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Model for asymptotic D-state parameters of light nuclei: application to ^4He *

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Model for asymptotic D-state parameters of light nuclei: Application to ^4He

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A simple method for calculating the asymptotic D state observables for light nuclei is suggested. The method exploits the dominant clusters of the light nuclei. The method is applied to calculate the ^4He asymptotic D to S normalization ratio ρ^α and the closely related D state parameter D_2^α. The study predicts a correlation between D_2^α and B_α, and between ρ^α and B_α, where B_α is the binding energy of ^4He. The present study yields $\rho^\alpha \simeq -0.14$ and $D_2^\alpha \simeq -0.12$ fm^2 consistent with the correct experimental η^D and the binding energies of the deuteron, triton, and the α particle, where η^d is the deuteron D state to S state normalization ratio.

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I. INTRODUCTION

The role of the deuteron asymptotic D to S normalization ratio η^d has been emphasized recently in making a theoretical estimate of the triton asymptotic D to S normalization ratio η^t[1,2]. There has been considerable interest for theoretical and experimental determination of the asymptotic D to S normalization ratio of light nuclei ever since Amado suggested that this ratio should be given the "experimental" status of a single quantity to measure the D state of light nuclei [3]. In this paper we generalize certain ideas used successfully in the two- and three-nucleon systems in order to formulate a model for the asymptotic D to S normalization ratio of light nuclei. We apply these ideas to the study of the asymptotic D to S normalization ratio, ρ^α, and the D state parameter, D_2^α, of ^4He.

Though a realistic numerical study of the asymptotic D to S normalization ratios of ^2H, ^3H, and ^3He is completely under control[4-9], the same can not be affirmed in the case of other light nuclei. Even in the case of ^4He, such a task, employing the Faddeev-Yakubovskii dynamical equations, is a formidable, but feasible, one. This is why approximate methods are called for. As the nucleon-nucleon tensor force plays a crucial and fundamental role in the formation of the D state of light nuclei[1,2,7], it is interesting to ask what are the dominant many body mechanisms that originate the D-state. The present study is aimed to shed light on the above questions.

In the case of the D state of the deuteron, exploiting the weak (perturbative) nature of the D state, Ericson and Rosa Clot[7] have demonstrated that the essential ingredients of the asymptotic D to S normalization ratio η^d are the long-range one-pion-exchange tail of the nucleon-nucleon interaction, the binding energy, B_d, and the S-state asymptotic normalization parameter (ANP), C_S^d, of the deuteron. In the case of ^3H we have seen that the long-range one-nucleon-exchange tail of the nucleon-deuteron interaction plays a crucial role in the formation of the trinucleon D state[1,2]. We have demonstrated that all realistic nucleon-nucleon potentials will virtually yield the same value of η^t provided that they also yield the same values for the S-state ANP and η^d of the deuteron and binding energies of ^2H and ^3H[1,2].

The purpose of the present study is to identify the dominant mechanisms for the formation of the D state in more complex situations. We do not consider the full dynamical problem for our purpose, but, rather, a cluster model exploiting the relevant long-range part of the cluster-cluster interaction supposed to be responsible for the formation of the relevant D state. The α particle or ^4He has a very important role in nuclear physics and a study of its structure deserves a special attention. One important aspect of its bound state is its D state admixture for the $^4He \to 2\,^2H$ channel. There have been a lot of theoretical and experimental activities for measuring the asymptotic D state to S state normalization ratio ρ^α for this channel. [11-21] In this paper we study the D state of ^4He and make a model independent estimate of ρ^α and the closely related parameter D_2^α.

All the observables directly sensitive to the tensor force of the nucleon-nucleon interaction, such as the deuteron quadrupole moment, Q^d, and η^t etc., have been found to be correlated in numerical calculations with C_S^d through the relation[1,2,7,10]

$$\frac{\mathcal{O}}{\eta^d} \sim (C_S^d)^2 f, \tag{1.1}$$

where $\mathcal{O}$ stands for Q^d, η^t, or the usual D state parameter, D_2^t, for the triton. The function f depends on the relevant binding energies, e.g., the binding energy of the deuteron in the case of Q^d, and the binding energies of the deuteron and triton in the case of η^t and D_2^t, while other low-energy on-shell nucleon-nucleon observables are held fixed. If correlation (1.1) were exact, no new information about the nucleon-nucleon interaction could be obtained from the study of Q^d, η^t, or D_2^t, which is not implicit in the values of B_d, B_t, C_S^d, and η^d.[1] However, this correlation is approximate and information about the nucleon-nucleon tensor interaction might be obtained from a study of these parameters from a breakdown of these correlations. In order that such informations could be extracted, however, one should require precise experimental measurements of these observables.[1, 2]

In this paper we shall be interested to see if correlation (1) extrapolates to the case of other light nuclei, specifically, to the case of ^4He. We provide a perturbative solution of the problem, which presents a good description of the D-state. We find that in order to reproduce the correct D-state parameters of ^4He, the minimum ingradients required of a model are the correct low-energy deuteron properties including C_S^d and η^d and the triton and ^4He binding energies, B_t and B_α.

The model also provides the essential behavior of D_2^α and ρ^α as function of the binding energy, B_α, of the α particle for fixed B_d, B_t, and η^d. Consistent with the experimental B_d, B_t, B_α, and η^d we find $\rho^\alpha = -0.14$, and $D_2^\alpha = -0.12$ fm^2. The model also predicts an approximate linear correlation between D_2^α (ρ^α) and B_α for fixed B_d, B_t, and η^d to be verified in realistic dynamical four-nucleon calculations.

The model for the formation of the D state is given in Sec. II. In Sec. III we present relevant notations for our future development of the D state. In Sec. IV the analytic model for the D state of ^4He is presented. Section V deals with the numerical investigation of our model. Finally, in Sec. VI brief summary and discussion are presented.

II. THE MODEL

As the exact dynamical studies of the D state for the light nuclear systems employing the connected kernel Faddeev-Yakubovskii equations are usually performed in the momentum space, we present our model in the momentum space in terms of the Green functions or propagators.

