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The Physical Content Of The Sp(1,R) Model

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Abstract

We analyse the properties of the Sp(1,R) model states using a basis obtained from the deformed harmonic oscillator wave functions. We make an Sp(1,R) calculation for ^{12}C and consider bases obtained from oblate, triaxial and prolate intrinsic states. The deformed basis is obtained by angular momentum projection of vibrational phonons, which are associated with giant monopole and quadrupole resonances.

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1 Introduction

The sympletic collective model[1] (SCM) is introduced to describe the collective motion of the atomic nucleus in a microscopic basis. The SCM is an algebraic model whose basis of states is associated with an irreducible representation (IR) of the Sp(3,R) group. The sp(3,R) algebra is realized through one-body operators defined in terms of harmonic oscillator annihilation and creation operators and contains[1] the six generators of monopole and quadrupole deformations, the three generators of angular momentum algebra, the six cartesian components of the quadrupole tensor and the six cartesian components of the momentum tensor. The realization of the sp(3,R) algebra in the Fock space is given by the 21 operators:

$$A^{\dagger}_{\mu\nu}(b_0) = \frac{1}{2}\sum_{i=1}^{A-1} \hat{a}^{\dagger}_{\mu i}(b_0)\hat{a}^{\dagger}_{\nu i}(b_0)$$

$$A_{\mu\nu}(b_0) = \frac{1}{2}\sum_{i=1}^{A-1} \hat{a}_{\mu i}(b_0)\hat{a}_{\nu i}(b_0) \tag{1}$$

$$C_{\mu\nu}(b_0) = \frac{1}{2}\sum_{i=1}^{A-1} \hat{a}^{\dagger}_{\mu i}(b_0)\hat{a}_{\nu i}(b_0) + \hat{a}_{\nu i}(b_0)\hat{a}^{\dagger}_{\mu i}(b_0)\ ,$$

where the creation and annihilation operators of harmonic oscillator quanta are defined by:

$$\hat{a}^{\dagger}_{\mu i}(b_0) = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}_{\mu i}}{b_0} + i\frac{b_0}{\hbar}\hat{p}_{\mu i}\right),\ \hat{a}_{\mu i}(b_0) = [\hat{a}^{\dagger}_{\mu i}(b_0)]^{\dagger}\ ,$$

where $\hat{x}_{\mu i}, \hat{p}_{\mu i}$ correspond to a set of jacobi coordinates and their associated canonical momenta respectively, the index μ stands for the three spatial directions, where by convention (123) is associated with the (zxy) axis and $b_0 = \sqrt{\frac{\hbar}{m\omega}}$ is the oscillator size parameter.

The operators $A^{\dagger}_{\mu\nu}(b_0)$ and the raising operators of u(3), $[C_{\mu\nu}(b_0), \mu > \nu]$ are defined as the raising operators of sp(3,R); The operators $A_{\mu\nu}(b_0)$ and the lowering operators of u(3), $[C_{\mu\nu}(b_0), \mu < \nu]$ are defined as the lowering operators of sp(3,R). The operators $C_{\mu\mu}(b_0)$, $\mu = 1-3$ are defined as being the weight operators of sp(3,R) which coincide with the u(3) weight operators. The Sp(3,R) lowest-weight

state (LWS) ,$|0>$, is defined as the state which is simultaneously eigenstate of the weight operators and annihilated by the lowering operators, i.e.:

$$C_{\mu\mu}(b_0)|0>= f_\mu|0>,\ f_\mu = N_\mu + \frac{A-1}{2}$$

$$C_{\mu\nu}(b_0)|0>=0\ \ (\mu<\nu),\ A_{\mu\nu}(b_0)|0>=0\ \ \forall\mu,\nu\ ,$$

where N_μ correspond to the number of oscillator quanta in the direction μ. A basis of states of a IR of Sp(3,R) is obtained by the repeated action of the raising operators on the LWS. u(3) is a subalgebra of sp(3,R) and the LWS of Sp(3,R) is also a LWS of SU(3). Therefore the Sp(3,R) model can be considered as a generalization of the Elliott[2] SU(3) model. In the Elliott model the states have the same harmonic oscillator energy. In the sympletic model, however, besides the states of the Elliott model we consider states with excitation energy $2n\hbar\omega(n \neq 0)$ relatively to the SU(3) model. Because of considerable difficulties concerning computation and algebraic manipulations wherever the SCM is utilized, calculations using submodels of the SCM with less degrees of freedom are often found in the literature[3-6]. When some degrees of freedom are suppressed, one obtains computational feasibility easier. One of the most used submodel is the Sp(1,R) model[3]. In this model the collective subspace is obtained by angular momentum projection of the basis states of a IR of the Sp(1,R) group. For prolate nuclei we use the realization of the sp(1,R) algebra associated with vibrations in the direction of the symmetry axis and in this case we have the $Sp_{\|}(1,R)$ model. For oblate nuclei we use the realization of the algebra associated with vibrations in the direction of the plane perpendicular to the symmetry axis and then we have the $Sp_{\perp}(1,R)$ model. The calculations found in literature using the Sp(1,R) model show that we need to consider high excitation energy states in order to describe the observables through this model. In these kind of calculations one often finds bases where the number of oscillator shells significantly contributing are of the order of 20. This fact makes the physical interpretation of the mechanisms responsible for the appearance of the excited configurations obscure, and imposes problems of convergence in the calculations. Furthermore this makes the systematic analysis of the calculations that take into account the mixing

