12

IFT

Instituto de Física Teórica
Universidade Estadual Paulista

Fermilab Library
0 1160 0049150 0

June/95 IFT–P.030/95

FERMILAB
OCT
BRAR

Limit on a heavy Dirac neutrino through oblique radiative corrections

A. A. Natale and P. S. Rodrigues da Silva

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 – São Paulo, S.P.
Brazil

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 251.5155
Telefax: 55 (11) 288.8224
Telex: 55 (11) 31870 UJMFBR
Electronic Address: LIBRARY@IFT.UESP.ANSP.BR
47553::LIBRARY

Limit on a heavy Dirac neutrino through oblique radiative corrections

A. A. Natale and P. S. Rodrigues da Silva

Instituto de Física Teórica, Universidade Estadual Paulista

Rua Pamplona, 145, 01405, São Paulo, SP, Brazil

Abstract

We compute the one-loop oblique corrections in a typical model with neutrino masses due to the see-saw mechanism. We verify that a Dirac neutrino mass up to 178 GeV is still allowed by the experimental data.

There has been recently some interest on the implications of one-loop corrections due to majorana particles on the electroweak measurements [1, 2, 3]. Most of this interest was focused on the consequences of the Hill and Paschos model [4], where it is introduced a fourth generation of leptons, which accomodates a heavy neutral member with a majorana mass of $O(G_\mu^{-1/2})$, and where the light neutrinos obtain small masses through the see-saw mechanism. Computing the oblique corrections for such a model, it was verified that a heavy majorana neutrino in the third generation decouples from electroweak physics and cannot be excluded experimentally [3].

The one-loop corrections in these type of models have been parametrized in terms of the heavy (or fourth generation) charged lepton mass, and of the light and heavy neutrino mass eigenstates (see, for instance, Refs. [2, 3]), and they show some intricate behavior as these masses are variated [2]. Although the introduction of a new generation is phenomenologically attractive [5], there is a much wider class of models, where we have majorana neutrinos without the company of the fourth generation charged lepton, and where we expect that the contribution of the new physics to the oblique parameters S, T and U [6], will be described only in terms of the light and heavy neutrino mass eigenstates. Therefore, we expect to constrain directly the neutrino masses, when we compute and compare their contribution to the radiative corrections with the experimental data.

Grand-unified see-saw models [7] are typical examples of the models we have in mind. They have a neutrino mass matrix given by the see-saw relation

$$m_\nu = m_D M_N^{-1} m_D^T, \tag{1}$$

where $m_D(M_N)$ is a Dirac(Majorana) neutrino mass matrix, which can be naturally of $O(m_U)$, i.e. the up-type quark mass matrix, and where all the heavy sector decouples from the low energy physics. With the evidences for a heavy top quark we may wonder about the effects of having a quite large Dirac neutrino mass! The relation given by Eq.(1) is also the case for many majoron models [8], where most

of the new physics is concentrated in the neutrino mass matrix m_ν, and where there is not much hope to detect the remaining physics of the model.

The question we want to address in this note is a very simple one: Which is the limit that present experimental data put on the Dirac neutrino masses (m_D) given in Eq.(1)? We certainly have an upper limit of few TeV on m_D coming from unitarity, however, we will see that the oblique corrections are more restrictive. The limit on m_D is directly translated into a constraint on the physical mass m_ν, and, obviously, for large Majorana masses we will have very small upper limits on m_ν.

Any model with the see-saw mechanism will be suitable for our calculation. As we expect that the most interesting limit on m_ν will come from a model with large Majorana masses, we will select one where M_N is obliged to be large. Therefore, we consider a model proposed by Shin [9], where besides the standard generations of quarks and leptons, it contains three generations of right-handed neutrinos (N_{0R}), one superheavy quark, and one $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ singlet complex scalar σ. One of the interesting points of the model is that the σ boson carry nonzero Peccei-Quinn charge [10], i.e. the σ boson will acquire a large vacuum expectation value (vev), which will be identified with the scale of Peccei-Quinn symmetry breaking, which resolve the strong CP problem. At the same time the σ will give large Majorana masses to the right-handed neutrinos (N_{0R}), therefore, as it carries lepton-number and PQ charge, the Goldstone boson associated with this symmetry breaking was called "majoraxion". As occurs with the ordinary axion, the vev of this scalar boson (V_{PQ}) will be limited to $10^{10} - 10^{12}$ GeV [11]. Notice that we could have chosen a GUT for our calculation, but the only difference would be a possible larger particle spectrum and values of the Majorana masses. Further details about the model can be found in Ref. [9], and here we will restrict ourselves only to what is necessary to compute the oblique corrections in this model.