Figures 1 and 2 represent a coupled set of dynamical equations between clusters valid for ^2H and ^3H, respectively. In the case of the deuteron the dashed line denotes the exchanged meson. In the case of the triton the exchanged particle is a nucleon and the double line denotes a deuteron. In both cases a single line denotes a nucleon. In the case of the deuteron these equations are essentially the homogeneous version of the momentum space Lippmann-Schwinger equations for the nucleon-nucleon system, which couples the S and the D states of the deuteron. Explicitly, these equations are written as

$$g_0 = V_{00}G_0g_0 + V_{02}G_0g_2, \tag{2.1}$$

$$g_2 = V_{20}G_0g_0 + V_{22}G_0g_2, \tag{2.2}$$

where $g_l = V|\phi_l\rangle$ ($l = 0, 2$) represent the relevant form factors for the two states denoted by the two-body bound

state wavefunction ϕ_l, G_0 is the free Green function for propagation, and V's are the relevant potential elements between the S and the D states. Figure 1(b) gives the two ways of forming the D state at infinity: (a) in the first term on the right-hand side (rhs), the deuteron breaks up first into two nucleons in the S state which gets changed to two nucleons in the D state via the one-pion-exchange nucleon-nucleon tensor force, (b) in the second term on the rhs, the deuteron breaks up first into two nucleons in the D state which continues the same under the action of the central one-pion-exchange nucleon-nucleon interaction. As the D state of the deuteron could be considered to be a perturbative correction on the S state, in Fig. 1(b) the first term on the rhs is supposed to dominate, with the second term providing small correction. Hence, the essential mechanism for the formation of the D state in this case is given by the following equation

$$g_2 = V_{20}G_0g_0. \tag{2.3}$$

Given a reasonable g_0 and the tensor interaction V_{20}, Eq. (2.3) could be utilized for studying various properties of the D state. This equation should determine the asymptotic D to S ratio of deuteron η^d provided that the model has the correct deuteron binding B_d and the one-pion-exchange tail of the tensor nucleon-nucleon interaction.

In the momentum space representation of Eq. (2.3), at the bound state energy, g's have the following structure

$$\langle i\mu|g_l\rangle \sim C_l^d, \tag{2.4}$$

with $\mu = \sqrt{2m_RB_d}$, m_R being the reduced mass and C_l^d the deuteron ANP's for the state of angular momentum l. The off-diagonal tensor potential V_{20} is proportional to $g_{\pi N}^2$, where $g_{\pi N}$ is the pion-nucleon coupling constant. From Eqs. (2.3) and (2.4), at the bound state energy one has

$$\eta^d \sim g_{\pi N}^2 \times Int, \tag{2.5}$$

where Int represents a definite integral determined by the deuteron binding B_d. Hence η^d is mainly determined by the deuteron binding energy and the pion-nucleon coupling constant[7].

This idea could be readily generalized to more complex situations. In the case of the triton D-state, Fig. 2 and Eqs. (2.1) and (2.2) are valid. The form-factors g_l are to be interpreted as the triton-nucleon-deuteron form factors, the Green function G_0 represents the free propagation of the nucleon-deuteron system, and the potentials V_{02} and V_{20} are the Born approximation to the rearrangement nucleon-deuteron elastic sacttering amplitudes representing the transition between the relative S and D angular momentum states of the nucleon-deuteron system. For example, for nucleon-nucleon separable tensor potential, V_{02} corresponds to the inhomogeneous term of the Amado model[10] for nucleon-deuteron scattering for the transition between S and D states of the nucleon-deuteron system. The essential mechanism for the formation of the D-state is again given by Eq. (2.3). Now in the momentum space representation of Eq. (2.3), at the bound state energy, g's have essentially the structure given by

$$\langle i\mu|g_l\rangle \sim C_l^t, \tag{2.6}$$

where C_l^t is the ANP of the triton for the angular momentum state l. In Eq. (2.3), V_{02} connects a relative nucleon-deuteron S state to a nucleon-deuteron D state in different subclusters via a nucleon exchange. Hence the amplitude

V_{02} involves two form-factors, one for the deuteron S state and the other for the deuteron D state. Consequently, at the triton pole the momentum space version of Eq. (2.3) has the following form:

$$C_D^t \sim C_S^d C_D^d C_S^t \; Int, \tag{2.7}$$

where Int represents the remaining definite integral now expected to be determined essentially by the deuteron and triton binding energies and other low-energy nucleon-nucleon observables. Recalling that $\eta^t \equiv C_D^t/C_S^t$, with η^d defined similarly, Eq. (2.7) reduces to Eq. (1.1). Hence, this simple consideration shows that the ratio η^t/η^d is a universal one satisfying Eq. (1.1) determined essentially by the deuteron and triton binding energies and the deuteron S wave ANP C_S^d.

Next let us consider the example of ^4He, where the two deuterons could appear asymptotically either in a relative S or a D state. However, asymptotically the nucleon and the trinucleon could exist only in the relative S state. In this case the lowest scattering thresholds are the nucleon-trinucleon and the deuteron-deuteron ones. If we include these two possibilities of breakup of ^4He, then the principal mechanisms for the formation of the asymptotic deuteron-deuteron states are given in Fig. 3. We have two equations of the type shown in Fig. 3, one for the S state and the other for the D state. In Fig. 3 the contribution of the last term on the rhs is expected to be small. The virtual breakup of ^4He first to two deuterons and their eventual breakup to four nucleons to form the four-nucleon-exchange deuteron-deuteron amplitude as in this term is much less probable at negative energies than the virtual breakup of ^4He to a nucleon and a trinucleon and its eventual transformation to the deuteron-deuteron cluster as in the first term on the right-hand side of this equation. For this reason we shall neglect the last term of Fig. 3 in the present treatment. As in the three-nucleon case the amplitudes in Fig. 3 are the Born approximations to rearrangement amplitudes between diffenent subclusters, which connect different angular momentum states, e.g., S and D.

