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HQET - A short review

B. E. Palladino

Departamento de Física da Univerdade de Rio Grande - FURG
Campus Carreiro - Av. Itália Km. 8
96200-000 - Rio Grande, R.S.
Brazil

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 - São Paulo, S.P.
Brazil

SKARHA 10716 318

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

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HQET - A SHORT REVIEW

Braz Edson Palladino

FURG - Fundação Universidade do Rio Grande
Departamento de Física

and

IFT - Instituto de Física Teórica - UNESP
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 - São Paulo, SP, Brazil

Abstract

A brief review about the Heavy Quark Effective Theory (HQET) is made. An introduction to the subject is presented shortly, followed by a sketch of the main results obtained with HQET.

(Work presented in the panel section of **HADRONS 94** - Brazilian meeting - to appear in the proceedings)

HQET - A SHORT REVIEW

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0.1 - OBJECTIVE

Far of being a complete review, or even a resumé on the subject, these tables intend to be a short introductory text for passing-by pedestrians. It may also work as a guideline, reminding the most expert ones of the recent developments in HQET.

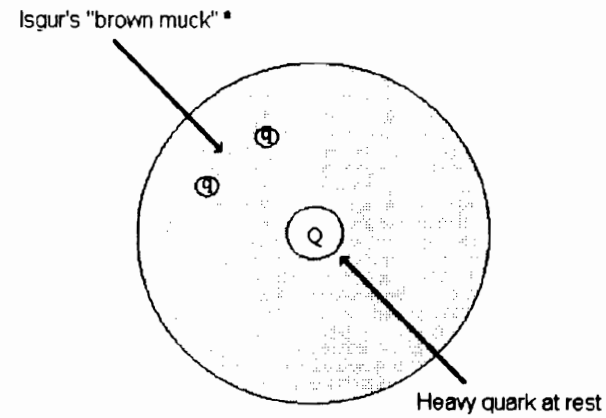
0.2 - INTRODUCTION

Heavy Quark Effective Theory (HQET) started being developed recently, basically beginning 5 or 6 years ago, with papers by N. Isgur, M.B. Wise and H. Georgi, amongst others. In the last years the theory became more and more popular, as it proved to be a powerful calculational tool, which may help to overcome usual difficulties appearing in quark models of QCD.

When at least one of the quarks in a hadron is heavy (quark *c*, *b* or *t*) the hadron can be thought as a system similar to an atom, with the heavy quark *Q* sat down over the system's center-of-mass. As compared to Λ, the hadronic scale parameter of QCD, the heavy quark mass M_Q is large ($M_Q >> \Lambda$) and allows a perturbative treatment in powers of Λ/M_Q.

HQET tells us that hadrons with heavy quarks have peculiar symmetry properties. When *Q* is sufficiently heavy ($M_Q \to \infty$), the heavy quark acts like a static source of gluons (color fields) in its rest frame and the internal dynamics of heavy hadrons becomes independent of M_Q and the quark *Q* spin.

<u>Atom-like, heavy quark hadron</u>



* The "slimy matter" in the problem: light quarks, hard gluons, etc.

1 - BASIC FORMULATION

The QCD Lagrangian for a heavy quark Q is

$$\mathcal{L}_{QCD} = \overline{\psi}_Q\left(i\not{D} - M_Q\right)\psi_Q$$

where the covariant derivative is $D_\mu = \partial_\mu - ig_s A_\mu^a T^a$.

Of course, it is not possible to take the limit $M_Q \to \infty$ directly in the Lagrangian, but remembering the heavy quark should exchange only small momenta ($k \lesssim \Lambda_{QCD}$) we can write

$$P_Q^\mu = M_Q v^\mu + k^\mu, \qquad \text{where } v_Q \approx v_{hadron} = v$$

"kinetic part" (large amount of momentum from hadron's motion concentrated on heavy quark)

"residual part" (small amount of momentum due to interactions of heavy quark with light quarks and gluons)

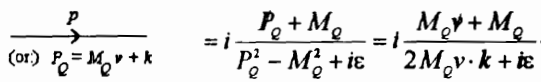
Now take $v^\mu = \frac{1}{\sqrt{1-v^2}}(1,\vec{v})$, which gives

$$P_Q^2 = M_Q^2 + 2M_Q v\cdot k + \ldots$$

P_Q^2 ↓ almost on-shell; M_Q^2 ↓ on-shell; $2M_Q v\cdot k$ ↓ off-shell

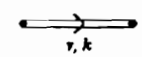
In propagator language:

