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Quark-antiquark correlation in the pion

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Abstract

The electromagnetic tensor for inclusive electron scattering off the pion, ($W^{\mu\nu}$), for momentum transfers such that $q^+ = 0$, ($q^+ = q^0 + q^3$) is shown to obey a sum-rule for the component W^{++}. From this sum-rule, one can define the quark-antiquark correlation function in the pion, which characterizes the transverse distance distribution between the quark and antiquark in the light-front pion wave-function. Within the realistic models of the relativistic pion wave function (including instanton vacuum inspired wave function) it is shown that the value of the two-quark correlation radius ($r_{q\bar{q}}$) is near twice the pion electromagnetic radius (r_π), where $r_\pi \approx 2/3$ fm. We also define the correlation length l_{corr} where the two - particle correlation have an extremum. The estimation of $l_{corr} \approx 0.3 - 0.5\ fm$ is very close to estimations from instanton models of QCD vacuum. It is also shown that the above correlation is very sensitive to the pion light-front wave-function models.

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Investigation of low-energy pion constants and pion form-factors at low and intermediate momentum transfers provides important information about internal dynamics of hadron constituents. At asymptotically high momentum transfers the behaviour of pion form-factors is defined by quark counting rules [1,2] and perturbative QCD gives rigorous predictions for exclusive amplitudes [3,4]. However, some time ago the applicability of the perturbative approach to exclusive processes at moderately high momentum transfers has been stood under question [5,6]. It turns out that the attempts to describe the pion form factor using only perturbative hard scattering mechanism is not successful and soft internal dynamics of pion constituents becomes important. Later on, in Refs.([7-9]), it was shown that it is necessary to include an intrinsic transverse momentum dependence of the soft pion wave function to justify perturbative QCD calculations of the pion form factors in the region of momentum transfers far below of asymptotic one. Moreover, numerical analysis show that at low and intermediate momentum transfers, $Q < 3-5$ GeV, the soft (overlaping) diagram dominates over the asymptotic (one - gluon exchange) ones, inspite of the fact that the first one is parametrically smaller by $1/Q^2$ factor at large Q.

So, detailed theoretical input and additional experimental information is needed to relate low and high energy properties of pion. In this work we want to consider a correlation function describing quark - antiquark correlations in transverse space direction that could be in principle measured in the electron inclusive scattering off pion at moderately high energy experiments. Considering the actual interest and convenience in applying the light cone formalism to investigate the hadronic structure at low and intermediate energies, it will be important to derive some useful sum rules in this new context. We derive for the pion a sum rule for the light cone component of the inclusive hadronic tensor W^{++} that is diagonal in the Fock state basis, which gives the quark - antiquark correlation. Also, applying this sum rule, we study a few relativistic models for the pion wave-function.

At energy scale less then few GeV, the pion seems to behave as a strongly bound system of constituent quarks of masses 250 - 350 MeV. In this picture, hadron amplitudes describe the transition of hadron states into quark - antiquark pairs. They are of nonperturbative origin and serve as absolute normalization (initial condition) of the large Q^2 behaviour calculated perturbatively. We will use the light - cone constituent quark models with wave-functions defined in the null-plane hypersurface ($x^+ = x^0 + x^3 = 0$) [10]. This approach allows a consistent truncation of the Fock-space, such that the boost transformations that keep the null-plane invariant, do not mix different Fock-components [11] (see [12] for a modern discussion of this problem). In that respect, we can work with a fixed number of constituents.

All light-cone operators, corresponding to physical quantities, are classified as "good" and "bad" ones, where the "good" operators are diagonal in the Fock state basis, as a consequence of suppression of pair creation processes [11,13]. We show that, by using the "good - good" component of the inclusive hadronic tensor, W^{++}, for momentum transfers such that $q^+ = 0$ and integrating in q^-, it is possible to introduce a sum rule for W^{++}, which is the relativistic generalization of the Coulomb sum rule for inelastic electron scattering. It is well known [14,15] that the Coulomb sum rule integral is the Fourier transform of the two-body density. As its non-relativistic counterpart, we show that the relativistic sum rule defines the correlation function characterizing the transverse distance distribution of quark and antiquark in the pion. This allows a simple interpretation of the observable in terms of

constituent $q\bar{q}$ composite light-front pion wave-function.