We notice that in the first term of Fig. 3 either of the vertices has to be a D state so that the passage from S to D state is allowed in this diagram. Consequently, at the pole of the ^4He bound state the momentum space version of Fig. 3 has two contributions corresponding to the deuteron (triton) vertex on the right hand side being the S-state and the triton (deuteron) vertex being the D-state so that we may write,

$$C_D^{\alpha\to dd} \sim C_S^{\alpha\to nt} C_S^d C_D^t \; Int_1 + C_S^{\alpha\to nt} C_D^d C_S^t \; Int_2, \tag{2.8}$$

where Int_1 and Int_2 are two definite integrals. The ^4He asymptotic D-state to S-state ratio ρ^α is defined by

$$\rho^\alpha \equiv \frac{C_D^{\alpha\to dd}}{C_S^{\alpha\to dd}}. \tag{2.9}$$

It is clear that, unlike in the case of triton, ρ^α is determined by two independent terms. Physically, it means that there are two mechanisms that construct the D state ANP of ^4He. Now recalling the empirical relation $\eta^t \equiv C_D^t/C_S^t \sim (C_S^d)^2\eta^d$, we obtain from Eq. (2.8)

$$\frac{\rho^\alpha}{\eta^d} \sim \xi_S \; Int, \tag{2.10}$$

where ξ_S is determined by the S-state asymptotic normalizations $C_S^{\alpha\to dd}$, $C_S^{\alpha\to nt}$, C_S^t, C_S^d, and Int represents integrals which are essentially determined by the binding energies B_d, B_t and B_α. Hence, in the case of ^4He Eq. (1.1) gets modified to one of the form of Eq. (2.10).

III. DEFINITIONS AND NOTATIONS

In this Section we present notations and definitions which we shall use for future development. The asymptotic wavefunction for a two-body bound state, ϕ_l, (binding energy B) in a potential V is given by

$$\lim_{r\to\infty} \langle rlj|\phi_l\rangle = -\frac{\sqrt{2\pi}\, m_R\, e^{-\mu r}}{r} \lim_{q\to i\mu} \langle qlj|V|\phi_l\rangle, \tag{3.1}$$

where l is the relative orbital angular momentum, j is the total final spin of the system (the intrinsic spin of the system is not shown) and $|qlj\rangle$ is the momentum space wave function. The asymptotic normalization parameter C_{jl} for this state is defined by

$$\lim_{r\to\infty} \langle rlj|\phi_l\rangle = \frac{C_{jl}\sqrt{2\mu}\, e^{-\mu r}}{\sqrt{N}\, r}. \tag{3.2}$$

Here N represents the number of ways a particular asymptotic configuration can be constructed from its constituents in the same channel. For example, in the channel, $^3H \to n + {}^2H$, as we can combine the proton with either of the two neutrons to form ^2H, $N = 2$. Similarly in the $^4He \to 2\,{}^2H$ channel, the deuteron can be formed in two different ways and $N = 2$. But in the channels $^4He \to {}^3H + {}^1H$ and $^4He \to n + {}^3He$, neglecting Coulomb interaction, there are four different possibilities for constructing the outgoing channel components, so that $N = 4$.

From Eqs. (3.1) and (3.2), we obtain

$$C_{jl} = -\frac{m_R\sqrt{\pi N}}{\sqrt{\mu}} \lim_{q\to i\mu} \langle qlj|V|\phi_l\rangle. \tag{3.3}$$

As the partial wave t matrix may be expressed as

$$\langle qlj|t|qlj\rangle = \lim_{q\to i\mu} \frac{|\langle qlj|V|\phi_l\rangle|^2}{E+B}, \tag{3.4}$$

the parameter C_{jl} is related to the residue at the t-matrix pole by

$$\langle qlj|t|qlj\rangle_{Res} \equiv \lim_{q\to i\mu} |\langle qlj|V|\phi_l\rangle|^2 = \frac{\mu}{\pi N m_R^2} C_{jl}^2 \tag{3.5}$$

With this definition, in the limit of $\mu \to 0$, $C_{jl} \to 1$.[1]

For the two-particle bound state, the vertex function $g(q)$ for a definite angular momentum l (and j) can be written as

$$g_{jl}(q^2) = -\sqrt{\frac{\mu}{\pi N m_R^2}}\, C_{jl} \left(\frac{q}{i\mu}\right)^l \hat{g}(q^2), \tag{3.6}$$

where the kinematical factor which takes into account the centrifugal barrier has been explicitly shown. The function $\hat{g}(q^2)$ essentially provides the momentum dependence of the vertex function. In the present qualitative study we set $\hat{g}(q^2) = 1$ so that we have

$$g_{jl}(q^2) = -\sqrt{\frac{\mu}{\pi N m_R^2}}\, C_{jl}\,(\frac{q}{i\mu})^l\,. \tag{3.7}$$

In Eq. (3.7) apart from a kinematical factor that takes into account the centrifugal barrier, the form factor is assumed to be independent of the relative momentum of the two components forming the bound state, consistent with the minimal three body model [1]. In particular, the form factors for the formation of ^4He from nucleon(N)−triton(t) channel and from two deuterons(dd) are respectively,

$$g_{00}^{Nt}(q) = -\frac{2}{3}\frac{\sqrt{\mu_{Nt}}}{\sqrt{\pi}} C_S^{\alpha\to Nt}, \tag{3.8}$$

and

$$g_{jl}^{dd}(q) = -\frac{\sqrt{\mu_{dd}}}{\sqrt{2\pi}} C_{jl}^{\alpha\to dd}(\frac{q}{i\mu_{dd}})^l, \tag{3.9}$$

with

$$\mu_{Nt} = \sqrt{\frac{3(B_\alpha - B_t)}{2}} \quad , \quad \mu_{dd} = \sqrt{2(B_\alpha - 2B_d)}, \tag{3.10}$$

where B_α and B_d are the binding energies of ^4He and ^2H respectively and we assume $\hbar = m_{nucleon} = 1$.