In a general way, we can write the neutrinos mass matrix as follows,

$$\begin{pmatrix} \bar{\nu}_{0L} & \bar{N}^c_{0L} \end{pmatrix} \begin{bmatrix} 0 & m_D \\ m_D^T & M_N \end{bmatrix} \begin{pmatrix} \nu^c_{0R} \\ N_{0R} \end{pmatrix} + H.c. \tag{2}$$

This procedure allows us to consider an arbitrary number of lepton families, i.e., for n families the mass matrix elements, m_D and M_N, are indeed $n \times n$ matrices, with

$$(m_D)_{ij} \equiv (\frac{v}{\sqrt{2}}) y^{(\nu)}_{ij} \;\; \text{e} \;\; (M_N)_{ij} \equiv (\frac{V_{PQ}}{\sqrt{2}}) \times \exp(i\theta_0) y^{(N)}_{ij}, \tag{3}$$

where θ_0 is the vev phase of σ, which must be fixed according to the strong CP problem.

In the case we want to deal only with the three Standard Model generations, the matrices m_D and M_N will assume a 3×3 form. In order to obtain the neutrinos mass eigenstates, we have to diagonalize the mass matrix in Eq.(2). This can be done introducing a 6×6 matrix, V, in the following way

$$V^T \begin{bmatrix} 0 & m_D \\ m_D^T & M_N \end{bmatrix} V = diag(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, M_1, M_2, M_3) \tag{4}$$

where $m_{\nu_i} \ll M_i$. Then, we can write the relations between the mass eigenstates and interaction ones,

$$\begin{bmatrix} \nu_L \\ N^c_L \end{bmatrix} = V^T \begin{bmatrix} \nu_{0L} \\ N^c_{0L} \end{bmatrix}, \quad \begin{bmatrix} \nu^c_R \\ N_R \end{bmatrix} = V^\dagger \begin{bmatrix} \nu^c_{0R} \\ N_{0R} \end{bmatrix}. \tag{5}$$

Defining de mixing matrix by

$$V = \begin{bmatrix} V_a & V_b \\ V_c & V_d \end{bmatrix} \equiv U^* \equiv \begin{bmatrix} U^*_a & U^*_b \\ U^*_c & U^*_d \end{bmatrix}, \tag{6}$$

where V_i and U^*_i ($i = a, b, c, d$) are 3×3 matrices. Now, we are able to write the interaction Lagrangian, describing the coupling between leptons and gauge bosons

$$\begin{aligned} \mathcal{L}_{Z\nu N} &= -\frac{1}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{\frac{1}{2}} Z_\mu \Big\{ \bar{\nu}\gamma^\mu (U^\dagger_a U_a)(1-\gamma_5)\nu + \bar{N}\gamma^\mu (U^\dagger_b U_a)(1-\gamma_5)\nu \\ &+ \bar{\nu}\gamma^\mu (U^\dagger_a U_b)(1-\gamma_5)N + \bar{N}\gamma^\mu (U^\dagger_b U_b)(1-\gamma_5)N \Big\}, \end{aligned} \tag{7}$$

and

$$\mathcal{L}_{W^{\pm}\nu N} = -\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} W_\mu^- \Big\{ \bar{l}^- \gamma^\mu (U_a)(1-\gamma_5)\nu$$

$$+\bar{l}^- \gamma^\mu (U_b)(1-\gamma_5) N \Big\} + H.c. \tag{8}$$

In order to make the computation of the diagonalization matrix feasible, we need to introduce some assumptions on the model proposed by Shin [9], which are summarized in the following. Only the tau neutrino (ν_τ) will acquire mass via the seesaw mechanism. The heavy neutrinos (N_1, N_2, N_3) are expected to have masses of the same order of V_{PQ} (the Peccei-Quinn symmetry scale). We will assume mixing just between ν_τ and N_3, and the mass matrix of charged leptons is assumed to be diagonal. These assumptions retain most of the physics of the problem, and at the same time greatly simplify the calculation, because we get rid of a complicated mixing matrix, which, if we do have a mass hierarchy in the Dirac neutrino sector, will not modify significantly our results. Therefore we arrive at the following diagonalizing matrix