of Sp(3,R) IR very difficult. This configuration mixing is essential in the study of the intruder states as the 0_2^+ of ^{12}C and ^{16}O. The key to elucidate this problem lays in the observation that in light nuclei the correlations due to deformation are very important[7]. Thus whenever we use single particle states of the spherical harmonic oscillator to describe the effects of the correlations due to deformations we need to introduce highly excited configurations. This suggests that a optimized basis must be built in terms of single particle states of a deformed harmonic oscillator instead of the spherical one. In ref.[7] we consider an $Sp_{\|}(1,R)$ calculation for the ^{8}Be using a deformed basis. We show that in that case, the basis associated with the ground state band can be very well described by the projection of few deformed states. We have also shown that in many situations, bases consisting of the projection of only one deformed state, are enough for the adequate description of the observables. In this way the Sp(1,R) model can be considered as a generalization of the method variation after projection of a deformed core. The aim of this work is to extend the analysis done in ref.[7] to the prolate basis associated with the ground state band of ^{8}Be, considering also oblate and triaxial bases. We perform an Sp(1,R) calculation for the ^{12}C using the $Sp_{\|}(1,R)$, $Sp_{\perp}(1,R)$ models in a deformed basis equivalent to the spherical one. The equivalence follows from the fact that the deformed and spherical basis are related by a unitary scale transformation, whose generator belongs to the sp(1,R) algebra. The method of calculation in the deformed basis is described in great detail in ref.[7]. Our paper is organized as follows: In section-2 we describe the Sp(1,R) calculation associated with the oblate, prolate and triaxial bases. In section-3 we describe the equivalent calculation in the deformed basis. In section-4 we discuss the results and show our conclusions.

2 The Sp(1,R) Calculation in the ^{12}C

2.1 $Sp_\perp(1,R)$ Model

The $sp_\perp(1,R)$ algebra associated with vibrations in the plane perpendicular to the symmetry axis, which we identify with axis 1, is given by the operators:

$$A_\perp^\dagger(b_0) = A_{22}^\dagger(b_0) + A_{33}^\dagger(b_0) \ (raising\ operator)$$

$$A_\perp(b_0) = A_{22}(b_0) + A_{33}(b_0) \ (lowering\ operator) \qquad (2)$$

$$A_{0\perp}(b_0) = \frac{1}{2}C_{22}(b_0) + \frac{1}{2}C_{33}(b_0) \ (weight\ operator) \ .$$

Using the same standard procedure discussed in the introduction of this paper to the construction of the IR of Sp(3,R), we define the LWS of the $Sp_\perp(1,R)$ group $|0;b_0>$ by the conditions:

$$A_{0\perp}(b_0)|0;b_0>= k_\perp|0;b_0>, \ A_\perp(b_0)|0;b_0>= 0 \ ,$$

hence a basis of states of the IR $k_\perp$ of $Sp_\perp(1,R)$ is obtained by the repeated action of the $sp_\perp(1,R)$ raising operators on the state $|0;b_0>$.

In order to describe the ground band of ^{12}C , Arickx et al[4] uses the $Sp_\perp(1,R)$ model where the LWS of $Sp_\perp(1,R)$, $|0(04);b_0>$, is given by the intrinsic state associated with the Slater determinant :

$$|\psi_0(04);b_0>= \frac{1}{\sqrt{12!}}det\ (000)^4(010)^4(001)^4 \ ,$$

where $(n_1n_2n_3)$ correspond to the harmonic oscillator wave functions in a cartesian basis with n_i quanta in the direction i (i=1,2,3); the upper index 4 indicates that the four spin-isospin states of each nucleon are occupied and $(\lambda\mu)$=(04) are the Elliott labels that determine the SU(3) IR.

