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Null-plane approach to the neutral pion decay and transition form-factor

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Null-plane approach to the neutral pion decay and transition form-factor

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Abstract

We study the $\pi^o \rightarrow \gamma^*\gamma$ process using different model wave-functions in the null-plane. We compute the π^o decay and electromagnetic transition form-factor, and we found out that the π^o width has a strong model sensitivity and should be used as an important constraint to the model wave function.

The neutral pion electromagnetic transition form-factor of the reaction $\pi^o \rightarrow \gamma^*\gamma$ has been recently measured by the CELLO Collaboration [1]. They measured $F(k_1^2, k_2^2)$, where k_1^μ and k_2^μ are the photon four-momentum, for $k_2^2 \approx 0$ and in the space-like region of k_1^μ. The extracted value of the neutral pion radius, $r_{\pi^o} = 0.65 \pm 0.03\ fm$, is in remarkable agreement with the charged pion radius $r_{em} = 0.657 \pm 0.012\ fm$ [2]. This new set of data provides further tests of the description of the pion wave-function in relativistic models. In particular we are concerned with the pion wave-function defined in the null-plane [3-8].

The relativistic approach to the wave-function based only on constituents quarks is possible in the null-plane due to the suppression of pair creation process. This property arises from the particular choice of the null-plane coordinates [9,10]. Also the center of mass coordinate is easily separated [3]. In specific processes involving a bound-state, the internal loop-momentum is integrated first in the null-plane energy, then the wave-function of the bound-state appears naturally [9a]. This procedure is the essence of the "diagrammatic approach" that was applied in ref. 8 to obtain the weak decay constant and electromagnetic form-factor of the charged pion. In this reference, it was used one-loop diagrams; the triangle diagram for the form-factor and the bubble diagram which expresses the Partial Conservation of the Axial Current (PCAC) [11]. The important point is the convergence of the null-plane energy integration. This is the case for the "good component of the current" [10]. For the weak decay constant, this integration is also convergent [8].

In ref. 8, the null-plane quark-antiquark bound state wave-function was introduced in the calculation of the electromagnetic form-factor of the pion, starting from the triangle diagram with constant pion vertex. The pion wave-function in the null-plane, for a constant vertex, goes asymptotically as the inverse of the difference between the squares of the pion and the free quark-antiquark pair masses. Then, it was obtained the normalization factor for the replacement of the asymptotic wave-function by a model wave-function in the expressions of the form-factor and weak decay constant. With the substitution, the formulae were the same as those obtained in the Hamiltonian Front-Form dynamics [6,7].

Our aim in this work is to study the π^o observables. We construct the electromagnetic transition form-factor of the neutral pion from the one-loop quark diagram with two external photons. As far we know, this observable has not yet been studied in the phenomenological null-plane approach. We follow the same procedure of ref.8. The integration over the null-plane energy in the triangle diagram is performed and the asymptotic wave-function of the bound quark-antiquark pair is replaced by phenomenological

pion wave-functions. We use three distinct model wave-functions: *i*) the Gaussian; [5-7]; *ii*) the hydrogen-atom [8] and *iii*) the wave-function model of ref. 12. This last model has the two characteristics that one believes belongs to Quantum Chromodynamics, i.e., confinement and short distance one gluon exchange. The Gaussian model has only the property of confinement and the Hydrogen model mimics the one gluon exchange at short distances. We study the consistence of the description of the recent neutral pion data and charged pion data within the relativistic approach.

In the next, we present the formulation of the neutral pion electromagnetic transition form-factor in the null-plane approach, followed by our numerical results for the transition form-factor. Finally we present our main conclusions.

We begin introducing the pseudo-scalar coupling of the pion to the constituents quarks, which can be expressed through the following effective interaction Lagrangian:

$$\mathcal{L}^{int}_{\pi q} = -i\frac{M}{f_\pi}\vec{\pi}.\,\bar{q}\gamma^5\vec{\tau}q\ , \tag{1}$$

where M is the constituent quark-mass, $f_\pi = 93MeV$ is the weak decay constant of the pion, $\vec{\pi}$ and q are respectively the pion field and quark wave function. Our units are such that $\hbar = c = 1$.