The electromagnetic tensor, $W^{\mu\nu}$, for the inclusive electron scattering off pion, is defined as the square of the amplitude for the photon absorption summed over all final hadron states:

$$W^{\mu\nu}(p,q) = \frac{(2\pi)^4}{m_\pi}\sum_n \int \frac{d^4x}{2\pi}\prod_{i=1}^{n}\left[\frac{d^3p_i}{(2\pi)^3 p_{i0}}\right] e^{iqx}\langle p_\pi|J^\gamma_\mu(x)|n\rangle\langle n|J^\gamma_\nu(0)|p_\pi\rangle\delta^4(p_n - p_\pi - q) =$$
$$= \frac{1}{m_\pi}\int \frac{d^4x}{2\pi} e^{iqx}\langle p_\pi|J^\gamma_\mu(x)J^\gamma_\nu(0)|p_\pi\rangle, \qquad (1)$$

where p denotes the four vector of the pion and q is the photon momentum transfer. As a tensor of second rank it is written in terms of two invariant structure functions W_1 and W_2, as follows from Lorentz, gauge and parity symmetries:

$$W^{\mu\nu}(p,q) = W_1(q^2,q.p)\left[\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right] + \frac{W_2(q^2,q.p)}{m_\pi^2}\left[p^\mu - \frac{p.qq^\mu}{q^2}\right]\left[p^\nu - \frac{p.qq^\nu}{q^2}\right]. \qquad (2)$$

In the rest frame of the pion, $p^+ = p^- = p^0 = m_\pi$ and $\vec{p} = 0$, the photon momentum can be choosen such that $q^+ = 0$, and the component W^{++} is given by:

$$W^{++}(p,q) \;=\; W_2(q^2,q.p)\;. \qquad (3)$$

We shall consider the sum rule $C(q_\perp^2)$, by integrating W^{++} in q^- at fixed $q_\perp^2$ and $q^+ = 0$[1]:

$$C(q_\perp^2) = \frac{1}{2e^2}\int_0^{+\infty} dq^- W^{++}(p,q)|_{(q^+=0)} = \frac{1}{2e^2}\int_0^{+\infty} dq^- W_2(q^2,q.p)|_{(q^+=0)}, \qquad (4)$$

where e is the electron charge. A similar sum-rule was first introduced by Gottfried [14], where W^{00} is integrated over the transfered energy, which is suitable for the use with instant form wave-functions.

The well known Dashen - Gell - Mann - Fubini current algebra sum rule (see for example, the book of De Alfaro et al. in Ref. [13]) differs from that suggested in Eq.(4) in that respect that the first one deal with the integration of the current commutator over q^- in the interval from $-\infty$ to $+\infty$. Due to crossing symmetry, it becomes trivial in the case of electron inelastic scattering and provides very important restrictions in the case of neutrino scattering. In the case of the sum rule given in Eq.(4), the integration over q^- is performed in the half axis interval, $q^- > 0$, where $W^{\mu\nu}(p,q)$ is not equal to zero, so it is not dominated by light - cone current algebra contribution.

It is well known that this kind of sum rule is unusual since it is of "wrong" signature. Really, the derivation of the sum rules of "right" signature is based on consideration of causal amplitude which is defined via time - ordered product of currents. In that case the absorptive part of the amplitude is expressed through the commutator of currents satisfying causality principle. However, the sum rules of "wrong" signature such as the Gottfried sum rule are

[1]It is assumed that permutation between $q^+ = 0$ and $\int_0^\infty dq^-$ is allowed.

constructed from amplitudes of opposite crossing symmetry properties and correspond to the matrix elements of the anti - commutators of currents [17]. Singular on the light cone, the contributions of the current anti - commutator provide parton sum rules (possessing scaling at $q^2 \to \infty$) and describe the $SU(3)$ structure of hadrons. We shall consider not only singular contributions but also regular two - particle contributions on the light cone, which describe correlations in transverse space and correspond to power corrections to parton sum rules.

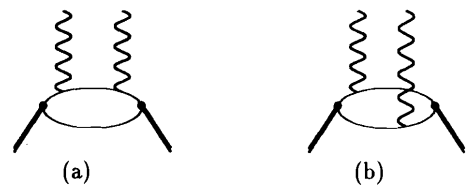


FIG.1 Photon absorption graphs.