A normalized basis of states of the $k_\perp = 19/2$ oblate IR of $Sp_\perp(1,R)$ is given by:

$$|n_\perp(04);b_0>= [A_\perp^\dagger(b_0)]^{n_\perp}|0(04);b_0>(\frac{\Gamma(19)}{\Gamma(n_\perp+1)\Gamma(n_\perp+19)})^{1/2} \ ,n_\perp = 0,1,\dots \ , \qquad (3)$$

The states above correspond to vibrations in the plane perpendicular to the symmetry axis with phonon oscillator energy equal to $2\hbar\omega$. This phonon is associated with the giant monopole and quadrupole resonances. The collective subspace of the $Sp_\perp(1,R)$ model is obtained by angular momentum projection of the states (2) :

$$|n_\perp(04),LM0;b_0>= P_{M0}^L|n_\perp(04);b_0> \ , \qquad (4)$$

where P_{MK}^L is the angular momentum projector[8]. The states (4) form an orthogonal basis of the $Sp_\perp(1,R)$ model. In (4) K=0, because the states $|n_\perp(04);b_0>$ are axially symmetric.

2.2 $Sp_\parallel(1,R)$ Model

2.2.1 Prolate Basis

The sp(1,R) algebra associated with vibrations in the direction of the symmetry axis, again identified with the axis 1, is given by the operators:

$$A_\parallel^\dagger(b_0) = A_{11}^\dagger(b_0) \ (raising\ operator)$$

$$A_\parallel(b_0) = A_{11}(b_0) \ (lowering\ operator)$$

$$A_{0\parallel}(b_0) = \frac{1}{2}C_{11}(b_0) \ (weight\ operator) \ .$$

The LWS of $Sp_\parallel(1,R)$ is defined as the state $|0;b_0>$, which satisfies the conditions :

$$A_{0\parallel}(b_0)|0;b_0>= k_\parallel|0;b_0>, \ A_\parallel(b_0)|0;b_0>= 0 \ .$$

A basis of states of the IR $k_\parallel$ of $Sp_\parallel(1,R)$ is obtained by the repeated action of the raising operator in the LWS. In the case of the prolate basis, we consider the state $|0(12\ 0);b_0>$ as the intrinsic state associated with the slater determinant :

$$|\psi_0(12\ 0);b_0>= \frac{1}{\sqrt{(12!)}}det(000)^4(100)^4(200)^4 \ . \qquad (5)$$

The state above was considered by Bouten et al[10] in a variational calculation as a possible candidate to describe the second 0^+ state of the ^{12}C spectrum. The

normalized basis of states of the $k_{\|} = 35/4$ prolate IR of $Sp_{\|}(1,R)$ is given by:

$$|n_{\|}(12\ 0); b_0 >= [A^{+}_{\|}(b_0)]^{n_{\|}} |0(12\ 0); b_0 > (\frac{\Gamma(35/2)}{\Gamma(n_{\|}+1)\Gamma(n_{\|}+35/2)})^{1/2} \quad , n_{\|} = 0, 1, \ldots . \tag{6}$$

The collective subspace of the $Sp_{\|}(1,R)$ model is obtained by the angular momentum projection of the states (6) :

$$|n_{\|}(12\ 0), LM0 >= P^{L}_{M0} |n_{\|}(12\ 0); b_0 > , \tag{7}$$

where the states above are orthogonal and correspond to vibrations in the direction of the symmetry axis, with phonon oscillator energy equal to $2\hbar\omega$. As in the oblate case, K=0 because the states $|n_{\|}(12\ 0); b_0 >$ are axially symmetric.

2.2.2 Triaxial Basis

In this case we have not a symmetry axis. Nevertheless we will still use the $Sp_{\|}(1,R)$ model. This is due to the fact that we restrict ourselves to the case of "prolate like" triaxial bases, i.e, bases where the LWS of $Sp_{\|}(1,R)$ has Elliott labels satisfying the condition $\lambda > \mu$.

The basis of the $Sp_{\|}(1,R)$ model is obtained in this case in a parallel way to the last section and thus we will only discuss the peculiarities related to the triaxial basis. The triaxial LWS we consider is given by the intrinsic state associated with the state below :

$$|\psi_0(62); b_0 >= \frac{1}{\sqrt{(4!)}} \sum_{P_{\alpha\beta\gamma\delta}} (-1)^{P_{\alpha\beta\gamma\delta}} \frac{1}{\sqrt{(12!)}} det(000)^4 (100)^4 (010)^{n_\alpha} (010)^{n_\beta} (200)^{n_\gamma} (200)^{n_\delta} , \tag{8}$$

where $n_\alpha n_\beta n_\gamma n_\delta$ is a shorthand for the spin-isospin ($n_\alpha = t_\alpha s_\alpha, etc$) labels. The sum $\sum_{P_{\alpha\beta\gamma\delta}}$ is done on the 4! permutations of the four spin-isospin labels n+,n-,p+,p- and $P_{\alpha\beta\gamma\delta}$ is the parity of the permutation of $n_\alpha n_\beta n_\gamma n_\delta$. The state above was considered by Bouten et al[10] in a variational calculation for the description of the ^{12}C spectrum. The collective subspace of the $Sp_{\|}(1,R)$ model associated with the triaxial basis is obtained by the angular momentum projection of the basis states of

the $k_{\|} = 27/4$ IR of $Sp_{\|}(1,R)$ associated with the triaxial LWS, $|0(62); b_0 >$:

$$|n_{\|}(62), LMK; b_0 >= P^{L}_{MK} |n_{\|}(62); b_0 >, \ n_{\|} = 0, 1, \ldots , \tag{9}$$

where the states above are not orthogonal in the label K. The label K in eq.(9) can assume the values K=0,2 according to the Elliott[2] rule. Using the Gram-Schmidt method we obtain the orthogonal basis of the $Sp_{\|}(1,R)$ model.