For the case where angular momentum states S and D states are mixed, the probability amplitude for a given l-value is proportional to the corresponding spherical harmonic $Y_{lm_l}(\hat{q})$. Defining the spin-angular momentum functions $\mathcal{Y}_{sljm}(\hat{q})$ as

$$\mathcal{Y}_{lsjm}(\hat{q}) = (Y_l(\hat{q}) \bigotimes \psi_s)_{jm} = \sum_{m_s,m_l} C^{lsj}_{m_l m_s m}\, Y_{lm_l}(\hat{q})\psi_{sm_s}, \tag{3.11}$$

where ψ_s is the spin state of the system, $\otimes$ denotes angular momentum coupling, the vertex functions in the minimal model for $t \to Nd$ and $d \to NN$ vertices take the form

$$g^{Nd}_{j=\frac{1}{2},m}(\vec{p}_1) = -\frac{3}{2}\sqrt{\frac{\mu_{Nd}}{2\pi}} C_S^t \left[\mathcal{Y}_{0\frac{1}{2}\frac{1}{2}m}(\hat{p}_1) - \frac{\eta^t p_1^2}{\mu_{Nd}^2}\mathcal{Y}_{2\frac{3}{2}\frac{1}{2}m}(p_1)\right] \tag{3.12}$$

$$g^{NN}_{j=1,m}(\vec{p}_3) = -\sqrt{\frac{4\mu_{NN}}{\pi}} C_S^d \left[\mathcal{Y}_{011m}(\hat{p}_3) - \frac{\eta^d p_3^2}{\mu_{NN}^2}\mathcal{Y}_{211m}(\hat{p}_3)\right] \tag{3.13}$$

Here C_S^d and C_S^t are the asymptotic normalization parameters for the S state of the deuteron and triton respectively, whereas η^d and η^t are the ratios of corresponding D-state ANP's with the S-state ANP's. The relative momentum of the nucleon with respect to the deuteron is $\vec{p}_1$, whereas the relative momentum of the two nucleon system is $\vec{p}_3$.

IV. D-STATE PARAMETERS OF ^4HE

Having defined the relevant vertex functions, we can write down the equation for constructing the S and D states of ^4He in the $^4He \to 2\,^2H$ channel. The first line of Fig. 3 represents diagrammatically the present model for the formation of the asymptotic S and D states of ^4He. Using the notation of previous Sec. the explicit partial wave form of the present model could be written down as:

$$\begin{aligned} g^{dd}_{0l_3}(q_3) = &\int_0^\infty dq_1 q_1^2 \int\int d\Omega_{q1} d\Omega_{q3} \sum_{L1,L3} \left(\left(\hat{g}^{NN}_{1L_3}(\vec{p}_3) \bigotimes \chi^d_1\right)^*_{l_3} \bigotimes Y^*_{l_3}(\hat{q}_3)\right)_{00} \\ &\times \frac{1}{B_d - B_\alpha - q_1^2 - \frac{3}{4}q_3^2 - \vec{q}_1\cdot\vec{q}_3} \left(\left(\hat{g}^{Nd}_{\frac{1}{2}L_1}(\vec{p}_1) \bigotimes \chi^N_{\frac{1}{2}}\right)_0 \bigotimes Y_0(\hat{q}_1)\right)_{00} \\ &\times \frac{1}{B_t - B_\alpha - \frac{2}{3}q_1^2} g^{Nt}_{00}(q_1), \end{aligned} \tag{4.1}$$

where $\vec{p}_1 = -(\frac{2}{3}\vec{q}_1 + \vec{q}_3)$ is the relative momentum between the the exchanged nucleon 2 and the structureless deutron 3, $\vec{p}_3 = (\vec{q}_1 + \frac{\vec{q}_3}{2})$ is the relative momentum between the spectator nucleon 1 and nucleon 2, q_1 is the momentum carried by nucleon 1 and q_3 is the momentum carried by the structureless deutron of momentum q_3. The indices 1, 2 refer to the spectator nucleon, the exchanged nucleon, and 3 refers to the structureless deuteron. Here l_3 is the angular momentum state of ^4He; $l_3 = 0$ (2) corresponds to the S (D) states of ^4He. L_3 is the relative angular momentum of the two nucleons forming the deuteron of momentum $-q_3$, L_1 is the relative angular momentum of the structureless deuteron of momentum q_3 and the nucleon 2 forming the triton of momentum $-q_1$, $\chi^N_{\frac{1}{2}}$ is the spin state of nucleon 1 with momentum q_1, and χ^d_1 is the spin state of the structureless deuteron with momentum q_3. We dropped the index m of the form-factors at NN, Nd and Nt vertices, because of the angular momentum coupling notation employed. Here $\hat{g}_{jL}$ is the L component of the vertex defined in Eqs. (3.12) and (3.13). For example,

$$\hat{g}^{NN}_{1L_3}(\vec{p}_3) = -\sqrt{\frac{4\mu_{NN}}{\pi}} C^d_{L_3} \mathcal{Y}_{L_3 11m}(\hat{p}_3), \tag{4.2}$$

where $C^d_{L_3} = C^d_S$ ($-C^d_S \eta^d p_3^2/\mu_{NN}^2$) for $L_3 = 0$ (2).

The rhs of Eq. (4.1) is the first term on the rhs of Fig. 3. The term g^{Nt}_{00} is the Nt form factor, $(B_t - B_\alpha - \frac{2}{3}q_1^2)^{-1}$ represents the propagation of the two-particle triton-nucleon state at a four particle energy $E = -B_\alpha$, the energy for propagation of the two-particle triton-nucleon state being $(B_t - B_\alpha)$. The term $(B_d - B_\alpha - q_1^2 - 3q_3^2/4 - \vec{q}_1\cdot\vec{q}_3)^{-1}$ represents the propagation of the three-particle nucleon-nucleon-deuteron state at a four particle energy $E = -B_\alpha$, the energy for propagation of the three-particle nucleon-nuclon-deuteron state being $(B_d - B_\alpha)$. There are two angular momentum-spin coupling coefficients. The one involving $\hat{g}^{Nd}_{1/2L_1}$ gives the angular momentum coupling to form the triton and its coupling to nucleon 1 to give the final zero total angular momentum of ^4He. The one involving $\hat{g}^{NN}_{1L_3}$ gives the spin-angular momentum coupling of nuclons 1 and 2 to form the deuteron of momentum $-q_3$ and its coupling to the structureless deuteron 3 to give the final zero total angular momentum of ^4He. Finally, there is summation over the internal angular momenta L_1 and L_3, and integrations over the internal loop momentum $\vec{q}_1$ and angles of $\vec{q}_3$.