Let us calculate $C(q_\perp^2)$ in the relativistic constituent quark framework. Since the photon can be absorbed by each of the constituents of the pion, two kind of terms arises, as shown in Fig.1. One corresponds to the direct term (D), which has the same quark absorbing and emitting the photon. The other one, the exchange term (E), has the photon absorbed by one quark and emitted by the other in its hermitian conjugate. Then, the electromagnetic tensor is written as

$$W^{\mu\nu} = W_D^{\mu\nu} + W_E^{\mu\nu}\;. \qquad (5)$$

In the Bjorken limit, $q_\perp^2 \to \infty$, just the direct term W_D survives, and the exchange term W_E is nonzero only as high twist correction. Correspondingly, the integral C, as given in Eq.(4), is expressed as the sum of a direct and an exchange term:

$$C(q_\perp^2) = \; C_D(q_\perp^2) \; + \; C_E(q_\perp^2) \qquad (6)$$

The coupling of the pion with quarks is given by the effective Lagrangian with vertex $L^{eff}_{\pi\to q\bar{q}} = (M/f_\pi)g_\pi(p^2,(p_1-p_2)^2)\bar{q}\gamma^5\vec{\tau}q$ defining the soft transition amplitude of the pion into quark - antiquark pair $\pi(p) \to \bar{q}(p_1)q(p_2)$, where M is the constituent quark mass, $\vec{\tau}$ the isospin matrices, and $f_\pi = 93\ MeV$.

From the direct component of the pion electromagnetic tensor (Fig. 1a), using Eq.(4), at $q^+ = 0$ and fixed $\vec{q}_\perp^2$ ($q^2 = -\vec{q}_\perp^2$), we obtain:

$$C_D(q_\perp^2) = I_D^\alpha \frac{N_c}{2\pi m_\pi}\left\{\frac{M^2}{f_\pi^2}\int dq^- \frac{d^4k d^4k'}{(4\pi)^2} k'^+\delta^4(k'-k+p_\pi+q)\delta(k^2-M^2)\delta(k'^2-M^2)\cdot\right.$$
$$\left.\cdot\frac{tr\,[\gamma^+(\not{k}-\not{p}_\pi+M)\gamma^5(\not{k}+M)\gamma^5(\not{k}-\not{p}_\pi+M)]}{((k-p_\pi)^2-M^2)^2} g_\pi^2((k-\frac{p}{2})^2)\right\}\;, \qquad (7)$$

where N_c is the number of colors, k is the 4-momentum of the spectator quark, $k' = k - p_\pi - q$, and $\gamma^+ = \gamma^0 + \gamma^3$. The trace over isospin space, considering the charges of quark and antiquark, gives I_D^α:

$$I_D^\alpha = Tr\left(QQ\tau^\alpha\tau^{\alpha\dagger}\right) + Tr\left(\bar{Q}\bar{Q}\tau^{\alpha\dagger}\tau^\alpha\right) = \frac{5}{9} \; for \quad \alpha = \pi^+,\ \pi^-,\ \pi^o,$$

where the charge matrices of quark and antiquark are $Q = -\bar{Q} = (1/6 + \tau_z/2)$.

The integrations over the four momentum k' and over the light cone variables q^- and k^-, in Eq. (7), provide the result:

$$C_D(q_\perp^2) \;=\; I_D^\alpha \frac{2N_c}{(2\pi)^3}\frac{M^2}{f_\pi^2}\int_0^1 dx \int \frac{d^2k_\perp}{x(1-x)}\frac{M_0^2}{(M_0^2 - m_\pi^2)^2} g_\pi^2(M_0^2)\,, \tag{8}$$

where the momentum fraction $x = k^+/m_\pi$ is introduced and the invariant mass of the $q\bar{q}$ system is given by

$$M_0^2(x, \vec{k}_\perp) = \frac{k_\perp^2 + M^2}{x(1-x)}\,. \tag{9}$$

From Eq. (8) it is easy to see that $C_D(q_\perp^2)$ is proportional to the normalization factor of the pion elastic electromagnetic form-factor [10,16,18] and the pion deep inelastic structure function [19]. Thus, we have

$$C_D(q_\perp^2) \;=\; \frac{5}{9},$$

that is the sum of the valence quark number weighted by the squared charges.