2.3 Results of the Sp(1,R) Calculation

In our calculations we use the semi-microscopic hamiltonian B1 of Brink-Boeker[9] in order to obtain the wave functions and energy levels of the three bases considered, i.e., oblate, triaxial and prolate bases. Sp(1,R) bases with 30 states were enough to assure the convergence of the observables considered.

The oscillator size parameter b_0, was fixed in order to minimize the energy of the lowest state, projected in L=0, for each one of the three considered bases. In fig.1 we show the energy levels calculated by the model when compared to the experimental data. For the second 0^+, our calculation is closer to the experimental data when compared with the Bouten[10] calculation. For each lowest 2^+ state of the three Sp(1,R) models studied, we show in fig.2 the partial contribution to the electric quadrupole moment when we consider systematically more model basis states, eqs.(4),(7) and (9) in the calculation of this observable. In fig.3 as in fig.2 we show the partial contribution to the $B(E2, 2^+_1 \rightarrow 0^+_1)$ between the L=0 and L=2 states of lowest energy in each of the Sp(1,R) bases taken into account. We note from these figures that the relevance of the excited configurations is evident, as in the case of ^{8}Be[7]. For these observables the dominant contributions come from states with $n \neq 0$.

3 Calculation in the Deformed Basis

In the following we make a brief discussion of the method we use to perform the calculation in the deformed basis; the details are described in ref.[7].

In this paper, as discussed in the introduction, we extend our former Sp(1,R) calculation in an optimised basis to the prolate basis of ^{8}Be to the ^{12}C nucleus, where now besides the prolate basis we consider oblate and triaxial bases. We show that, as in ^{8}Be case, the use of an optimized basis constructed from deformed harmonic oscillator wave functions, solves the problems of convergence and physical interpretation of the Sp(1,R) model in the spherical basis.

The generator of scale transformation in the direction 1 is given by $S_{\|} = i(A^{+}_{\|}(b_0) - A_{\|}(b_0))$ and belongs to the $sp_{\|}(1,R)$ algebra. Similarly the generator of scale transformation in the direction of the plane perpendicular to axis 1 is given by $S_{\perp} = i(A^{+}_{\perp}(b_0) - A_{\perp}(b_0))$ and belongs to the $sp_{\perp}(1,R)$ algebra. Through the action of the operator $S_{\perp}(S_{\|})$ we obtain a deformed basis of the $Sp_{\perp}(1,R)(Sp_{\|}(1,R))$ model. The basis of states of the $k_{\perp}$ IR of the deformed $Sp_{\perp}(1,R)$ model is given by:

$$|n_{\perp}(04); b_0 b_{\perp} >= exp(-iln(\frac{b_{\perp}}{b_0})S_{\perp})|n_{\perp}(04); b_0 >$$

$$= \sum_{n'_{\perp}=0}^{\infty} U_{n'_{\perp} n_{\perp}} |n'_{\perp}(04); b_0 > , \qquad (10)$$

which is a scale transformation of the state $|n_{\perp}(04); b_0 >$, and

$$|n_{\perp}(04); b_0 b_{\perp} >= [A^{+}_{\perp}(b_{\perp})]^{n_{\perp}} |0(04); b_0 b_{\perp} > (\frac{\Gamma(2k_{\perp})}{\Gamma(n_{\perp}+1)\Gamma(n_{\perp}+2k_{\perp})})^{1/2} ,$$

where

$$A^{+}_{\perp}(b_{\perp}) = exp(-iln(\frac{b_{\perp}}{b_0})S_{\perp})A^{+}_{\perp}(b_0)exp(iln(\frac{b_{\perp}}{b_0})S_{\perp})$$

and

$$|0(04); b_0 b_{\perp} >= exp(-iln(\frac{b_{\perp}}{b_0})S_{\perp})|0(04); b_0 > \ .$$