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & ic_\theta & 0 & 0 & s_\theta \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -is_\theta & 0 & 0 & c_\theta \end{bmatrix}. \tag{9}$$

yelding the mixing angle as a function of the ν_τ mass (m_1) and N_3 mass (m_2),

$$s_\theta^2 = \frac{m_1}{m_1+m_2} \quad \text{and} \quad c_\theta^2 = \frac{m_2}{m_1+m_2} \tag{10}$$

which are related to Dirac and Majorana masses by

$$m_D = \sqrt{m_1 m_2} \quad \text{and} \quad M_N = m_2 - m_1. \tag{11}$$

With these results we can rewrite the interaction Lagrangian in terms of these mixing angles, recalling some properties of majorana particles, which lead us to

$$\mathcal{L}_{Z\nu N} = -\frac{g_2}{4c} Z_\mu \Big\{ -c_\theta^2 \bar{\nu}\gamma^\mu\gamma_5\nu - s_\theta^2 \bar{N}_3\gamma^\mu\gamma_5 N_3 + 2is_\theta c_\theta \bar{N}_3\gamma^\mu\nu \Big\} \tag{12}$$

and,

$$\mathcal{L}_{W^{\pm}\nu N} = -\frac{g_2}{\sqrt{2}} W_\mu^- \Big\{ -ic_\theta \bar{l}^-\gamma^\mu(1-\gamma_5)\nu + s_\theta \bar{l}^-\gamma^\mu(1-\gamma_5)N \Big\} + H.c. \tag{13}$$

Now we are able to compute the oblique radiative corrections to this model. We will adopt the parametrization given by Kniehl e Kohrs [3], where the independent parameters S, T and U are determined through

$$\alpha S = 4s^2c^2 \left\{ \frac{\Pi_{ZZ}(0)}{z} - Re\frac{\Pi_{ZZ}(z)}{z} \right\}, \tag{14}$$

$$\alpha T = \frac{\Pi_{ZZ}(0)}{z} - \frac{\Pi_{WW}(0)}{w}, \tag{15}$$

$$\alpha U = 4s^2 \left\{ \frac{\Pi_{WW}(0)}{w} - Re\frac{\Pi_{WW}(w)}{w} \right\} - \alpha S, \tag{16}$$

where we are abbreviating the notation for the weak mixing angle and gauge boson masses, $c = \cos\theta_w$, $s = \sin\theta_w$, $w = M_w^2$, and $z = M_z^2$.

In Eqs.(14) to (16) the nonrenormalized vector boson self-energies (Π_{VV}) can be written as a function of the transferred moment q^2:

$$\Pi_{WW}(q^2) = \frac{e^2}{8s^2} \Big[c_\theta^2(\Pi^V + \Pi^A)(q^2, m_1, m_\tau) + s_\theta^2(\Pi^V + \Pi^A)(q^2, m_2, m_\tau) \Big], \tag{17}$$

and

$$\Pi_{ZZ}(q^2) = \frac{e^2}{16c^2s^2} \Big[2c_\theta^4 \Pi^A(q^2, m_1, m_1) + 2s_\theta^4 \Pi^A(q^2, m_2, m_2) + 4s_\theta^2 c_\theta^2 \Pi^V(q^2, m_1, m_2) \Big]. \tag{18}$$

where

$$\Pi^{V,A}(q^2,m_1,m_2) = \frac{1}{12\pi^2}\left\{\left[q^2 - \frac{m_1^2+m_2^2}{2}(+,-)\,3m_1m_2 - \frac{(m_1^2-m_2^2)^2}{2q^2}\right]B_0(q^2,m_1^2,m_2^2)\right.$$

$$+m_1^2\left[-1+\frac{m_1^2-m_2^2}{2q^2}\right]B_0(0,m_1^2,m_1^2) + m_2^2\left[-1+\frac{m_2^2-m_1^2}{2q^2}\right]B_0(0,m_2^2,m_2^2)$$