Next, in analogous manner, we evaluate the exchange component $C_E(q_\perp^2)$ (Fig. 1b):

$$C_E(q_\perp^2) \;=\; I_E^\alpha \frac{2N_c}{(2\pi)^3}\frac{M^2}{f_\pi^2}\int_0^1 dx \int \frac{d^2k_\perp}{x(1-x)}\frac{\left(M_0^2 - \vec{k}_\perp.\vec{q}_\perp/[x(1-x)]\right)}{(M_0'^2 - m_\pi^2)(M_0^2 - m_\pi^2)} g_\pi(M_0^2) g_\pi(M_0'^2), \tag{10}$$

where $M_0^2 \equiv M_0^2(x, \vec{k}_\perp)$ and $M_0'^2 \equiv M_0^2(x, \vec{k}_\perp - \vec{q}_\perp)$ are given by Eq.(9), and

$$\begin{aligned} I_E^\alpha = 2Tr\left(Q\tau^\alpha\bar{Q}\tau^{\alpha\dagger}\right) &= -5/9 \; for \;\; \pi^0; \\ &= +4/9 \; for \;\; \pi^\pm. \end{aligned} \tag{11}$$

From Eq.(10), we can define the quark - antiquark density $C_{q\bar{q}}(q_\perp^2)$,

$$C_{q\bar{q}}(q_\perp^2) = \frac{C_E(q_\perp^2)}{I_E^\alpha}. \tag{12}$$

We can see now that $C_E(q_\perp^2)$ is proportional to the normalization factor only at $q_\perp^2 = 0$, such that $C_{q\bar{q}}(0) \;=\; 1$.

To define the correlation function let us separate out the elastic contribution of the sum rule, Eq.(4). The matrix elements of the electromagnetic current between pion states are expressed as

$$\langle\pi(p')|J_\mu^\gamma(0)|\pi(p)\rangle = (p_\mu + p'_\mu)F_\pi(q^2), \tag{13}$$

where $F_\pi(q^2)$ is the electromagnetic form factor of the pion normalized at the origin: $F_\pi(0) = 1$. It is easy to see, from the definition in Eq.(1), that in the elastic limit the contribution to the sum rule will be

$$C_{elastic}(q_\perp^2) \;=\; F_\pi^2(q_\perp^2). \tag{14}$$

We define the correlation function characterizing the deviation of the exchange sum from the elastic contribution as

$$C_{corr}(q_\perp^2) \;= C_{q\bar{q}}(q_\perp^2) - C_{elastic}(q_\perp^2). \tag{15}$$

From this definition it follows that the absolute value of the correlation function $C_{corr}(q_\perp^2)$ is zero at $q_\perp^2 = 0$ and as $q_\perp^2 \to \infty$, and has an extremum at $q_\perp^2 = \bar{q}_\perp^2$. The maximum of the absolute value of the correlation function defines the quark - antiquark correlation length:

$$l_{corr} = 1/\sqrt{\bar{q}_\perp^2}. \tag{16}$$

Then, following Refs. [15], the total sum rule can be expressed as the sum of the elastic contribution ($C_{elastic}$), inelastic contribution in the absence of correlations ($C_D - (5/9)F_\pi^2$), and inelastic contribution in presence of correlations, C_{corr}:

$$\begin{aligned} C^\pm(q_\perp^2) &= F_\pi^2(q_\perp^2) + \tfrac{5}{9}[1 - F_\pi^2(q_\perp^2)] + \tfrac{4}{9}C_{corr}(q_\perp^2) = \frac{5}{9} + \frac{4}{9}C_{q\bar{q}}(q_\perp^2), \\ C^0(q_\perp^2) &= \tfrac{5}{9}\{[1 - F_\pi^2(q_\perp^2)] - C_{corr}(q_\perp^2)\} \;\; = \frac{5}{9} - \frac{5}{9}C_{q\bar{q}}(q_\perp^2). \end{aligned} \tag{17}$$

Here we have to note that in our calculations we didn't take into account the Pomeron exchange contribution. To exclude it we consider the difference between the total sum rules for charged and non-charged pions which directly defines the quark - antiquark density, $C_{q\bar{q}}$:

$$C^\pm(q_\perp^2) - C^0(q_\perp^2) = F_\pi^2(q_\perp^2) + C_{corr}(q_\perp^2) = C_{q\bar{q}}(q_\perp^2). \tag{18}$$

The light-front pion wave-function can be introduced by modifying the vertex as discussed in Refs. [10,16,18]. In this scheme the composite pion has the correct quantum numbers, which is equivalent to constructing the pion wave-function as in Ref. [20]. The light-front pion wave-function in terms of the relative coordinate, can be introduced as in Ref. [18][2], by the following

$$\Phi_\pi(x, \vec{k}_\perp) \;=\; \frac{1}{\pi^{\frac{3}{2}}}\frac{M}{f_\pi}\frac{\sqrt{M_0 N_c}}{M_0^2 - m_\pi^2} g_\pi(M_0^2). \tag{19}$$