The amplitude for the electromagnetic process $\pi^o \to \gamma^*\gamma$ is diagrammatically represented in Fig.1. The tensor, $T^{\mu\nu}$ has two components, due to the bosonic symmetrization of the amplitude:

$$T_{\mu\nu}(k_1,k_2) = t_{\mu\nu}(k_1,k_2) + t_{\mu\nu}(k_2,k_1)_{\textcircled{1}}\ . \tag{2}$$

The tensor $t^{\mu\nu}(k_1,k_2)$ is obtained after the traces in the spinor and flavour space are performed and the result is [11]

$$t_{\mu\nu}(k_1,k_2) = \frac{4}{3}\frac{M^2}{f_\pi}e_o^2 N_c \varepsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \mathcal{I}(k_1^2) \tag{3}$$

where

$$\mathcal{I}(q^2) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{[(k_2-k)^2-M^2+i\varepsilon]} \times \frac{1}{(k^2-M^2+i\varepsilon)[(k_\pi-k)^2-M^2+i\varepsilon]}. \tag{4}$$

N_c is the number of colors (= 3), $k_\pi^\mu = k_1^\mu + k_2^\mu$ is the π^o momentum, $q^\mu = k_1^\mu$ is the space-like momentum transfer, and e_o is the unit charge. The factor of 1/3, in eq. 3, comes from the trace in the flavour space.

The integration in eq. 4 is performed after changing to the null-plane variables, $\vec{k}_\perp$, $k^+ = k^o + k^3$ and the null-plane energy $k^- = k^o - k^3$. After the integration over the k^- momentum, the asymptotic π^o wave-function appears explicitly, and from that the model wave-function is introduced in the expression [8]. We have two cases for the analysis of the k^- integration, the processes $\pi^o \to \gamma\gamma$ and $\pi^o \to \gamma^*\gamma$. In the following we will discuss in detail the last case.

The first step in the k^- integration of eq. 4, is the factorization of k^+, $k_\pi^+ - k^+$ and $k_2^+ - k^+$ in the energy denominator. The system of reference is chosen such that $q^+ = q^- = 0$ and the momentum transfer is transversal, $\vec{q}_\perp$. This frame is always possible to find due the the space-like character of q^μ. In the new set of variables

$$\mathcal{I}(q^2) = \frac{1}{2(2\pi)^4}\int dk^+ d^2k_\perp dk^- \left(k^+(k_\pi^+-k^+)(k_2^+-k^+)(k^- - \frac{k_\perp^2+M^2-i\varepsilon}{k^+})\right)^{-1}$$
$$\times\left((k_2^- - k^- - \frac{(\vec{k}_2-\vec{k})_\perp^2+M^2-i\varepsilon}{k_2^+-k^+})(k_\pi^- - k^- - \frac{(\vec{k}_\pi-\vec{k})_\perp^2+M^2-i\varepsilon}{k_\pi^+-k^+})\right)^{-1}, \tag{5}$$

where $k_\pi^+ = k_2^+$ and $k_\pi^- = k_2^-$.

The integration in k^- is convergent, and is performed using Cauchy theorem. The position of the poles in the k^- complex-plane depends on the value of k^+. Due to the fact that $k_\pi^+ = k_2^+$, only one region of k^+ values makes the integration non-vanishing. The set $k^+ < 0$ and $k^+ > k_\pi^+$ does not contribute to the integral because the three poles have imaginary parts with the same sign. The only contribution comes from the region where the + component of the momentum of the quark cannot exceed the pion momentum ($0 < k^+ < k_\pi^+$). This corresponds to the on-mass shell quark. The result is

$$\mathcal{I}(q^2) = -\frac{i}{2(2\pi)^3}\int \frac{dx d^2k_\perp}{x(1-x)^2}\left(k_2^- k_2^+ - \frac{k_\perp^2+M^2}{x} - \frac{(\vec{k}_2-\vec{k})_\perp^2+M^2}{1-x}\right)^{-1}$$
$$\times\left(k_\pi^- k_\pi^+ - \frac{k_\perp^2+M^2}{x} - \frac{(\vec{k}_\pi-\vec{k})_\perp^2+M^2}{1-x}\right)^{-1}, \tag{6}$$

where the momentum fraction $x = k^+/k_\pi^+$ and $0 < x < 1$.