$$\left.-\frac{q^2}{3}+\frac{(m_1^2-m_2^2)^2}{2q^2}\right\}. \tag{19}$$

The function $B_0(q^2,m_1^2,m_1^2)$ is one of the scalar Passarino-Veltman coeficients [12], which contains an ultraviolet divergent part that in the dimensional regularization scheme can be fixed as

$$\Delta = \frac{2}{\epsilon} - \gamma + ln(4\pi), \tag{20}$$

where ϵ is the deviation from the four dimensional space-time and γ is the Euler constant. The function B_0 is written as

$$B_0(q^2,m_1,m_2) = \Delta - \int_0^1 dx\; log\frac{x^2q^2 - x(q^2+m_1^2-m_2^2)+m_1^2-i\epsilon}{\mu^2}, \tag{21}$$

The finite part of the above equation can be found in Ref. [13].

We can now compute the oblique parameters, and after some algebra we obtain the analytic form of S, T and U

$$\begin{aligned}
\alpha T &= \frac{\alpha}{16\pi w s^2}\left\{-\frac{m_\tau^2}{3} + \frac{m_1m_2}{(m_1+m_2)^2}\left[6m_1m_2\right.\right.\\
&\left.+\frac{m_1^4+m_2^4-2m_1m_2(m_1^2+m_2^2)}{m_1^2-m_2^2}ln\frac{m_1^2}{m_2^2}\right]\\
&+\frac{m_1m_2}{m_1+m_2}\left[\left(m_2 - m_1\frac{m_1^2+m_\tau^2}{m_1^2-m_\tau^2}\right)ln\frac{m_1^2}{m_\tau^2}\right.\\
&\left.\left.+\left(m_1 - m_2\frac{m_2^2+m_\tau^2}{m_2^2-m_\tau^2}\right)ln\frac{m_2^2}{m_\tau^2}\right]\right\}; \qquad (22)
\end{aligned}$$

$$\begin{aligned}
\alpha S &= \frac{\alpha}{12\pi z}\left\{\frac{1}{(m_1+m_2)^2}\left[16m_1^2m_2^2 - m_1m_2(m_1^2+m_2^2) + 2\frac{m_1}{m_2}(m_1^2-m_2^2)^2\right]\right.\\
&+\frac{z}{(m_1+m_2)^2}\left[2\frac{m_1^3}{m_2} - \frac{20m_1^2}{3} + 6m_1m_2 - 4z\frac{m_1}{m_2} - \frac{z}{3}\frac{m_1^2}{m_2^2}\right]\\
&+2\frac{m_2^2}{(m_1+m_2)^2}(z-4m_1^2)ln\frac{z}{m_1^2} - 2zln\frac{z}{m_\tau^2}\\
&+\frac{2z}{m_1+m_2}\left[m_2ln\frac{m_1^2}{m_\tau^2} + m_1ln\frac{m_2^2}{m_\tau^2}\right]\\
&\left.+\frac{2m_1m_2}{(m_1+m_2)^2(m_1^2-m_2^2)}\left[z(m_1^2+m_2^2)+m_1^2m_2^2\right]ln\frac{m_1^2}{m_2^2}\right\}; \qquad (23)
\end{aligned}$$

$$\begin{aligned}
\alpha U &= \frac{\alpha}{\pi w}\frac{1}{(m_1+m_2)}\left\{+\frac{m_1}{2}w + \frac{m_1m_\tau^2}{6m_2^2}w - \frac{1}{3}\frac{m_1}{m_2^2}w^2 + \frac{m_2}{6w}(m_1^2-m_\tau^2)^2\right.\\
&-\frac{m_\tau^4}{6w}(m_1+m_2) + \frac{1}{3}(m_1+m_2)\left(w - \frac{m_\tau^2}{2} - \frac{m_\tau^4}{2w}\right)\times I\\
&+\frac{m_2}{12}(m_1^2+m_\tau^2) - \frac{m_1}{12}(m_2^2+m_\tau^2) + \frac{m_1}{6m_2^2}(m_2^2-m_\tau^2)^2\\
&+\left[\frac{m_2}{6}(m_\tau^2-m_1^2) + \frac{m_2}{4}\frac{(m_1^4+m_\tau^4)}{(m_1^2-m_\tau^2)} + \frac{m_2w}{6} + \frac{m_2}{12w}(m_1^4-m_\tau^4)\right]ln\frac{m_1^2}{m_\tau^2}\\
&+\left[\frac{m_1}{6}(m_\tau^2-m_2^2) + \frac{m_1}{4}\frac{(m_2^4+m_\tau^4)}{(m_2^2-m_\tau^2)} + \frac{m_1w}{6} + \frac{m_1}{12w}(m_2^4-m_\tau^4)\right.\\
&\left.-\frac{m_1}{12}\frac{(m_\tau^2+m_2^2)}{(m_\tau^2-m_2^2)}\left(2w - (m_\tau^2+m_2^2) - \frac{(m_2^2-m_\tau^2)^2}{w}\right)\right]ln\frac{m_2^2}{m_\tau^2}\\
&\left.+\frac{m_2}{6}\left[2w - (m_1^2+m_\tau^2) - \frac{(m_1^2-m_\tau^2)^2}{w}\right]ln\frac{w}{m_1m_\tau}\right\} - \alpha S. \qquad (24)
\end{aligned}$$