The action of the operator $exp(-iln(\frac{b_{\perp}}{b_0})S_{\perp})$ on eq.(10) consists in transforming the basis obtained from the spherical harmonic oscillator wave functions with $b_z = b_x = b_y = b_0$ into a basis obtained from the deformed harmonic oscillator wave functions with $b_z = b_0$, $b_x = b_y = b_{\perp}$. The states $|n_{\perp}(04); b_0 b_{\perp} >$ correspond to states of $n_{\perp}$ deformed vibrational phonons. Our procedure in the deformed basis calculation consists in using the expansion (10) of the deformed states in the spherical ones,

where we truncate the sum in (10) in a n_{max} value that assure the convergence of the calculations. The coefficients $U_{n'_{\perp} n_{\perp}}(b_0 b_{\perp})$ are calculated by the technique described in ref.[7]. The basis of states of the deformed $Sp_{\perp}(1,R)$ model is given by angular momentum projection of the states $|n_{\perp}(04); b_0 b_{\perp} >$, i.e. :

$$|n_{\perp}(04), LM0; b_0 b_{\perp} >= P^{L}_{M0} |n_{\perp}(04); b_0 b_{\perp} > \ . \qquad (11)$$

In the case of the $Sp_{\|}(1,R)$ model we have a completely analogous expression to eq.(10). In this case the states $|n_{\|}(\lambda\mu); b_0 b_{\|} >$ with $(\lambda\mu) = (12\,0)$ for the prolate basis and (62) for the triaxial one, are obtained by scale transformation in the direction 1 generated by $S_{\|}$, and are obtained from the deformed harmonic oscillator wave functions where $b_z = b_{\|}$, $b_x = b_y = b_0$. The basis of states of the deformed $Sp_{\|}(1,R)$ model is given by the angular momentum projection of the states $|n_{\|}(\lambda\mu); b_0 b_{\|} >$ i.e. :

$$|n_{\|}(\lambda\mu), LMK; b_0 b_{\|}(K) >= P^{L}_{MK} |n_{\|}(\lambda\mu); b_0 b_{\|}(K) > , \qquad (12)$$

where in the prolate basis case, as $\mu = 0$ we project only K=0 and in the triaxial basis case, we use the Elliott rule to obtain the possible K values. The bases (11) and (12) are not orthogonal and we use the Gram-Schmidt method to orthogonalize them.

The bases of the $Sp_{\|}(1,R)$ and $Sp_{\perp}(1,R)$ models depend on the parameters $b_{\|}(K)$ e $b_{\perp}$ respectively. For the oblate(prolate) case the parameter $b_{\perp}(b_{\|}(0))$ is determined in order to minimize the expected value of the energy for $n_{\perp} = 0 (n_{\|} = 0)$

$$\frac{\partial}{\partial b_{\perp}} \frac{< 0(04), LM0; b_0 b_{\perp} | H | 0(04), LM0; b_0 b_{\perp} >}{< 0(04), LM0; b_0 b_{\perp} | 0(04), LM0; b_0 b_{\perp} >} = 0 \ .$$

We have shown in ref.[7] that the condition above implies that the matrix element of the hamiltonian between the L projected vacuum and one deformed phonon state orthogonalized by the Gram-Schmidt method vanishes. Analogous expressions are valid in the case of the prolate basis of $Sp_{\|}(1,R)$ and thus we will omit this discussion. In the case of triaxial basis of $Sp_{\|}(1,R)$, to determine the oscillator size parameter $b_{\|}(K)$, K=0,2 we have to generalize the method described above to the axially symmetric case. The procedure now consists in building from the vacuum state

of the deformed phonon, the most general possible state consistent with rotational invariance, i.e.:

$$|\tau LM >= \sum_{K=0,2} C_K^\tau |0(62), LMK; b_0 b_{||}(K) > ,$$

where the coefficients C_K^τ are obtained by the diagonalization of the hamiltonian in the non-orthogonal basis $|0(62), LMK; b_0 b_{||}(K) >$. Note that C_K^τ depends also on $b_{||}(K)$. Thus we fix the parameters $b_{||}(K)$ so that the energy, for each L value, of the lowest state $|\tau_< LM >$ be a minimum as function of $b_{||}(K)$, i.e. :

$$\frac{\partial}{\partial b_{||}(K)} \frac{< \tau_< LM|H|\tau_< LM >}{< \tau_< LM|\tau_< LM >} = 0, \quad K = 0, 2 .$$

The condition above is equivalent to impose that the matrix element of the hamiltonian between the state $|\tau_< LM >$ and the state $|1(62), LMK; b_0 b_{||}(K) >$ (orthogonalized by the Gram-Schmidt method relatively to the state $|\tau_< LM >$) vanishes for all values of K, i.e. :

$$< \tau_< LM|H|1(62), LMK; b_0 b_{||}(K) >_{GS} = 0, \quad K = 0, 2 ,$$

where

$$|1(62), LMK; b_0 b_{||}(K) >_{GS} = |1(62), LMK; b_0 b_{||}(K) > - \frac{|\tau_< LM >< \tau_< LM|1(62), LMK; b_0 b_{||}(K) >}{< \tau_< LM|\tau_< LM >} .$$

The basis of states of the model, $|\bar{n}, LM\bar{K} >_{GS}$ is obtained by orthonormalizing the non-orthogonal states $|n(62), LMK; b_0 b_{||}(K) >$ and we can show that the above minimum condition implies that the matrix elements of H between the state $|\tau_< LM >$ and the $\bar{n}$=1 basis states of the model vanishes.