Introducing in eq. 6 the relative quark-antiquark perpendicular momentum, defined by [3]

$$\vec{K}_\perp = (1-x)\vec{k}_\perp - x(\vec{k}_\pi - \vec{k})_\perp \ , \tag{7}$$

we have

$$\mathcal{I}(q^2) = \frac{i}{2(2\pi)^3}\int \frac{dxd^2K_\perp}{(1-x)}\frac{1}{((\vec{K}+x\vec{q})_\perp^2 + M^2)(m_\pi^2 - M_0^2)} \tag{8}$$

where m_π is the pion mass. The free mass operator for $q-\bar{q}$ pair is written in terms of the momentum fraction and the relative perpendicular momentum as

$$M_0^2 = \frac{K_\perp^2 + M^2}{x(1-x)} \ . \tag{9}$$

The matrix element of the electromagnetic transition current is obtained from eqs. 2, 3 and 8

$$j_\mu(q^2) = e_o^2 \varepsilon_{\mu\nu\alpha\beta}\epsilon_\gamma^\nu q^\alpha k_\pi^\beta F_{\pi^o}(-q^2) \ , \tag{10}$$

where the neutral pion electromagnetic transition form-factor is

$$F_{\pi^o}(-q^2) = \frac{N_c}{6\pi^3}\frac{M^2}{f_\pi}\int \frac{dxd^2K_\perp}{(1-x)}\frac{1}{((\vec{K}+x\vec{q})_\perp^2 + M^2)(M_0^2 - m_\pi^2)} \ . \tag{11}$$

and ϵ_γ^ν is the polarization of the real photon.

In the soft-pion limit, following respectively refs. 11 and 13, we have

$$F_{\pi^o}(0) = \frac{1}{4\pi^2 f_\pi} \ and \ r_{\pi^o} = \frac{1}{\sqrt{2}M} \ . \tag{12}$$

The neutral pion decay width is [11]

$$\Gamma_{\pi^o} = \frac{\alpha^2 m_\pi^3 \pi}{4} F_{\pi^o}^2(0) \ , \tag{13}$$

where α is the fine structure constant.

Our next task is the introduction of the null-plane pion wave-function in eq.11. According to ref. 8, the asymptotic wave-function in the null-plane model is replaced as below:

$$\frac{1}{-m_\pi^2 + M_0^2} \rightarrow \frac{\pi^{\frac{3}{2}} f_\pi}{M\sqrt{M_0 N_c}}\Phi_\pi(K^2) \ , \tag{14}$$

where the wave-function $\Phi_\pi(K^2)$ is normalized to one:

$$\int d^3K \Phi_\pi^2(K^2) \ = \ 1. \tag{15}$$

This is the normalization condition in the framework of the Hamiltonian Front-Form dynamics [6,9c]. The functional dependence of K^2 with $\vec{K}_\perp$ and x comes from the free mass operator [6,9c]

$$K^2 = \frac{M_0^2}{4} - M^2 \ . \tag{16}$$

From the above eqs. 11-16, we obtain the central formula of the present work, the neutral pion transition form-factor for a null-plane model of the quark bound-state wave-function:

$$F_{\pi^o}(-q^2) = \frac{\sqrt{N_c}M}{6\pi^{\frac{3}{2}}}\int \frac{dxd^2K_\perp}{(1-x)\sqrt{M_0}}\frac{\Phi_\pi(K^2)}{((\vec{K}-x\vec{q})_\perp^2 + M^2)} \ . \tag{17}$$

In the limit of $-q^2 \rightarrow \infty$, the form-factor behaves as $\sim \ q^{-2}$ and we define

$$\Lambda_{\pi^o} = \lim_{-q^2\rightarrow\infty}[-q^2 F_{\pi^o}(-q^2)] = \frac{\sqrt{N_c}M}{6\pi^{\frac{3}{2}}}\int \frac{dxd^2K_\perp}{x^2(1-x)\sqrt{M_0}}\Phi_\pi(K^2) \ . \tag{18}$$

The calculation of ref.14 within Perturbative Quantum Chromodynamics, gives $2f_\pi$ for the above limit. This value should be compared with eq. 18 for the null-plane models of the pion.