Substituting in Eq. (10) the mass denominator by the bound-state wave-function given by Eq.(19), we have the result for the quark-antiquark density function in the pion:

[2]In this reference, $g_\pi(M_0^2) = 1$.

$$C_{q\bar{q}}(q_\perp^2) = \int_0^1 dx \int \frac{d^2k_\perp}{4x(1-x)} \sqrt{\frac{1}{M_0 M_0'}} \left[M_0^2 - \frac{\vec{k}_\perp . \vec{q}_\perp}{x(1-x)} \right] \Phi_\pi(x, \vec{k}_\perp)\, \Phi_\pi(x, \vec{k}_\perp - \vec{q}_\perp) \ . \quad (20)$$

With this choice of a phenomenological wave-function, we have the usual non-relativistic normalization, that is obtained at $\vec{q}_\perp = 0$, according to the context of the Hamiltonian Front Form of the dynamics [20,21]:

$$\int d^3k \left[\Phi_\pi(\vec{k}) \right]^2 = 1. \quad (21)$$

Here and in the following expressions, we use a dual notation, when writing our functions in terms of the instant form variables and in terms of the light-cone variables, such that

$$\Phi(\vec{k}) \equiv \Phi(x, \vec{k}_\perp).$$

The third component of the momentum is given in terms of x and $\vec{k}_\perp$ by [20]

$$k_z = \left(x - \frac{1}{2} \right) \sqrt{\frac{k_\perp^2 + M^2}{x(1-x)}} = \left(x - \frac{1}{2} \right) M_0 \ , \quad (22)$$

and the Jacobian of the transformation between $(x, \vec{k}_\perp)$ and $\vec{k}$ is

$$\frac{\partial(x, \vec{k}_\perp)}{\partial(k_z, \vec{k}_\perp)} = 4 \left[\frac{[x(1-x)]^3}{k_\perp^2 + M^2} \right]^{1/2} = \frac{4x(1-x)}{M_0}. \quad (23)$$

The transverse momentum, in the argument of the light-front wave-functions in Eq.(20), is given in the pion center of mass. The transverse photon momentum is subtracted from the center of mass momentum of one of the quarks, as a consequence of the absorption of the photon by the quark (antiquark) and the subsequent emission by the antiquark (quark).

In the non-relativistic limit ($M \to \infty$), $C_{q\bar{q}}(q_\perp^2)$ reduces to the Fourier transform of the two-body density, which appears in the Coulomb sum-rule [15], and it is given by:

$$C_{q\bar{q}}^{NR}(q_\perp^2) = \int d^3k \Phi_\pi(\vec{k})\, \Phi_\pi(\vec{k} - \vec{q}) \ . \quad (24)$$

For completeness, we present below the expressions of the pion charge form-factor, $F_\pi(q_\perp^2)$, and weak decay constant, f_π, using the light-front wave-function [10,16,18]:

$$F_\pi(q_\perp^2) = \int_0^1 dx \int \frac{d^2k_\perp}{4x(1-x)} \sqrt{\frac{1}{M_0 \tilde{M}_0}} \left[M_0^2 + \frac{\vec{k}_\perp . \vec{q}_\perp}{x} \right] \Phi_\pi(x, \vec{k}_\perp)\, \Phi_\pi(x, \vec{k}_\perp') \ , \quad (25)$$

where $\vec{k}_\perp' \equiv \vec{k}_\perp + (1-x)\vec{q}_\perp$ and $\tilde{M}_0 \equiv M_0(x, \vec{k}_\perp')$.

We have to emphasize that, inspite of similarity in the form of the expressions given in Eqs.(20) and (25) (just change $-q_\perp \to (1-x)\vec{q}_\perp$), they have very different physical interpretations. In the expression for $F_\pi(q_\perp^2)$, one of the pion is boosted such that it absorbs all the photon momentum in the form factor, whereas in the exchange term $C_E(q_\perp^2)$ the wave functions are calculated in the rest frame of pion. Covariance under kinematical boost

garantee that we can obtain the boosted wave function from the wave function in the center of mass frame (see Ref. [22] and references therein).