1st) Heavy quark propagator:



$$= i\frac{\not{P}_Q + M_Q}{P_Q^2 - M_Q^2 + i\varepsilon} = i\frac{M_Q\not{v} + M_Q}{2M_Q v\cdot k + i\varepsilon} + \ldots$$

which gives, for $M_Q \to \infty$: $\to \frac{i}{v\cdot k + i\varepsilon'}\left(\frac{1+\not{v}}{2}\right) =$

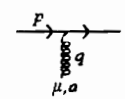


(effectively)

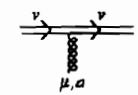
corresponding to the 2-point Green function

$$G^{(2)}(p) = \frac{i}{\not{P} - M_Q} \quad \to \quad i\left(\frac{1+\not{v}}{2}\right)\frac{1}{v\cdot k} + O\left(\frac{\Lambda}{M_Q}\right)$$

2nd) Quark-gluon vertex:



$$= ig_s T_a \gamma^\mu \quad \to \quad ig_s T_a v^\mu =$$



(effectively)

corresponding to the 3-point Green function

$$G_{\nu a}^{(2,1)}(P,q) = \frac{i}{\not{P} - M_Q}(+ig_s T_a\gamma^\mu)\frac{i}{\not{P} + \not{q} - M_Q}\Delta_{\mu\nu}(q) \to$$

$$\to \left(\frac{1+\not{v}}{2}\right)\frac{i}{v\cdot k}(ig_s T_a v^\mu)\frac{i}{v\cdot(k+q)}\Delta_{\mu\nu}(q) + O\left(\frac{\Lambda}{M_Q}\right)$$

since $\frac{1+\not{v}}{2}\gamma^\mu\frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}v^\mu\frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}v^\mu$ (this follows because we took v^μ such that $v^2 = \not{v}^2 = 1$.)

Above: the $\Delta_{\mu\nu}$ is the gluon propagator and the double line stands for the heavy quark propagator in HQET.

The factor $\frac{1+\not{v}}{2}$ works as a projection operator.

In the rest frame of Q, the heavy quark wavefunction would have a component

$$\psi_Q = \sqrt{\frac{E+M_Q}{2E}} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M_Q}\chi_s \end{pmatrix} e^{-i(Et-\vec{p}\cdot\vec{x})}$$

where χ_s is the two-component spinor.

For a large M_Q mass we have $E \approx M_Q$, the square root prefactor in the wavefunction is equal to one and the lower component tends to disappear.

The upper component can be described by a projection operator

$$\frac{1+\gamma_0}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

such that

$$\left(\frac{1+\gamma_0}{2}\right)\psi_Q = \sqrt{\frac{E+M_Q}{2E}} \begin{pmatrix} \chi_s \\ 0 \end{pmatrix} e^{-i(Et-\vec{p}\cdot\vec{x})} \approx \psi_Q(M_Q \text{ large})$$

In a similar way, in a general Lorentz frame labeled by the velocity four-vector v^μ, the leading behaviour of the wavefunction would be obtained with a projection operator $\frac{1+\not{v}}{2}$, instead of $\frac{1+\gamma_0}{2}$.

$$\psi_Q \approx \left(\frac{1+\not{v}}{2}\right)\psi_Q$$

This leads to the definition of a new heavy quark field, where the leading mass dependence is removed

$$\psi_Q(x) \equiv e^{-iM_Q v\cdot x} h_v^Q(x)$$

Thus, we have $h_v^Q = \frac{1+\not{v}}{2} h_v^Q$ and the heavy quark is almost on-shell:

$$(\not{P}_Q - M_Q)\psi_Q = M_Q(\not{v}-1)\psi_Q \approx 0 \rightarrow (\not{v}-1)h_v^Q = 0 + O(k/M_Q)$$

With the new heavy field the Lagrangian becomes

$$\mathcal{L}_Q = \bar{\psi}_Q(i\not{D} - M_Q)\psi_Q = \bar{h}_v^Q\left[i\not{D} + M_Q(\not{v}-1)\right]h_v^Q \approx \bar{h}_v^Q i\not{D} h_v^Q + O(k/M_Q)$$

Then, the effective Lagrangian of the Heavy Quark Effective Theory reads

$$\boxed{\mathcal{L}_{HQET} = \bar{h}_v^Q iv\cdot D h_v^Q}$$

or

$$\mathcal{L}_{HQET} = \bar{h}_v^Q v^\mu\left[i\partial_\mu + gT_a A_{a\mu}\right]h_v^Q$$

in agreement with the Green functions writen above.