Substituting the values of vertex functions and rearranging Eq. (4.1), we obtain the properly normalized function $\Lambda^{dd}_{\ell_3}(q_3)$ given by

$$\Lambda^{dd}_{l_3}(q_3) \equiv g^{dd}_{0l_3}(q_3)\sqrt{\frac{2\pi}{\mu_{dd}}} = \frac{2C^d_S C^t_S C^{\alpha\to Nt}_S}{\pi}\sqrt{\frac{\mu_{NN}\mu_{Nd}\mu_{Nt}}{\mu_{dd}}} \int_0^\infty dq_1 q_1^2 \frac{\mathcal{I}_{l_3}(q_3, q_1)}{B_t - B_\alpha - \frac{2}{3}q_1^2} \tag{4.3}$$

such that at the ^4He pole it gives the asymptotic normalization parameters of ^4He:

$$\Lambda^{dd}_{\ell_3}(i\mu_{dd}) = C^{\alpha\to dd}_{\ell_3}.$$

In Eq. (4.3)

$$\mathcal{I}_{l_3}(q_3,q_1) = \int d\Omega_{q_3} d\Omega_{q_1} \left((\hat{g}^{NN}_{1L_3}(\vec{p}_3) \otimes \chi^d_1)^*_{l_3} \otimes Y^*_{l_3}(q_3) \right)_{00}$$
$$\times \frac{1}{B_d - B_\alpha - q_1^2 - \frac{3}{4}q_3^2 - \vec{q}_1 \cdot \vec{q}_3} \left(\left(\hat{g}^{Nd}_{\frac{1}{2}L_1}(\vec{p}_1) \otimes \chi^N_{\frac{1}{2}} \right)_0 \otimes Y_0(\hat{q}_1) \right)_{00} \tag{4.4}$$

The values of $l_3 = 0, 2$ yield the S and D state of ^4He respectively. For evaluating the integral $\mathcal{I}$, we expand the energy propagator in terms of spherical harmonics as below,

$$\frac{1}{B_d - B_\alpha - q_1^2 - \frac{3}{4}q_3^2 - \vec{q}_1 \cdot \vec{q}_3} = 2\pi \sum_{L=0}^{\infty} (-1)^L K_L(q_1,q_3) \sqrt{2L+1} \, (Y_L(\hat{q}_1) \times Y_L(\hat{q}_3))_{00}, \tag{4.5}$$

where

$$K_L(q_1,q_3) = \int_{-1}^{1} \frac{P_L(x)\,dx}{B_d - B_\alpha - q_1^2 - \frac{3}{4}q_3^2 - q_1 q_3 x}. \tag{4.6}$$

By using the angular momentum algebra techniques [22], the intrinsic spin dependence of the integrand and the part containing the spherical harmonics in Eq. (4.3) are easily separated out and evaluated independent of each other. Next the same procedure is adopted to separate the q_1 and q_3 dependent parts of the integrand. After integrating over angles we get the following result for $\Lambda^{dd}_{l_3}$

$$\Lambda^{dd}_{l_3}(q_3) = \frac{C^d_S C^t_S C^{\alpha\to Nt}_S}{\pi} \sqrt{\frac{\mu_{NN}\,\mu_{Nd}\,\mu_{Nt}}{\mu_{dd}}} \int_0^\infty dq_1 \frac{q_1^2}{B_t - B_\alpha - \frac{2}{3}q_1^2} (-1)^{l_1+\frac{L_1}{2}+1+L_3-\beta-\gamma+S_1-\frac{1}{2}}$$
$$\times \sum_{L_1,L_2,\alpha,\beta,\gamma} \sum_{L=0}^{\infty} K_L(\vec{q}_3,\vec{q}_1) \left(\frac{\eta_d}{\mu^2_{NN}}\right)^{\frac{L_2}{2}} \left(\frac{\eta_t}{\mu^2_t}\right)^{\frac{L_1}{2}} \frac{q_1^{\alpha+\beta}\, q_3^{L_1+L_3-\alpha-\beta}}{3^\beta} \frac{2^{\alpha+\beta-L_3+1}}{}$$
$$\times C^{\beta L\alpha}_{000} C^{L_1-\beta L\gamma}_{000} C^{L_3-\alpha l_3\gamma}_{000} U(1L_3 1l_3; 1L_1)$$
$$\times U(\frac{1}{2}\frac{1}{2}L_1 1; 1S_1) U(\alpha L_3 - \alpha L_1 l_3; L_3\gamma) U(\alpha\gamma\beta L_1 - \beta; L_1 L)$$
$$\times \left[\frac{(2\beta+1)(2l_3+1)}{(2L+1)(2\gamma+1)}\right]^{\frac{1}{2}} \left[\frac{(2L_3+1)!2L_1!}{(2\alpha+1)!(2L_3-2\alpha)!(2\beta+1)!(2L_1-2\beta)!}\right]^{\frac{1}{2}}. \tag{4.7}$$

Here $U(j_1j_2j_3j_4;JK) \equiv (2J+1)(2K+1)W(j_1j_2j_3j_4;JK)$ are renormalized $6j$ symbols.

As $L_1 = 0$ or 2 and $L_3 = 0$ or 2, the left hand side of the above equation contains four terms. We retain the three terms linear in D-state and neglect the term containing a product of η^d and η^t. [In the limit $q_3 \to i\mu$, analytic expressions are easily obtained for $K_L(q_1,q_3)$ (relevant L values in the present context being $L = 0,1,2$).] The asymptotic D state to S state ratio for ^4He is defined by

$$\rho^\alpha \equiv \frac{\Lambda^{dd}_2(i\mu_{dd})}{\Lambda^{dd}_0(i\mu_{dd})}. \tag{4.8}$$

After substituting numerical values of various angular momentum coupling coefficients for allowed values of angular momenta in Eq. (4.7), we evaluate ρ^α as,

$$\rho^\alpha = \mu^2_{dd}\left(\frac{\eta^d}{4\mu^2_{NN}} - \frac{\eta^t}{\mu^2_{nd}}\right) - i\mu_{dd}\frac{F_1}{F_0}\left(\frac{\eta^d}{\mu^2_{NN}} - \frac{4}{3}\frac{\eta^t}{\mu^2_{Nd}}\right) - \frac{F_2}{F_0}\left(\frac{\eta^d}{\mu^2_{NN}} - \frac{4}{9}\frac{\eta^t}{\mu^2_{Nd}}\right), \tag{4.9}$$

where

$$F_L = \int_0^\infty \frac{dq_1 q_1^{L+2}}{B_t - B_\alpha - \frac{2}{3}q^2,} K_L(q_1, i\mu_{dd}).$$