It is straightforward to derive the neutral pion radius from eq. 17,

$$r_{\pi^o}^2 = \frac{\sqrt{N_c}M}{F_{\pi^o}(0)\pi^{\frac{3}{2}}}\int \frac{x^2 dxd^2K_\perp}{(1-x)\sqrt{M_0}}\frac{-\vec{K}_\perp^2 + M^2}{(\vec{K}_\perp^2 + M^2)^3}\Phi_\pi(K^2) \ . \tag{19}$$

For the purpose of completeness, we return the neutral pion decay. The amplitude for this process can be formulated using the same reasoning that led to eq. 17. And the result is

$$F_{decay} = \frac{2}{3}\frac{\sqrt{N_c}M}{\pi^{\frac{3}{2}}}\int \frac{dxd^2K_\perp}{(1-x)\sqrt{M_0}}\frac{\Phi_\pi(K^2)}{((\vec{K}-2x\vec{k}^\gamma)_\perp^2 + M^2)} \ , \tag{20}$$

where $|\ \vec{k}_\perp^\gamma\ | = m_\pi/2$ and $0 < x < \frac{1}{2}$.

In the soft-pion limit and for constant pion-vertex,

$$F_{decay} = F_{\pi^o}(0) \ . \tag{21}$$

In practice the calculation of the neutral pion width with either F_{decay} or $F_{\pi^o}(0)$, through eqs. 20 or 17, give the same results within the null-plane approach. This arises because of the small pion mass compared to the inverse of the typical size scale of the wave function.

The calculations of the neutral pion electromagnetic transition form-factors are performed with three different quark-antiquark wave-functions. The Gaussian, in which the

main characteristic is confinement and is frequently used [4-7], an Hydrogen atom type that mimics one gluon exchange at short-distances and the pion wave-function model of ref. 12. This last model presents the (iterated) one-gluon exchange and confinement. The quark-mass in this model is 220 MeV. We will explore the model sensitivity of the neutral pion observables with this set of wave-functions.

The phenomenological Gaussian and Hydrogen-atom wave-functions are function of two independent parameters, the quark-mass and the non-relativistic charge radius

$$r_{NR}^2 = -6\frac{d}{dq^2}\int d^3K\Phi_\pi(|\vec{K}+\frac{\vec{q}}{2}|)\Phi_\pi(K)\ . \tag{22}$$

This equation defines the range parameter. The wave functions Φ_π used are, respectively, $e^{-(4/3)(r_{NR}k)^2}$ and $1/(r_{NR}^{-2}+k^2)^2$.

The quark-mass influences strongly r_{π^o}, as is suggested by the soft-pion limit [13], $r_{\pi^o} = (\sqrt{2}M)^{-1}$. We chose for r_{NR} the values 0.195fm and 0.4fm. The first value is the non-relativistic pion radius of ref.12. The results for the neutral pion radius as a function of the constituent quark mass are presented in Fig.2. The trend of the curves follows $(\sqrt{2}M)^{-1}$ for both values of r_{NR}. Also, the result of ref. 12 is consistent with the Gaussian and Hydrogen-atom models calculated with the same quark mass. The experimental value of r_{π^o} is an important constraint for the fitted constituent quark mass.

The calculation of the neutral pion width as a function of the weak decay constant is shown in Fig. 3. The increase of Γ_{π^o} with f_π^{-2}, as suggested by eq. 13, is qualitatively followed by our results. But now, the model dependence is important and Γ_{π^o} is not completely determined by f_π. Thus, we conclude that this observable is a test of the pion wave-function beyond the value of f_π.

The asymptotic value of the transition form-factor multiplied by the squared momentum transfer,Λ_{π^o}, is shown as a function of f_π in Fig. 4. The set of values are obtained by changing the quark mass. In the framework of Perturbative QCD [14] this asymptotic limit is $2f_\pi$. Our calculations have the nice feature of following this trend.

The discussion of r_{π^o}, Γ_{π^o}, Λ_{π^o} and f_π, shows that these quantities are to some extend independent tests for the wave-function. To make this statement more precise, we performed our calculations by using the Gaussian and Hydrogen-atom models such that f_π is fixed at 93MeV.

The results for several quark-masses and r_{NR} for the Hydrogen-atom model are shown in Table I. The charge radius (r_π^{em}) is very stable against the variation of the mass, in

agreement with previous findings [8]. In accordance with such observation, in ref.5 after f_π was fitted, r_π^{em} reproduced the experimental data.