With the definition given by Eq.(19), the weak decay constant is given by:

$$f_\pi = \frac{M\sqrt{N_c}}{4\pi^{\frac{3}{2}}} \int \frac{dx d^2k_\perp}{x(1-x)} \frac{\Phi_\pi(x, k_\perp)}{\sqrt{M_0}} \ . \quad (26)$$

Let us make some predictions of the pion quark - antiquark density and correlation function, by performing numerical calculations with four models of light-front wave-functions: i) instanton, ii) Gaussian, iii) hydrogen atom and iv) the model wave-function of Ref. [23]. A recent discussion about light-front pion wave-functions can be also found in Ref. [24]. / Except for the total elastic contribution, as one can see from Eq.(15), the correlation function is closely related to the quark - antiquark density.

The hadron wave functions are defined by low energy quark dynamics. Within the realistic QCD vacuum approach, like QCD sum rules or instanton liquid model, the hadrons are considered as low energy excitations of nonperturbative QCD vacuum. As it has been shown by 't Hooft [25], for small size instanton the interaction generated by instanton - antiinstanton configurations induces a chirally invariant four - quark interaction, whose contributions to the Lagrangian is of the form

$$G \left[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5 \vec{\tau} \Psi)^2 \right], \quad (27)$$

where Ψ is the quark field and G is the interaction constant.

The quark then acquires a momentum dependent mass $M \cdot g(p^2)$ (where M is the constituent quark mass and $g(0) = 1$), via the Nambu-Jona-Lasinio mechanism [26], and in addition bound states appear in the pseudo scalar channel of $q\bar{q}$ system. From quark - antiquark scattering in the field of instanton, the non-local vertex function $g(p^2)$ is derived, with p^2 being invariant mass of quark - antiquark system [9,19,27].

The instanton inspired wave function is defined by the vertex function

$$g_{inst}(x, k_\perp) = exp\left(-\sqrt{\lambda} \frac{M_0(x, k_\perp)}{2} \right), \quad (28)$$

such that, within the normalization given by Eq.(21), we have

$$\Phi_{inst}(\vec{k}) = N_{inst} \frac{\left[4(\vec{k}^2 + M^2) \right]^{1/4}}{\left[4(\vec{k}^2 + M^2) - m_\pi^2 \right]} exp\left(-\sqrt{\lambda(\vec{k}^2 + M^2)} \right), \quad (29)$$

where N_{inst} is the normalization factor. In terms of the invariant $q\bar{q}$ mass we have

$$\Phi_{inst}(x, \vec{k}_\perp) = N_{inst} \frac{\sqrt{M_0}}{[M_0^2 - m_\pi^2]} exp\left(-\sqrt{\lambda} \frac{M_0}{2} \right). \quad (30)$$

The gaussian wave-function is given by

$$\Phi_\pi(\vec{k}) = \left(\frac{8r_{NR}^2}{3\pi} \right)^{3/4} exp\left(-\frac{4}{3} r_{NR}^2 \vec{k}^2 \right) \ ; \quad (31)$$

and the hydrogen atom wave-function by

$$\Phi_\pi(\vec{k}) = \frac{1}{2\pi}\left(\frac{\sqrt{3}}{r_{NR}}\right)^{5/2}\left(\frac{3}{4}r_{NR}^{-2}+\vec{k}^2\right)^{-2} . \tag{32}$$

In these last two wave-functions, r_{NR} is the scale defining the size properties of the wave function and the constituent quark mass we fixed at the value of 220 MeV, as in the model of Ref. [23].

The electromagnetic pion radius is given by Eq.(25), through the expression

$$r_\pi = \sqrt{-6(dF_\pi/dq_\perp^2)_{q_\perp^2=0}}; \tag{33}$$

and the quark - antiquark density radius $r_{q\bar{q}}$ is given by Eq.(20):

$$r_{q\bar{q}} = \sqrt{-6(dC_{q\bar{q}}(q_\perp^2)/dq_\perp^2)_{q_\perp^2=0}}. \tag{34}$$

In the same way, we have the correlation radius, defined from Eq.(15):

$$r_{corr} = \sqrt{-6(dC_{corr}(q_\perp^2)/dq_\perp^2)_{q_\perp^2=0}} = \sqrt{r_{q\bar{q}}^2 - 2r_\pi^2}. \tag{35}$$