Similarly, the D^α_2 parameter of ^4He is defined as

$$D^\alpha_2 = -\lim_{q_3\to 0} \frac{g^{dd}_{02}(q_3)}{q_3^2\, g^{dd}_{00}(q_3)} \equiv -\lim_{q_3\to 0} \frac{\Lambda^{dd}_2(q_3)}{\Lambda^{dd}_0(q_3)\, q_3^2}. \tag{4.10}$$

The integrals appearing in Eq. (4.10), are preformed analytically for $q_3 \to 0$ and the result for D^α_2 is

$$D^\alpha_2 = \left(\frac{\eta^d}{4\mu^2_{NN}} - \frac{\eta^t}{\mu^2_{Nd}}\right) - \frac{1}{6}\frac{2\mu_{Nt} + \sqrt{B_\alpha - B_d}}{\mu_{Nt} + \sqrt{B_\alpha - B_d}}\left(\frac{\eta^d}{\mu^2_{NN}} - \frac{4}{3}\frac{\eta^t}{\mu^2_{Nd}}\right)$$
$$+ \frac{2}{15}\frac{\mu^2_{Nt} + \frac{9}{8}\mu_{Nt}\sqrt{B_\alpha - B_d} + \frac{3}{8}(B_\alpha - B_d)}{(\mu_{Nt} + \sqrt{B_\alpha - B_d})^2}\left(\frac{\eta^d}{\mu^2_{NN}} - \frac{4}{9}\frac{\eta^t}{\mu^2_{Nd}}\right). \tag{4.11}$$

Equations (4.9) and (4.11) are the principal results of the present study.

V. NUMERICAL RESULTS

The numerical results for the D state parameters of ^4He based on Eqs. (4.9) and (4.11) are expected to be more reasonable than that for ^3H of Ref. [1] because of three reasons. Firstly, the approximate analytical treatment of Ref. [1] employing the diagramatic equation of Fig. 2 for ^3H is more approximate than the present treatment employing Fig. 3 for ^4He. This is because in the former case the neglect of the spin singlet two nucleon state as an intermediate state is too drastic; whereas in the latter case there are no other competing channels if we permit only exchange of one nucleon as shown in Fig. 2. The exchange of two nucleons is possible but is much less likely and is usually neglected in the treatment of four nucleon dynamics [23] Secondly, the minimal cluster model we are using is expected to work better when the nucleus is strongly bound and the constituents (^2H and ^3H) are loosely bound. As ^4He is strongly bound this approximation is more true in ^4He than in ^3H. Finally and most importantly, in the present model we are taking the different vertices to be essentially constants as in Eqs. (3.13) and (4.1) which corresponds to taking the vertex form factors unity. This reduces the dynamical equations essentially to algebraic relations between the asymptotic normalization parameters. In so doing systematic errors are introduced. The calculation of the triton asymptotic D to S ratio η^t in Ref. [1] will have the above error. But ^4He asymptotic D to S ratio ρ^α of Eq. (4.9) and D^α_2 of Eq. (4.11) are obtained by dividing two equations of type (4.4) – one for $l_3 = 0$ and the other for $l_3 = 2$ – where exactly identical approximations are made. This division is expected to reduce the above systematic error and Eqs. (4.9) and (4.11) are likely to lead to a more reliable estimate of ^4He D state compared to the estimate of ^3H D state obtained in Ref.[1].

Equation (4.9) or (4.11) yields that for fixed B_t, B_d, η^d, and η^t, ρ^α and D^α_2 are correlated with B_α. Specification of B_α alone is not enough to determine the ^4He D state parameters. We have established in Refs. [1, 2, 12] that in a dynamical calculation η^t is proportinal to η^d for fixed B_d and B_t, from which the theoretical estimate of η^t/η^d was made. This was relevant because of the uncertainty in the experimental value of η^d. If this result is used in Eqs. (4.9) or (4.11) it follows that ρ^α and D^α_2 are proportional to η^d for fixed B_d, B_t, and B_α.

Next the results of the present calculation using Eqs. (4.9) and (4.11) are presented. In Eqs. (4.9) and (4.11), in actual numerical calculation both η^d and η^t are taken to be positive. The positive sign of η^t is consistent with the order of angular momentum coupling we use in the present study [12]. In Fig. 4 we plot ρ^α versus B_α calculated using Eq. (4.9) for different values of B_t and for $\eta^d = 0.027$ and $B^d = 2.225$ MeV. The value $\eta^d = 0.027$ is the average

experimental value reported in Ref. [7]. There is a recent experimental finding: $\eta^d = 0.0256$ in Ref. [9]. For the present illustration we shall, however, use $\eta^d = 0.027$. Though the final estimate of the asymptotic D state parameters of ^4He will depend on the value of η^d employed, the general conclusions of this paper will not depend on the choice of this experimental value of η^d. The five lines in this figure correspond to $B_t = 7.0, 7.5, 8.0, 8.5$, and 9.0 MeV. The numerical value of η^t for a particular B_t is taken from the correlation in Ref. [1]. We find that the magnitude of ρ^σ increases with the increase of B_α for a fixed B_t and with the decrease of B_t for a fixed B_α. This should be compared with the correlation of η^t with B_t in Ref. [1]. We also calculated D_2^σ using Eq. (4.11) for different values of B_t and B_α.