In table-1 we show the harmonic oscillator size parameter values which minimize the expectation value of the hamiltonian in the state obtained by angular momentum projection of the vacuum of the deformed phonon. The oblate, prolate and triaxial states of the deformed Sp(1,R) model, are obtained by the diagonalization of the hamiltonian in the bases of states (11), (12) orthogonalized by the Gram-Schmidt

method and are given in the oblate basis case by :

$$|\nu, LM >= \sum_{\bar{n}=0}^{N} C_{\bar{n}}^\nu(L) |\bar{n}(04), LM0; b_0 b_\perp >_{GS} , \tag{13}$$

and in the case of prolate and triaxial bases by :

$$|\nu, LM >= \sum_{\bar{n}=0}^{N} \sum_{\bar{K}} C_{\bar{n},\bar{K}}^\nu(L) |\bar{n}(\lambda\mu), LM\bar{K}; b_0 b_{||} >_{GS} , \tag{14}$$

where $(\lambda\mu) = (12\ 0)$ and $\bar{K} = 0$, for the prolate basis and $(\lambda\mu) = (62)$ and $\bar{K} = 0, 2$ for the triaxial basis. In table-2 we show the probability of finding the state with $\bar{n} = 0, 1, 2$ in the expansion of the two lowest energy states with a given L, in the oblate basis eq.(13) and prolate and triaxial bases eq.(14).

In figures 4 and 5 we show the contribution of the states with $n \neq 0$ to the $Q_{el}(2_1^+)$ and $B(E2, 2_1^+ \to 0_1^+)$ for the three considered bases. These figures are obtained as the figures 2 and 3.

In tables 3,4,5 we show the spectroscopic properties of the L=0,2,4 lowest energy states, associated with the oblate, triaxial and prolate bases calculated diagonalizing the hamiltonian in a deformed basis truncated in N=0-5.

4 Conclusions and Discussion of the Results

We see in table-2 that for the three considered bases, the two lowest energy states with L=0,2,4 of each basis, are dominated by the angular momentum projection of the vacuum and one deformed phonon state orthogonalized by the Gram-Schmidt method. We also note in this same table that the probability of finding in the wave function of the first two 0^+ and 2^+ states the corresponding vacuum project states is larger than 99% .

The contribution of the vacuum projection to the $B(E2, 2^+ \to 0^+)$ between the 2^+ and 0^+ lowest energy states is larger than 98% for the oblate and prolate bases and larger than 96% for the triaxial basis. In fig.5 we show this dominance of vacuum projection for the three bases considered. For the electric quadrupole moment, the contribution of the vacuum projection is larger than 98% in the case of the oblate

and prolate bases. Nevertheless for the triaxial basis the contribution of the vacuum projection to the $Q_{el}(2_1^+)$ is less than 50% of the exact value. This is due to a cancelling that occurs among the dominant terms of the wave function, because in contrast to the former cases, we have now for $\bar{n} = 0$ two states, one with $\bar{K} = 0$ and the other with $\bar{K} = 2$. This can be understood, noting that in the deformed vacuum projected case, the basis is obtained considering in eq.(12) only the $n_{\|} = 0$, and thus the operator $Q_{el}(2_1^+)$ is given by four matrix elements where the two diagonal are, in general, dominant. However in the case where L=2 the dominant terms almost cancel each other and thus the contribution of terms with $N \neq 0$ in eq.(14) becomes important.

In all cases considered, the contribution coming from states with more than five phonons is negligible, as shown in figures 4 and 5. This can be demonstrated in a still more evident way through the tables 3-5. We observe from these tables that for oblate and prolate bases, considering only the deformed vacuum projection, we obtain results which are less than 2% different from the exact ones, being the B(E2) the most sensible observable. In the case of the triaxial basis, the projection of the deformed vacuum state, excepting the $Q_{el}(2_1^+)$, gives results that disagree in less than 6% with the exact ones. The triaxial deformed basis with five phonons yields results that disagree in less than 3% relatively to the exact ones. Making the same for the oblate and prolate bases we practically reproduce the exact results.

We also note that for the lowest energy states, bases consisting only of the projection of the deformed vacuum state are an excellent approximation to the exact calculation. We show that exceptions to the rule above occur in triaxial basis when there is a large cancelling of the dominant terms. This occurs because in this case there are states nearly degenerate with the same angular momentum. The electric quadrupole moment $Q_{el}(2_1^+)$ of the triaxial basis is an example of this exception, where the basis obtained by the projection of the deformed vacuum state contributes with less than 50% of the exact value.