It is well known that the square radius of the transverse space correlation function can be related with the total photoproduction cross section σ_γ^T. For the charged - neutral pion difference one has

$$\frac{d}{dq_\perp^2}(C^\pm(q_\perp^2) - C^0(q_\perp^2))|_{q_\perp^2=0} = -\frac{1}{3}\langle r_\pi^2\rangle + \frac{1}{4\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}(\sigma_{\gamma\pi^\pm}^T - \sigma_{\gamma\pi^0}^T), \tag{36}$$

where ν_0 is the threshold for inelastic photon absorption. and

$$-\frac{1}{m_\pi}\frac{W_2(q^2,\nu)}{dq^2} = \frac{\sigma_{\gamma\pi}^T}{4\pi^2\alpha\nu}. \tag{37}$$

This sum rule has been first derived by Gerasimov [28]. In order to obtain the estimation of the integral of the difference of the cross sections, that appears in the left hand side of Eq. (36), we can rewrite this sum rule in a form that it is related with r_{corr}:

$$\frac{1}{4\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}(\sigma_{\gamma\pi^\pm}^T - \sigma_{\gamma\pi^0}^T) = -\frac{1}{6}\langle r_{corr}^2\rangle \tag{38}$$

Analogous considerations in nucleon case has been done in Refs. [14,29].

The wave function model given in Ref. [23] has a non-relativistic pion radius of 0.195 fm and it gives for the electromagnetic pion radius the value of 0.456 fm [18]. For the same r_{NR} the Gaussian and the hydrogen-atom models give r_π values of 0.476 fm and 0.463 fm, respectively [18]. None of these three models were able to describe the experimental electromagnetic pion radius of $r_\pi^{exp} = 0.660 \pm 0.024$ fm [30]. We observe, in this case, that the experimental values of $F_\pi(q_\perp^2)$ [31] are not reproduced

by the models, and in Fig. 2 this fact is represented by the model of Ref. [23].

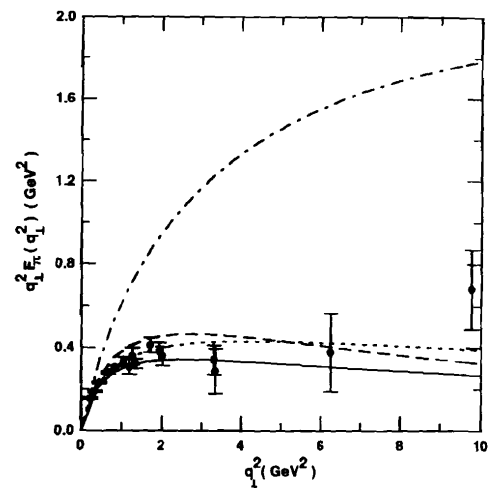


FIG.2 Pion form-factor for $q_\perp^2 < 10\ GeV^2$. The four curves represent four models considered in this paper. Godfrey and Isgur model (dashed) uses the non-relativistic radius of 0.195 fm that does not fit f_π. The other three curves uses parameters such that fit $f_\pi = 93$MeV. In the Instanton model (short-dashed) we use $M = 200$ MeV and $\lambda = 0.1153/M$; for the Hydrogen-atom model (solid) we use $M = 220$ MeV and r_{NR}= 0.456 fm; and for the Gaussian model (long-dashed) M = 220 MeV and r_{NR}= 0.321 fm. Experimental data from Ref. [31].

In order to have a model reasonable consistent with the experimental form factor data, we choose to fit the experimental pion decay constant $f_\pi = 93$ MeV, since this also should produce a reasonable pion radius [18]. We obtain $r_{NR} = 0.321$ fm and $r_\pi = 0.64$ fm for the Gaussian model, and $r_{NR} = 0.456$ fm and $r_\pi = 0.76$ fm for the hydrogen-atom model. With these parameters the calculated pion form factor for both models are in good agreement with the experimental data, as we observe in Fig. 2. Here we also show the results for the instanton model, that fit f_π with $M = 200$ MeV and $\lambda = 0.1153/M^2$. The instanton model gives $r_\pi = 0.77$ fm, and shows a behaviour similar to the hydrogen atom model, with a good fitting of the experimental data.

The calculated quark - antiquark density and correlation radius for the different models we consider, using Eqs.(34) and (35), are: $r_{q\bar{q}} = 1.12$ fm and $r_{corr} = 0.66$ fm for the Gaussian model, $r_{q\bar{q}} = 1.37$ fm and $r_{corr} = 0.85$ fm for the Hydrogen-atom model, and $r_{q\bar{q}}$= 1.39 fm and $r_{corr} = 0.86$ fm for the Instanton model. As we see, the quark - antiquark density radius are near twice the pion radius, in agreement with the nonrelativistic expectation.