The neutral pion radius presents a correlation with the quark mass as already discussed. The width is strongly dependent on the mass even for f_π fixed. The consistence of the theoretical width and the experimental value is reached together with the radii. The asymptotic value of the transition form-factor is approximately stable against the mass variation for fixed f_π and it agrees with ref. 14. These conclusions also applies to the Gaussian model as shown in Table II. From the comparison of both tables we observe the model dependence of the behavior of the width, with the variation of the mass.

In summary, we explore numerically the consequences of three different wave-functions (ref. 12, Gaussian and Hydrogen-atom) for the neutral pion radius, decay width and asymptotic value of the transition form-factor. The decay width is an important source of information of the wave-function. It is independent to some extend of f_π, which determines it in the limit of constant pion vertex. The neutral pion radius depends strongly on the constituent quark-mass. The present study points to an average constituent mass of the up and down quarks of about 200 MeV.

Although we obtained an overall consistence with the experimental data, there are still some discrepancies that one could solve by using a wave-function from a more fundamental level. We show the importance of the neutral pion data for testing null-plane models.

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References

[1] H.-J. Behrend et al., Z. Phys. C 49 (1991) 401.

[2] S.R. Amendolia et al., Phys. Lett. B 146 (1984) 116; S.R.Amendolia et al., Phys. Lett. B 178 (1986) 435.

[3] M. V. Terent'ev, Sov. J. Nucl. Phys. 24 (1976) 106.

[4] Z. Dziembowski, Phys. Rev. D 37 (1988) 778 and references therein.

[5] I. G. Aznauryan and K.A. Oganesyan, Sov. J. Nucl. Phys. 47 (1988) 1097; ibid., Phys. Lett. B 249 (1990) 309.

[6] P. L. Chung , F. Coester and W. N. Polizou , Phys. Lett. B 205 (1988) 545.

[7] C.R. Ji and S. Cotanch, Phys. Rev. D 21 (1990) 2319.

[8] T. Frederico and G.A. Miller, to appear in Phys. Rev. D.

[9] L.L.Frankfurt and M.I.Strikman, Nucl. Phys. B 148 (1979) 107; ibid., Phys. Rep. 76 (1981) 215; P. L. Chung, F. Coester , B. D. Keister and W. N. Polizou, Phys. Rev. C 37 (1988) 2000; M. G. Fuda, Ann. Phys. 197 (1990) 265.

[10] R. Dashen and M. Gell-Mann, 1966, in Proceedings of the 3rd Coral Gables Conference on Symmetry Principles at High-Energy (Freeman); S. Fubini, G. Segré and D. Walecka, Ann. Phys. 39 (1966) 381; V. de Alfara, S. Fubini, G. Furlan, C. Rossetti, "Currents in Hadron Physics", North Holland, Publishing Amsterdam 1973.

[11] C. Itzykson and J. B. Zuber, "Quantum Field Theory"; McGraw-Hill, 1980.

[12] S. Godfrey and N. Isgur, Phys. Rev. D 32 (1985) 189.

[13] Ll. Ametller, L. Bergström, A. Bramon and E. Massó, Nucl.Phys. B 228 (1983) 301.

[14] S. J. Brodsky and G. P. Lepage, Phys. Rev. D 24 (1981) 1808.

[15] Particle Data Group, "Review of Particle Properties", in Phys. Lett. B 239 (1990).

Table Captions

Table I Results for the null-plane Hydrogen-atom model of the pion for f_π= 93 MeV. The last column contains the experimental data, where we have also included the Perturbative QCD result for the asymptotic value of the form-factor .

$M\ (MeV)$	200	250	300	350	400	
$r_{NR}\ (fm)$	0.404	0.513	0.572	0.606	0.626	
$r_\pi^{em}\ (fm)$	0.737	0.769	0.768	0.759	0.749	0.657 ± 0.012[2]
$r_{\pi^o}\ (fm)$	0.773	0.686	0.607	0.541	0.487	0.65 ± 0.03[1]
$\Gamma_{\pi^o}\ (eV)$	8.40	5.91	3.78	2.44	1.61	7.8 ± 0.6[15]
$\Lambda_{\pi^o}\ (GeV)$	0.216	0.186	0.174	0.166	0.160	0.186[14]

Table II Results for the null-plane Gaussian model of the pion for f_π= 93 MeV. The last column contains the experimental data, where we have also included the Perturbative QCD result for the asymptotic value of the form-factor .