More results of our calculation using Eqs. (4.9) and (4.11) are exhibited in Table 1. We employed different values of B_t and B_α. The value $B_\alpha = 28.3$ MeV and $B_t = 8.48$ MeV are the experimental values. The other values of B_α and B_t are considered as they are identical with results of theoretical calculation of Ref. [20]. As the values of the binding energies are crucial[1, 2] for a correct specification of the D state parameters we decided to consider these binding energies obtained in Ref. [20]. For example, $B_t = 8.15$ MeV is the mean of ^3H and ^3He binding energies obtained in Ref.[20] with the Urbana potential. For the same potential they obtained $B_\alpha = 28.2$ MeV, and $D_2^\sigma = -0.24$ fm^2, to be compared with the present $D_2^\sigma = -0.15$ fm^2. For the Argonne potential they obtained mean $B_t = 8.04$ MeV, $B_\alpha = 27.8$ MeV, and $D_2^\sigma = -0.16$ fm^2, to be compared with the present $D_2^\sigma = -0.12$ fm^2. But the large change of D_2^σ in Ref. [20] from one case to the other is in contradiction with the present study. The first row of Table 1 is the result of our calculation for ρ^σ and D_2^σ consistent with the correct experimental B_t and B_α and using $\eta^d = 0.027$, $\eta^t/\eta^d = 1.68$:

$$\begin{aligned} &\rho^\sigma \simeq -0.14, \\ &D_2^\sigma \simeq -0.12 fm^2, \\ &\rho^\sigma/\mu_{dd}^2 D_2^\sigma \simeq 1. \end{aligned} \tag{5.1}$$

In Ref. [24] it has been estimated that $\rho^\sigma/(\mu_{dd}^2 D_2^\sigma) \simeq 0.9$ in agreement to the present finding.

Next we would like to compare the present result with other ('experimental') evaluations of these asymptotic parameters. Santos et al. [14] evaluated ρ^σ from an analysis of tensor analyzing powers for (d,α) reactions on S and Ar. They employed a simple one step transfer mechanism, plane-wave scattering states, zero-range or asymptotic bound states. Keeping only the dominant angular momentum states for the transferred deuteron they predicted $\rho^\sigma = -0.21$.

In another study Santos et al.[15] considered the tensor analyzing power of reaction $^2H(\vec{d},\gamma)^4He$ and concluded that agreement with experiment could be obtained for $-0.5 < \rho^\sigma < -0.4$.

Karp et al.[16] studied tensor analyzing powers for (d,α) reactions on ^{89}Y. They employed simple shell-model configurations for the nuclei involved, performed a finite-range DWBA calculation, and represented the $^4He \to 2^2H$ overlap by an effective two-body model. From an analysis of the experimental data the authors concluded $D_2^\sigma = -0.3 \pm 0.1$ fm^2.

Tostevin et al.[17] studied tensor analyzing powers for (d,α) reactions on ^{40}Ca. They performed a DWBA calculation with local energy approximation and concluded $D_2^\sigma = -0.31$ fm^2 and $\rho^\sigma = -0.22$. But Tostevin[18] has later warned that this value of D_2^σ may have large error.

Merz et al.[19] has performed an analysis in order to make a more reliable estimate of these parameters. From a study of the $^{40}Ca(d,\alpha)^{38}K$ reaction at 20 MeV bombarding energy employing a full finite-range DWBA calculation they predicted $D_2^\sigma = -0.19 \pm 0.04$ fm^2.

Recently, Piekarewicz and Koonin[21] performed a phenomenological fit to the experimental data of the $^2H(d,\gamma)^4He$ reaction and predicted $\rho^\sigma = -0.4$. From a study of cross section of the same reaction, however, Weller et al. [13] predicted $\rho^\sigma = -0.2 \pm 0.05$.

Cosidering the qualitative nature of the present study we find that there is reasonably good agreement between the present and other studies. This assures that we have correctly included the essential mechanisms of the formation of the D state.

Unlike in the case of ^3H, there are two distinct ingredients for the formation of the D state of ^4He: η^t and η^d. This is clear from expressions (4.9) and (4.11). This possibility did not exist in the case of ^3H where η^t is determined uniquely by η^d apart from the binding energies. In the usual optical potential study of the D state of ^4He as in Ref. [24] the dependence of ρ^σ on η^t is always neglected. This dependence will be explicit in a microscopic four-particle treatment of ^4He. Such microscopic calculations are welcome in the future for establishing the conclusions of the present study.

VI. SUMMARY

We have calculated in a simple model the asymptotic D state parameters for ^4He. The present investigation generalizes the consideration of universality as presented in Refs. [1] and [2] for the trinucleon system. The universal trend of the theoretical calculations on the trinucleon system and the consequent correlations are generalized here to the case of the D-state observables of ^4He. The essential results of our calculation appear in Eq. (5.1). We have used a minimal cluster model in our calculation where essentially the bound state form factors are neglected, thus transforming the dynamical equation into an algebraic relation between the different asymptotic parameters. Dividing two such equations – one for the asymptotic S state of ^4He and other for the asymptotic D state of ^4He – the estimates of Eq. (5.1) are arrived. As we have pointed out in Sec. V, such a division should reduce the systematic error of the approximation scheme. Dynamical calculation using realistic four-body model should be performed in order to see whether the present estimate (5.1) is reasonable. At the same time accurate experimental results are called for. We have predicted correlations between ρ^σ and B_α, and between D_2^σ and B_α to be found in actual dynamical calculations. Such correlations though appear to be extremely plausible in view of the calculation of η^t of [1], can only be verified after performing actual dynamical calculations.

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Table 1

B_α (MeV)	B_t (MeV)	η^d	η^t	D_2^α (fm^2)	ρ^α	$\frac{\rho^\alpha}{\mu_{dd}^2 D_2^\alpha}$
28.3	8.48	0.027	0.0454	-0.12	-0.14	1.07
28.2	8.15	0.025	0.0514	-0.15	-0.17	0.98
27.8	8.04	0.0266	0.0430	-0.12	-0.14	1.05

Table Caption: Results for D state parameters of ^4He calculated using Eqs. (33) and (35).

Figure Captions

1. The coupled Schrödinger equation for the formation of the D state in ^2H.

2. The coupled Schrödinger equation for the formation of the D state in ^3H.

3. The present model for the formation of the S and D states in ^4He. In numerical calculation only the first term on the right hand side of this diagram is retained.

4. The ρ^α versus B_α correlation for fixed B_t = 7, 7.5, 8, 8.5, and 9 MeV and with η^d = 0.027 using Eq. (33). The curves are labelled by the E_t values. The η^t values for each line are taken from the η^t versus E_t correlation of Ref. [1].