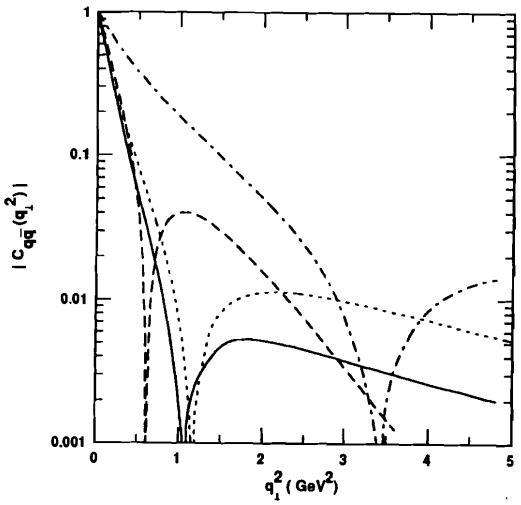
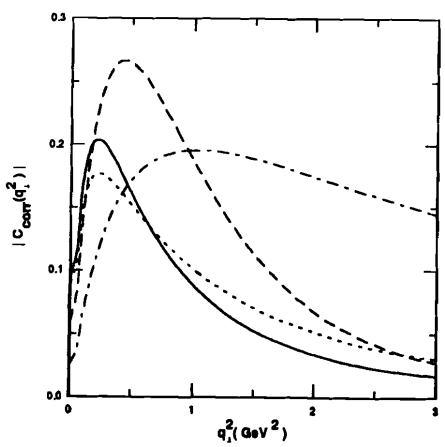


FIG.3 The absolute value of the quark - antiquark density in the pion, $C_{q\bar{q}}(q_\perp^2)$ for $q_\perp^2 < 5\ GeV^2$. The curves represent the models with the parametrization and line conventions as in Fig. 2.



'IG.4 The absolute value of the correlation function of the pion, $C_{corr}(q_\perp^2)$, is plotted against $1/q_\perp$ n Fms), showing the corresponding correlation lengths l_{corr} (as given by Eq. (16)) of the four iodels we consider: ≈ 0.42 fm for the instanton and the hydrogen atom models, ≈ 0.30 fm for the laussian model and ≈ 0.20 fm for the model of Ref. [23]. The parametrization and line conventions re the same as in Fig. 2.

At this point, we have four models for which we calculate the quark - antiquark density function $C_{q\bar{q}}(q_\perp^2)$ in the pion. In Fig. 3, we plot the results obtained for the instanton, the Gaussian and the hydrogen-atom models. We also show the model of Ref. [23], for reference, but this model does not fit f_π. The observable $C_{q\bar{q}}(q_\perp^2)$ has a zero for these three models, and it depends strongly on the choice of the wave-function. In Fig. 4, we show the results for the correlation function C_{corr}, given by Eq.(15), for those four models. The corresponding correlation lengths, as seen from the maxima of the curves, are $l_{corr} = 0.42$ fm for the instanton and the hydrogen atom models, and $l_{corr} = 0.30$ fm for the Gaussian model. The model of Ref. [23] gives $l_{corr} = 0.20$ fm. In this figure we observe the model dependence of C_{corr}.

The Gaussian model, which gives f_π and fits reasonably the elastic form-factor experimental data, does not give the same quark - antiquark density and the correlation functions as the other two models we have used (the instanton and the hydrogen atom models). This was shown in Figs.3 and 4. This is an indication that the correlation between the quarks in the pion, as seen by $C_{q\bar{q}}(q_\perp^2)$ or $C_{corr}(q_\perp^2)$, can bring more physical information about the wave-functions not completely contained in the elastic form factor data.

In conclusion, we have shown that a light-front sum-rule can be defined from the W^{++} component of structure tensor for inelastic electron scattering, by integrating it in the q^- component of the momentum transfer for $q^+ = 0$. This sum-rule is the light-front generalization of the non-relativistic Coulomb sum-rule, and it allows to define the quark-antiquark correlation function in the pion. We made some model calculations and it turned out that this function is very sensitive to the model of the pion bound-state wave-function. In general, it can be a useful source of information on the relativistic constituent quark wave-function of the hadrons. Further investigation about spin - flavor correlations in proton and deuteron can provide very important informations about the hadron structure, with a better interpretation of existing exclusive and DIS data on spin and flavour content of hadrons; it also can suggest an answer to the very intriguing question on the role of nonvalence degrees of freedom.

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