$M\ (MeV)$	200	250	300	350	400	
$r_{NR}\ (fm)$	0.269	0.374	0.425	0.451	0.464	
$r_\pi^{em}\ (fm)$	0.602	0.666	0.663	0.645	0.625	0.657 ± 0.012[2]
$r_{\pi^o}\ (fm)$	0.683	0.636	0.578	0.523	0.477	0.65 ± 0.03[1]
$\Gamma_{\pi^o}\ (eV)$	5.59	5.28	3.77	2.58	1.77	7.8 ± 0.6[15]
$\Lambda_{\pi^o}\ (GeV)$	0.221	0.183	0.167	0.158	0.152	0.186[14]

Figure Captions

Fig. 1 One loop diagrams for the neutral pion electromagnetic transition form-factor.

Fig. 2 Neutral pion radius as a function of the inverse constituent quark-mass. Hydrogen-atom model with r_{NR}= 0.4 fm (dashed line) and r_{NR}= 0.195 fm (dotted-dashed line). Gaussian model with r_{NR}= 0.4 fm (solid line) and r_{NR}= 0.195 fm (dotted line). The straight line is the soft-pion result [13], $r_{\pi^o} = (\sqrt{2}M)^{-1}$. Data of ref.1 is inside the limits shown by the two horizontal lines. The model of ref. 12 is shown by the square.

Fig. 3 Neutral pion decay width as a function of f_π^{-2}. The same description as in Fig. 2. The soft-pion limit is the straight solid line. Data from ref. 14.

Fig. 4 The asymptotic neutral pion transition form-factor time the squared momentum transfer as a function of f_π. The same description as in Fig. 2. The triangle is the Perturbative QCD result of ref. 14.