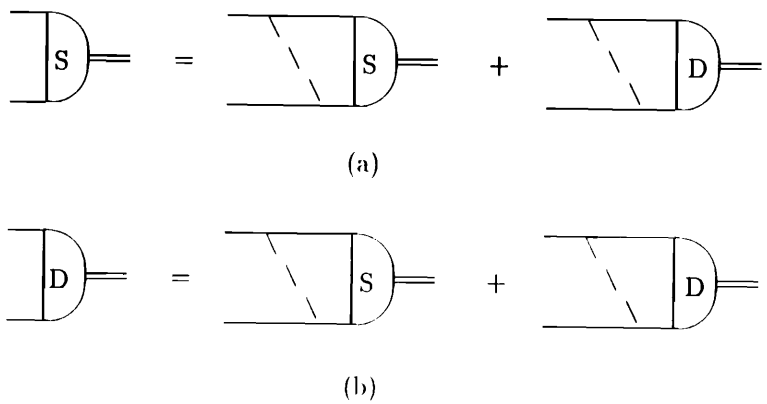

Fig. 1

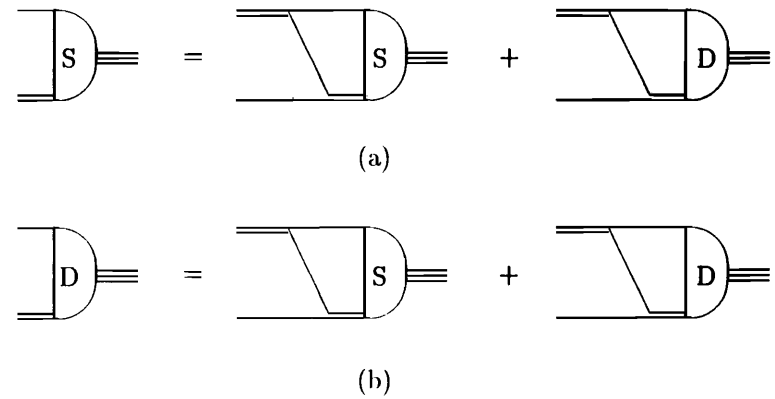

Fig. 2

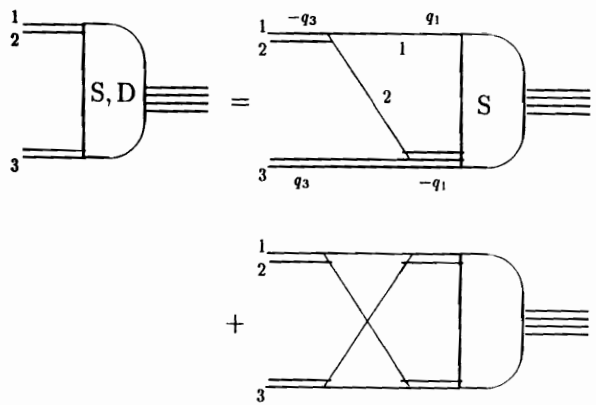
S, D
S
1
2
3
$-q_3$
q_1
q_3
$-q_1$

Fig: 3

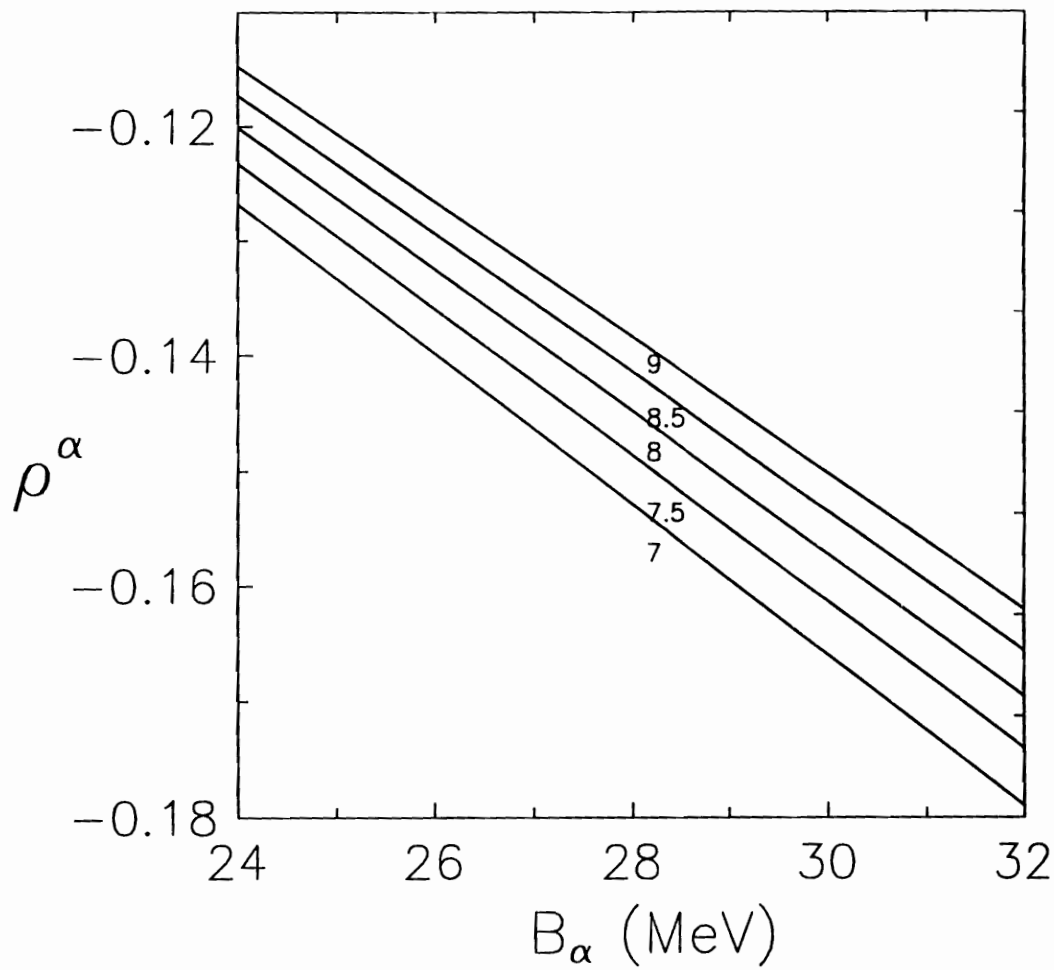
ρ^{α}
B_{α} (MeV)
−0.12
−0.14
−0.16
−0.18
24
26
28
30
32
9
8.5
8
7.5
7

Fig. 4