theta}\sinh\Lambda]H^R(z,w^*) \tag{46}$$

with which we rewrite the time evolution equation for $H^R(z,w)$ that are not explicitly time dependent

$$\dot{\rho} = -\mathcal{L}(z,w^*)\rho(z,w^*,t) \ . \tag{47}$$

It is then direct to obtain the formal solution

$$\rho(z,w^*,t) = e^{-\mathcal{L}(z,w^*)(t-t_0)}\rho(z,w^*,t_0) \ . \tag{48}$$

These results are extremely important since they permit us to write the time evolution of equations for the expectation value of any given operator, namely

$$(\frac{d}{dt}\langle\hat{O}\rangle)(z,w^*) \equiv h\int d\mu(z)d\mu(w^*)O^*(z,w^*)\frac{\partial}{\partial t}\rho_h(z,w^*,t) \ , \tag{49}$$

or, in an alternative form,

$$(\frac{d}{dt}\langle\hat{O}\rangle)(z,w^*) = -h\int d\mu(z)d\mu(w^*)[\mathcal{L}^*(z,w^*)O^*(z,w^*)]\rho_h(z,w^*,t) \ , \tag{50}$$

which is extremely useful in obtaining the classical limit of this equation. Thus, for example, we get for the harmonic oscillator

$$\frac{d}{dt}\langle\hat{q}\rangle \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} -\int d\mu(z)d\mu(w^*)[ib\Omega\frac{z^*-w}{\sqrt{2}}e^{z^*w}]\rho_h(z,w^*,t) \tag{51}$$

and

$$\frac{d}{dt}\langle\hat{p}\rangle \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} -\int d\mu(z)d\mu(w^*)[\frac{\hbar\Omega}{b}\frac{z^*+w}{\sqrt{2}}e^{z^*w}]\rho_h(z,w^*,t) \ , \tag{52}$$

respectively, which are the holomorphic phase space version of the Ehrenfest equations for the expectation values of the canonical operators. If we use now the mapped expression of the minimum wave packet associated to the Schrödinger squeezed state, eq.(36), with the proper normalization for our density function, we obtain

$$\frac{d}{dt}\langle\hat{q}\rangle_{mwp} \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} \frac{p_0}{m}\beta \tag{53}$$

and

$$\frac{d}{dt}\langle\hat{p}\rangle_{mwp} \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} \frac{-m\Omega^2 q_0}{\beta} \tag{54}$$

respectively. One sees that only in the particular case of $\beta = 1$ one gets the usual expressions in agreement with the classical equations. These results, however, indicate the genuine quantal character of the canonical pair $\hat{q}-\hat{p}$, since, due to the Heisenberg principle, if one tries to define a sharp q - representation (a wave packet completely localised at $q = q_0$) the center of the wave packet in the p coordinate, p_0, is completely lost. This effect of squeezing in the phase space cells is revealed in the equations (53) and (54) through the parameter β, i.e., when $\beta \longrightarrow 0$ (sharp q - representation) one doesn't have the center of the wave packet, p_0, at all and when $\beta \longrightarrow \infty$ (sharp p - representation) we have a plane wave in q. Hence, in this context, we will have classical - like equations only when the widths of the wave packet and that associated to the phase space descriptions coincides, i.e., $\beta = 1$. It is very interesting to observe that the same structure as those of eqs. (53) and (54) is obtained in the context of quantum algebras [16]. This also strongly suggests us to look for the classical limit of the Hamilton's equations for these results through the use of eq. (49). We can easily obtain the classical limit by simply putting the mapped expression of the minimum wave packet in eq. (49) for an arbitrary Hamiltonian; in this form we use $\beta = 1$ so as to obtain the classical limits

$$\frac{d}{dt}\langle\hat{q}\rangle_{mwp} \begin{matrix}\rightarrow\\ \leftarrow\end{matrix} \frac{2ib_0}{\hbar\sqrt{2}}\frac{\partial}{\partial d}H^*(\alpha,\alpha^*) \tag{55}$$

and

$$\frac{d}{dt}(\hat{p})_{mwp} \begin{matrix} \rightarrow \\ \leftarrow \end{matrix} \frac{-2}{b_0\sqrt{2}}\frac{\partial}{\partial s}H^*(\alpha,\alpha^*) \ , \tag{56}$$

where again $d = \alpha - \alpha^* = 2iIm\alpha$ and $s = \alpha + \alpha^* = 2Re\alpha$ respectively, thus showing exactly the form of the Hamilton's equations. This is to be compared to a result presented several years ago by Strocchi [17]. It must be stressed, however, that these equations are in fact quantal in its very nature since the mapped Hamiltonian may still present typical quantum features. These expressions in the form of Hamilton's equations are not surprising at all since it is well known [6] that the quantum phase space is endowed with a presymplectic structure which guarantees the existence of a Poisson bracket in the $\hbar \longrightarrow 0$ limit. Again, for the particular case of a harmonic oscillator those forms are exactly the classical ones.

4 Summary and Conclusions

It has been shown in this paper how to implement a squeezed state holomorphic phase space representation of quantum mechanics related to bosonic degrees of freedom. Its main virtue is to take into account the second quantization nature of the quantum description of bosonic physical systems in a natural way, i.e., closely paraleling the procedure by Bargmann [10] we were able to construct the integral kernel that maps operators onto functions in the holomorphic phase space in such a form to depend explicitely on the squeeze parameter. The complex structure of the corresponding phase space embodies the particular features of the algebra of creation and annihilation operators but also contains the information associated to the squeezing described by the width of the harmonic oscillator which generates the mapping kernel. In this sense, the mapping of products of operators exhibits some new ingredients when compared to the standard q - p quantum phase spaces, namely, the ordering of creation and annihilation operators gives rise to additional terms in the corresponding holomorphic phase space representatives. The mapped expression of commutators is directly obtained which are extremely important in the study of genuine quantum effects. In this connection, the von Neumann - Liouville equation for the time evolution of the density operator is mapped onto the holomorphic phase space and some particular cases are studied. The equivalent holomorphic Wigner function is presented and the time evolution in the phase space description of stationary states is discussed; the resulting equations characterizing the stationarity are discussed in connection with the concept of curves preserving the form of the Wigner function. For the harmonic oscillator case those equations are the well known time solutions for the creation and annihilation operators respectively. Also in that context we have presented the equivalent equations for the time evolution of expectation values of operators which enabled us to write the Hamilton - like equations. Although of similar structure as the classical ones, the present equations are still of quantum nature. They preserve the original content through the mapped expression of the Hamiltonian which depends on quantum operators and, besides, through the quantum squeezing effect embodied in the parameter describing the deformation of the fundamental cell in the phase space while preserving its volume. This result is extremely

interesting since it draws attention to the inherent entanglement of the time solution of the canonical pair in the original quantum description through the Heisenberg principle; the canonical - like equation is only attained when one uses the minimum wave packet for the calculation of the expectation value, being the classical case then obtained in the usual $\hbar \longrightarrow 0$ limit. This result shows us explicitely that the classical structure of the equations of motion has its roots already in the quantum phase space and that the classical limit consists, besides considering $\hbar \longrightarrow 0$, in ignoring the cellular character of the quantum phase space, as obtained by the assumption $\beta = 1$ in eqs. (53) and (54).

The present formal scheme can be used in a large variety of physical processes, in particular the excitation process of giant resonances of nuclear targets by Coulomb interaction is under investigation and will appear in a future communication.

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