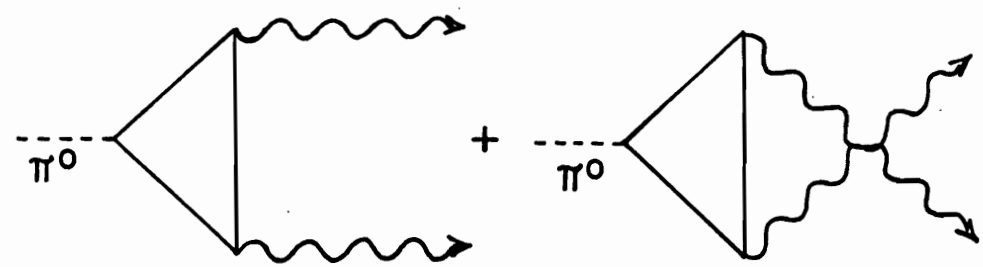

Fig. 1

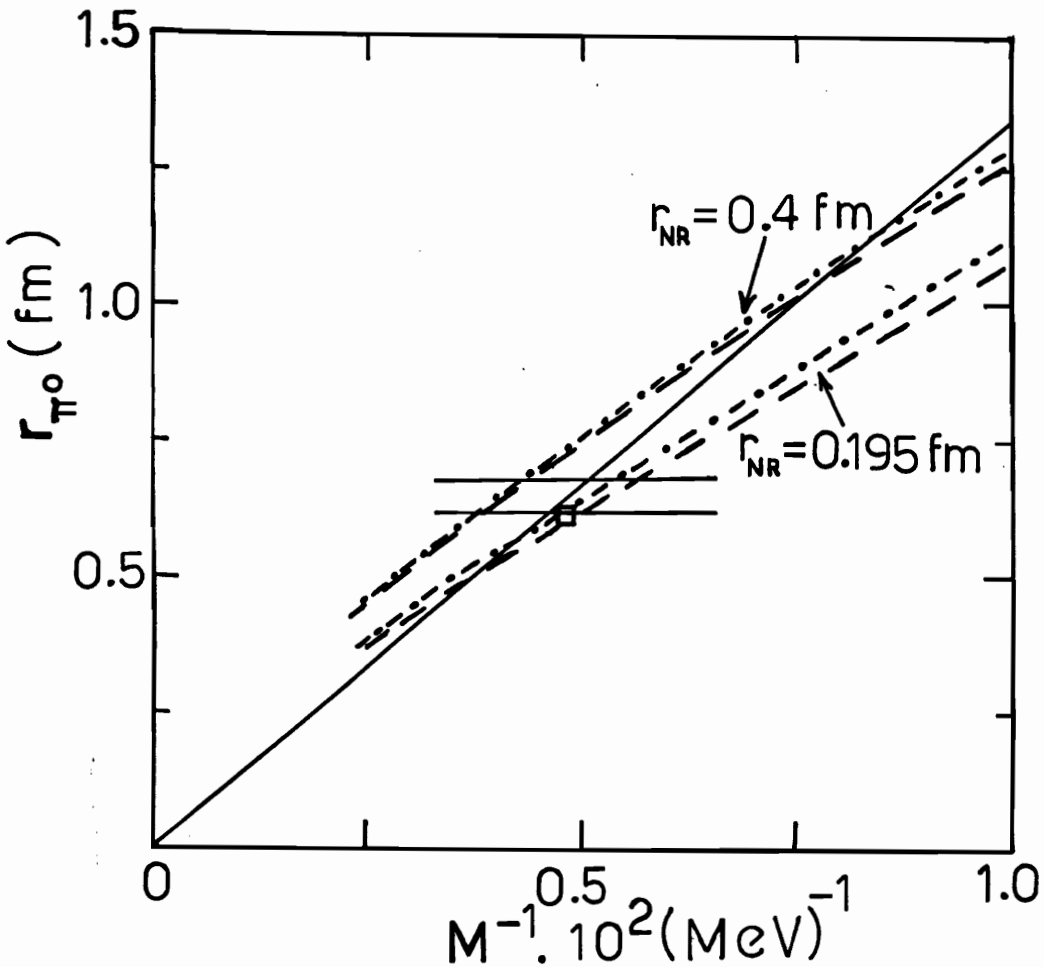
1.5
1.0
0.5
0
0.5
1.0
r_{π^0} (fm)
r_{NR} = 0.4 fm
r_{NR} = 0.195 fm
$M^{-1}.10^2 (MeV)^{-1}$

Fig. 2

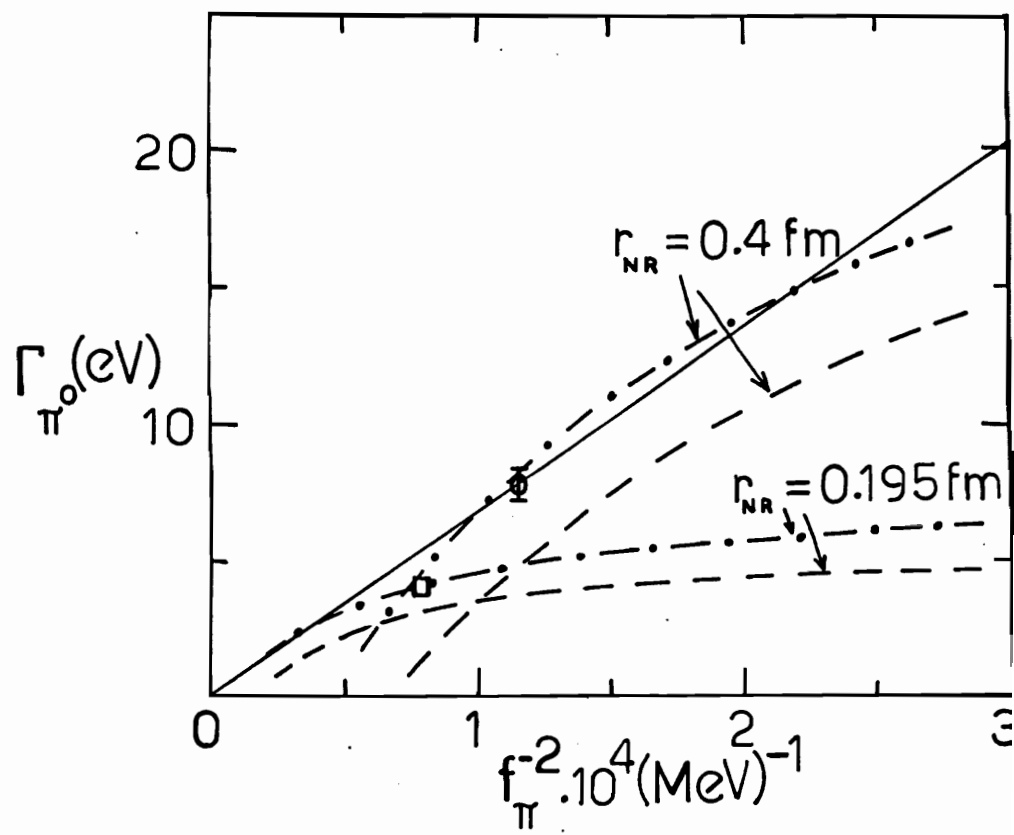
20
10
0
1
2
3
Γ_{π^0}(eV)
r_{NR} = 0.4 fm
r_{NR} = 0.195 fm
$f_\pi^{-2}.10^4 (MeV)^{-1}$