2 - SYMMETRIES

2.0 - Let us summarize some conclusions which can be taken from our preciding discussions:

→ The heavy quark is at rest in the hadron's center-of-mass, almost on-shell. This static source of gluons at the center of the hadron is characterized by the quark's Q flavor and color-SU(3) quantum numbers.

→ Spin effects, falling like g_s/M_Q (g_s = strong interactions coupling constant), become negligible, and the spin quantum number itself decouples in the large M_Q limit.

→ With $M_Q \rightarrow \infty$, the binding potential in the hadron is flavor independent.

→ The heavy-quark state is characterized by the quark's Q velocity v, rather than by its Q-flavor (*c*, *b* or *t*) mass. Thus, all momenta, and therefore energies and masses in HQET, are obtained (and measured) according to $M_Q v_\mu$. (This is in essence what has been called by Howard Georgi as a "velocity superselection rule").

The internal symmetries of the effective Lagrangian will be explicitely realized as peculiar degeneracies[1] in the spectrum and also as presumed relations between transition amplitudes of heavy quark hadrons.

2.1 - FLAVOR SYMMETRY

The Lagrangian for N heavy quarks with same velocity v is

$$\mathcal{L}_{HQET}^{(v)} = \sum_{j=1}^{N} \bar{h}_v^{(j)} iv\cdot D h_v^{(j)}$$

This Lagrangian is invariant under unitary flavor transformations

$$h_v^j \rightarrow U^{jk} h_v^k \qquad , \; U \in SU(N)$$

Flavor conservation is a good symmetry of strong interactions, corresponding to a subgroup $U(1)^N$. The new aspect of HQET is the nonabelian nature of the symmetry group, which leads to relations between the properties of hadrons of different heavy quark flavors, but with the same velocity.

For example, with the b and c quarks in HQET we have N = 2 and thus B and D mesons form a doublet under flavor SU(2) transformations. This flavor SU(2) is an approximate symmetry of QCD, but it is a good symmetry in HQET, as we have $m_c \gg \Lambda$, $m_b \gg \Lambda$. Thus, follows for instance the expectation of similarities in the spectroscopy of B and D mesons.

[1] About the words degeneracy or degenerate, please note in many HQET papers the following jargon: (example) "degenerate B and D mesons", actually meaning that: physically, both mesons differ in their masses by an amount around $m_b - m_c$.

2.2 - SPIN SYMMETRY

In HQET only the two upper components of the spinor ψ enter the Lagrangian. As $\mathcal{L}_{HQET}$ contains no Dirac matrices they are diagonal and we have gain an SU(2) spin symmetry.This spin symmetry is actually an internal symmetry, in a sense that no transformation on the coordinates is needed to have it.

The Lagrangian is invariant under transformations

$$h_v^Q \rightarrow (1+\vec{\varepsilon}\cdot\vec{S})h_v^Q$$

and the SU(2) spin symmetry generators

$$\vec{S} = \frac{1}{2}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$$

rotate the two components on the heavy quark wavefunction among them.

If we write the total angular momentum operator

$$\vec{J} = \vec{L} + \vec{S}$$

we see that $\vec{S}$ is the angular momentum operator for the heavy quark, and is also the spin-SU(2) generator, while $\vec{L}$ is the angular momentum generator related to the light degrees of freedom. Since the total angular momentum $\vec{J}$ is conserved, and $\vec{S}$ is conserved, $\vec{L}$ is also conserved and the states can be labeled by the quantum numbers (l, m_l; s ,m_s), with s = 1/2, m_s = ±1/2 only. Some spectroscopical implications will follow from this peculiarity in the classification os states.

→ The combination of the results exposed in the two last sections gives the symmetry group ruling the behaviour of N heavy fermions in HQET. Results:

$$\boxed{\text{SU(2N) spin-flavor symmetry}}$$

3 - SPECTRUM PREDICTIONS

Let us report in this section some spectroscopical results as examples of application of HQET.

1st. - Prediction: From spin symmetry , there is a degenerate doublet of states with J = j ± 1/2, for each j ≠ 0.