where

$$I = Re\left[\left(\frac{m_\tau^2}{w} - 1\right) ln\left(1 - \frac{w}{m_\tau^2} - i\epsilon\right)\right].$$

Notice that we subtracted in these parameters the contribution of the massless ν_τ to avoid double counting. To compare these expressions with the experimental results we may recall that these variables are related to the ϵ_i of Altarelli and Barbieri[14] by:

$$\begin{aligned} \alpha S &= 4s_W^2\epsilon_3 , \\ \alpha T &= \epsilon_1 , \\ \alpha U &= -4s_W^2\epsilon_2 . \end{aligned} \tag{25}$$

Considering the results of a very recent fit of the ϵ parameters for the experimental data[15], with $m_t = 175$ GeV and the Higgs mass varying between $M_H = 50 - 1000$ GeV,

$$\begin{aligned} \epsilon_1 \times 10^3 &= 4.8 \pm 2.2 , \\ \epsilon_2 \times 10^3 &= -7.0 \pm 5.3 , \\ \epsilon_3 \times 10^3 &= 3.5 \pm 3.0 , \end{aligned} \tag{26}$$

we may obtain the values of S, T and U, due to some new physics, subtracting from these the main contribution of the Standard Model [16].

In the model we are discussing the heavy Majorana mass is related to V_{PQ}, and we have naturally that $m_2 \approx M_N = O(V_{PQ})$, therefore, when we computed numerically the values of S, T and U, we assumed m_2 to be in the interval $10^{10} - 10^{12}$ GeV. The parameter S gives the most stringent limit on the mass m_1, and our results are displayed in Fig.(1). In these figures we show the values of the S parameter, due to the new physics, as a function of m_1 and m_2. In Figs.(1a) and (1b) we show the surfaces of the allowed values of S for different ranges of m_1 and m_2, and in Figs.(1c) and (1d) we present a cut of these surfaces at $S = 0.3$, wich is the maximum value

of S due to the new physics that is allowed by the experimental data. The minimum value of S does not give any constraint for this model. For $m_2 = 10^{10}$ GeV we obtain that $m_1 \leq 3.2$ KeV, whereas for $m_2 = 10^{12}$ GeV we obtain $m_1 \leq 32$ eV, which is a limit smaller than the experimental value of the ν_τ mass. This limit is a direct consequence of the fact that the experimental data still allow for a heavy Dirac neutrino with $m_D \leq 178$ GeV. With the U parameter we obtain a limit that is at least one order of magnitude worse, whereas the T calculation does not give any meaningful result.

As the Majorona mass is of the order of the Peccei-Quinn scale we obtain an extremely small mixing angle between the neutrino eigenstates, therefore our limits can be easily related to a limit on the Dirac neutrino mass. If we had a much lighter Majorana mass we would have the same constraint on the Dirac one, however, no limit would be obtained for the ν_τ, i.e. better than the present experimental limit on m_{ν_τ}. Notice that even if we had not simplified the mixing matrix we would obtain a similar result, as long as there is a mass hierarchy for the Dirac neutrino masses.

In conclusion, through the computation of the oblique parameters, we verified that a heavy Dirac neutrino with mass up to 178 GeV is allowed by the experimental data. The stronger constraint comes from the analysis of the S parameter. In the particular model that we considered, the Majorana neutrino mass is of the order of the Peccei-Quinn symmetry breaking scale, therefore, from the constraint on the Dirac neutrino mass, we obtained a limit on the ν_τ mass.