Fig. 3

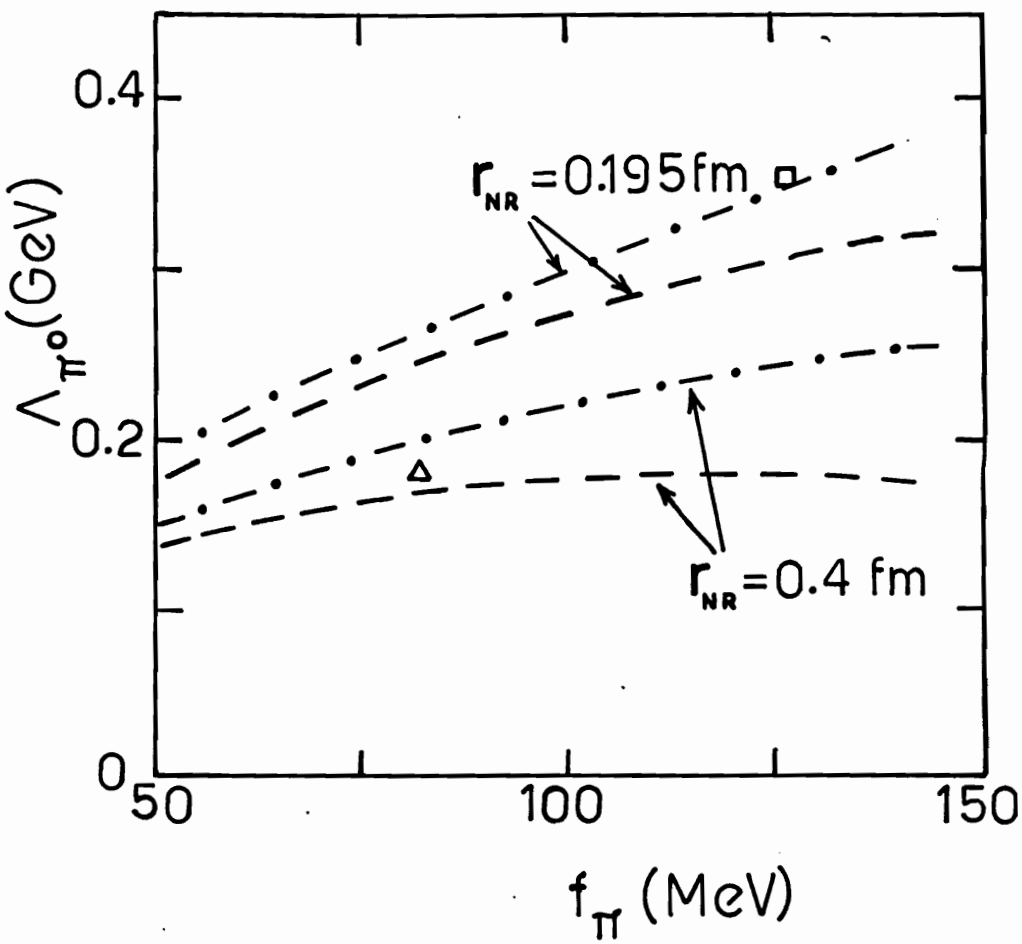

Fig. 4