Thus we see that the Sp(1,R) model can be considered as a generalization of the deformed harmonic oscillator variation after projection method of Bouten et

al[11] and Abgral et al[12]. In this model we only consider the states obtained by the angular momentum projection of the deformed vacuum state, which is a lowest(highest) weight state of the deformed SU(3). In our paper we show that for representations where $\lambda, \mu \neq 0$ this might not be enough. For example, we need in our analysis of the triaxial basis, to consider bases with phonons of the order of N=5 .

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References

[1] Rowe D.J., Rep.Prog.Phys.48(1985)1419-80.

[2] Elliott J.P., Proc.R.Soc.A245(1958)128-45,562-81.

[3] Arickx F. et al, Nucl. Phys. A318(1979)269-86.

[4] Arickx F. et al, Nucl. Phys. A377(1982)121-36.

[5] Okhrimenko et al, Sov. J. Nucl. Phys.34(1981)488-94

[6] Ovcharenko et al, Sov. J. Nucl. Phys.35(1982)372-77

[7] Avancini S. S. and de Passos E.J.V., J. Phys.G:Nucl. Part.Phys.15(1989)161-72,933

[8] L.C. Biedenharn and J.D. Louck, Angular Momentum in Quantum Physics (Encyclopedia of Mathematics and its Applications, v.8) (Addison-Wesley , 1981)

[9] Brink D.M. and Boeker E., Nucl. Phys. A91(1967)1-26.

[10] Bouten M. et al, Nucl. Phys. A158(1970)217-224.

[11] Bouten M. et al, Prog. in Part. and Nucl. Phys.5(1981)55-78.

[12] Abgrall Y. et al,Nucl.Phys.A131(1969)609-22.

[13] Ajzenberg-Selove F.,Nucl.Phys.A433(1985)55

Table Captions

Table I: Oscillator size parameters that minimize the expectation value of the hamiltonian in the L projected deformed vacuum of the oblate, triaxial and prolate bases. In parentheses we show the oscillator size parameters of the spherical bases. Note that we have for the triaxial basis two size parameters due to the projection in K=0 and K=2. All sizes are in fm.

Table II: Probability of finding the states with $\bar{n} = 0, 1, 2$ in the expansion of the two lowest energy states of the oblate eq.(13), prolate and triaxial eq.(14) bases. Note that for the prolate and oblate bases we have only $\bar{K} = 0$.(Deformed bases with N=9).

Table III: Excitation energy and geometric properties of the ^{12}C ground band calculated in a deformed basis truncated in N=0-5. Energies in MeV, $< r^2 >^{1/2}$ in fm, B(E2,L→L-2) in $e^2.fm^4$ and Q_{el} em $e.fm^2$.

Table IV: Excitation energy and geometric properties of the lowest energy states calculated in a deformed basis truncated in N=0-5. Energies in MeV, $< r^2 >^{1/2}$ in fm, B(E2,L→L-2) in $e^2.fm^4$ and Q_{el} em $e.fm^2$.

Table V: As in table IV

Figure Captions

Figure 1: First excitation energy levels of ^{12}C calculated in the Sp(1,R) model when compared to the experimental data[13]. The 0_1^+ and 2_1^+ states are associated with the oblate basis of Sp(1,R) and the 0_2^+ with the prolate one.

Figure 2: Partial contribution to the $Q_{el}(2_1^+)$, due to a systematic inclusion, in the expansion $|\nu LM>=\sum_{\bar{n}=0}^{n_{max}} C^{\nu}_{\bar{n},K}(L)|\bar{n}(\lambda\mu), LM\bar{K};b_0>_{GS}$ of the basis states $|n(\lambda\mu), LM\bar{K};b_0>_{GS}$, associated with the oblate, triaxial and prolate bases. The value of Q_{el} at the point n is found by considering, in the above expansion, the contribution of all terms up to n, inclusive. Note that the GS index and $\bar{K}\neq 0$ refer only to the triaxial basis. The height of each block at the point n of the figure corresponds to the gain due to the inclusion of the Sp(1,R) n basis state in the $Q_{el}(2_1^+)$ calculation. We link all the points obtained to guide the eyes.

Figure 3: As in figure 2, but for the B(E2,$2_1^+\rightarrow 0_1^+$) between the lowest energy states of the Sp(1,R) models associated with the oblate, prolate and triaxial bases.

Figure 4: Partial contribution to the $Q_{el}(2_1^+)$, due to a systematic inclusion of the deformed basis states in the expansion of the deformed Sp(1,R) model states, eq.(13) and eq.(14). This figure is obtained as the figure 2. Note the change of scale.