Acknowledgments

This research was supported in part by the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico (CNPq) (AAN), and in part by Fundacao de Amparo a Pesquisa do Estado de São Paulo (FAPESP) (PSRS).

References

[1] S. Bertolini and A. Sirlin, Phys. Lett. **B257**, 179 (1991).

[2] E. Gates and J. Terning, Phys. Rev. Lett. **67**, 1840 (1991).

[3] B. A. Kniehl and H. -G. Kohrs, Phys. Rev. **D48**, 225 (1993).

[4] C. T. Hill and E. A. Paschos, Phys. Lett. **B241**, 96 (1990).

[5] W. Buchmuller, C. Greub, and H. -G. Kohrs, Nucl. Phys. **B370**, 3 (1992).

[6] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. **D46**, 381 (1992); D. C. Kennedy and B. W. Lynn, Nucl. Phys. **B322**, 1 (1989).

[7] P. Langacker, Phys. Rep. **72**, 185 (1981).

[8] Y. Chikashige, R. N. Mohapatra and R. Peccei, Phys. Rev. Lett. **B98**, 265 (1981); G. Gelmini, S. Nussinov and T. Yanagida, Nucl. Phys. **B219**, 31 (1983).

[9] Michael Shin, Phys. Rev. Let. **59**, 2515 (1987).

[10] R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D16**, 1791 (1977).

[11] M. S. Turner, Phys. Rep. **197**, 67 (1990).

[12] G. Passarino and M. Veltman, Nucl. Phys. **B160**, 151 (1979).

[13] B. A. Kniehl, Nucl. Phys. **B352**, 1 (1991).

[14] G. Altarelli and R. Barbieri, Phys. Lett. **B253**, 161 (1990).

[15] R. Barbieri, Talk given at the Rencontres de la Vallee d'Aoste, La Thuile, Italy, March 1994, Preprint IFUP-TH 28/94.

[16] G. Altarelli, R. Barbieri and F. Caravaglios, Nucl. Phys. **B405**, 3 (1993).

Figure Caption

Fig. 1. Figure (a) and (b) show the behavior of the S parameter as a function of m_1 and m_2 for different ranges of these masses. In figure (c) and (d) the shaded area is the one allowed for m_1 and m_2 by the experimental data, they were obtained by a cut of the surfaces shown in (a) and (b) at $S = 0.3$, the maximum value of S permitted to the new physics.

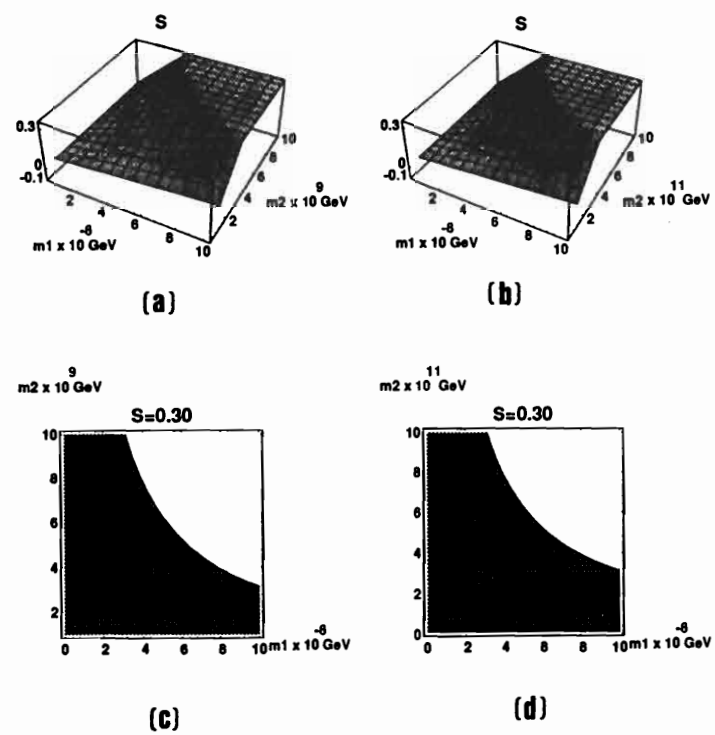

FIG. 1