Expectation: For ground-state mesons with j = 1/2:

J = 0 → pseudoscalar mesons B, D

J = 1 → vector mesons B*, D*

expect: $M_{B^*} \cong M_B$, $M_{D^*} \cong M_D$

Experiment: M_{B^*} = 5,33 GeV, M_{D^*} = 2,01 GeV

M_B = 5,28 GeV, M_D = 1,87 GeV

($\Delta M_B \approx 0{,}1\%\ M_B$, $\Delta M_D \approx 7\%\ M_D$)

2nd - Prediction: From spin symmetry, for excited mesons with angular momentum l = 1

Expectation: For mesons j = 3/2:

J = 1 → meson D_1 (2420)

J = 2 → meson D_2^* (2460)

expect: $M_{D_2^*} \cong M_{D_1}$

Experiment: ΔM = 35 MeV (ΔM ≈ 1,4%M)

3rd - Prediction: The splittings of the B, B* mesons and of D, D* result from symmetry breaking effects and should be corrections of order Λ/M_Q to the HQET predictions.

Expectation:

$$\frac{M_{B^*}-M_B}{M_{D^*}-M_D} = \frac{\Lambda^2/m_b}{\Lambda^2/m_c} = \frac{m_c}{m_b} \approx \frac{M_D}{M_B} \cong \frac{1}{3}$$

Experiment: 0,36

4th - Prediction: From flavor symmetry, the excitation energy for states with different quantum numbers is independent of the heavy quark flavor.

Expectations: For different light quark flavor:

$$M_{B_s} - M_B \cong M_{D_s} - M_D \cong 100\ \text{MeV}$$

For different spin-parity:

$$M_{B_1} - M_B \cong M_{D_1} - M_D \cong 555\ \text{MeV}$$

Experiment: Waiting for observation in the b-systems

Etc.

4 - FURTHER APPLICATIONS

As we have seen, HQET is a powerful calculational tool, enabling the theorists to make predictions about the properties of heavy hadrons. Some of these properties are actually important in the context of the Standard Model (SM) physics. We list below some of these important topics, to which HQET has been applied.

1st. - Standard Model Physics: Predicts small CP-asymmetries in B-meson decays.
HQET: Study decays of heavy mesons and baryons using HQET to calculate the elements of the Kobayashi-Maskawa (KM) matrix accounting of CP-violation in the SM.

2nd. - SM Physics: The precise knowing of KM elements constrain the models that address to the family problem.
HQET: Again, check on the standard KM matrix elements; through the evaluation of semileptonic decays of $\overline{B}$ mesons $\overline{B} \to D\, e\, \upsilon$, $\overline{B} \to D^*\, e\, \upsilon$, which makes possible to extract the mixing angle $|V_{cb}|$.

3rd. - SM Physics: Some "magic" properties in semileptonic form-factors decompositions are observed. (The universality of the Isgur-Wise function $\xi(v \cdot v')$.)
HQET: Form-factors predictions for heavy mesons semileptonic decays. Better understanding of the properties appearing in the form-factors decompositions.

4th. - SM Physics: Meson decay constants, f_D and f_B have not been measured yet. Uncertain predictions exist from Lattice gauge theory and Sum rules calculations.
HQET: Predictions for f_D, f_{D^*}, f_B and f_{B^*} based on heavy quark symmetry.
One finds

$$\frac{f_B}{f_D} \cong \sqrt{\frac{M_D}{M_B}} \to f_B \cong 0{,}6 f_D$$

and from heavy quark spin symmetry it is expected

$$f_{D^*} = f_D + O(1/m_c)$$
$$f_{B^*} = f_B + O(1/m_b)$$

5th. - SM Physics: Rare decays processes start at 1-loop order and are good probes for new, non-SM physics.
HQET: Calculate the partial widths for rare decays of heavy mesons, such as $\Gamma(B \to K^*\gamma)$ or $\Gamma(B \to K^* l^+ l^-)$.

6th. - SM Physics: The $B^0 - \overline{B}^0$ mixing depends strongly on the top quark mass. (A large mixing parameter was observed in 1987, indicating a very heavy top quark.)
HQET: Calculate the decay rates in semileptonic decays $B \to D^{(*)} l\nu$, $\overline{B} \to D^{(*)} l\overline{\nu}$ and try to estimate the $B^0 - \overline{B}^0$ mixing.

7th. - Others:

heavy baryon decays
decays of excited states
strong decays of heavy hadrons
nonleptonic weak decays and factorization
combination of heavy-quark and chiral symmetry, etc.

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