Figure 5: As in fig.4, but for the B(E2,$2_1^+\rightarrow 0_1^+$) between the lowest energy states of the deformed Sp(1,R) models associated with the oblate, triaxial and prolate bases. Note the change of scale.

	oblate basis ($b_0=1.36$)	triaxial basis ($b_0=1.58$)		prolate basis ($b_0=1.45$)
L	$b_\perp$	$b_\parallel(0)$	$b_\parallel(2)$	$b_\parallel$
0	1.95	2.39	—	2.62
2	1.93	2.38	2.40	2.62
4	1.90	2.34	2.36	2.63

Table I

$|C^{\nu}_{\bar{n},K}(L)|^2$

		oblate	basis	triaxial		basis		prolate	basis
	ν	1	2	1		2		1	2
L	$\bar{n}$			$\bar{K}=0$	$\bar{K}=2$	$\bar{K}=0$	$\bar{K}=2$		
0	0	0.998	—	0.999	—	—	—	0.999	—
	1	—	0.937	—	—	0.962	—	—	0.969
	2	0.001	0.045	0.001	—	0.029	—	0.001	0.023
2	0	0.997	—	0.629	0.369	0.369	0.629	0.999	—
	1	—	0.927	—	—	—	—	—	0.969
	2	0.002	0.049	0.001	—	0.001	0.001	0.001	0.023
4	0	0.993	0.001	0.523	0.473	0.468	0.520	0.998	—
	1	—	0.904	—	—	0.001	0.001	—	0.967
	2	0.006	0.051	0.002	0.001	0.005	0.004	0.002	0.022

Table II

Oblate Basis

	EXACT				VACUUM			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.60	—	—	—	2.60	—	—
2	3.07	2.59	15.89	7.75	3.09	2.59	15.70	7.70
4	11.49	2.58	23.01	8.70	11.71	2.57	22.55	8.54
	2-PHONON				3-PHONON			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.60	—	—	—	2.60	—	—
2	3.07	2.59	15.83	7.74	3.07	2.59	15.95	7.77
4	11.55	2.58	22.81	8.65	11.57	2.58	23.04	8.69
	4-PHONON				5-PHONON			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.60	—	—	—	2.60	—	—
2	3.07	2.59	15.91	7.76	3.07	2.59	15.90	7.75
4	11.58	2.58	23.01	8.70	11.59	2.58	23.01	8.70

Table III

Triaxial Basis

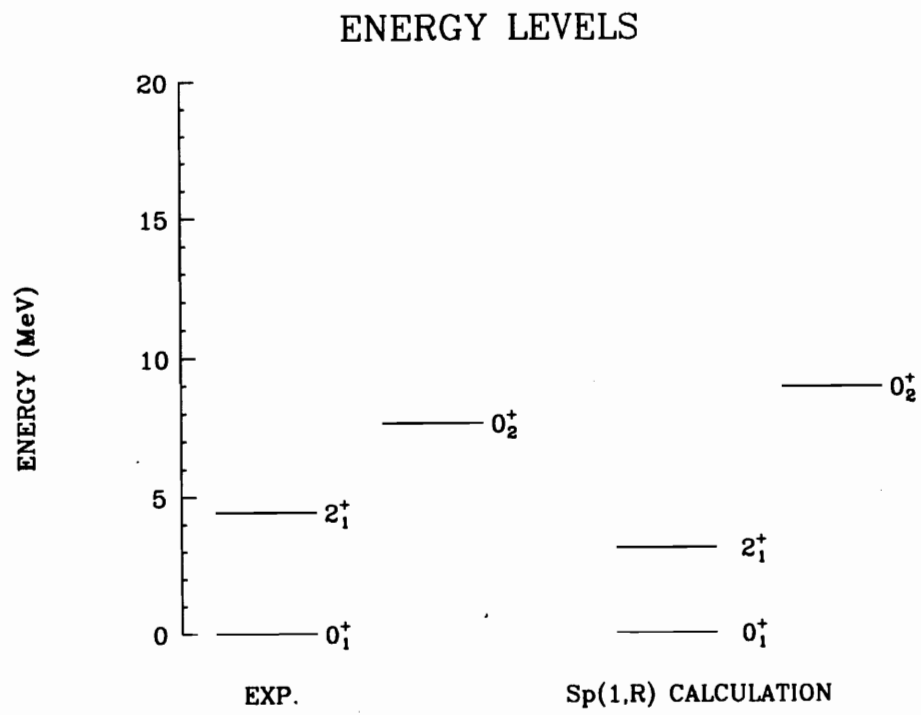
	EXACT				VACUUM			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.99	—	—	—	2.99	—	—
2	1.37	2.99	55.09	-2.50	1.38	2.99	51.98	-1.14
4	4.54	2.98	82.91	-12.87	4.59	2.97	80.17	-12.43
	2-PHONON				3-PHONON			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.99	—	—	—	2.99	—	—
2	1.37	2.99	56.16	-3.10	1.37	2.99	55.73	-2.78
4	4.53	2.98	82.82	-12.83	4.53	2.98	83.03	-12.86
	4-PHONON				5-PHONON			
L	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}	E.E.	$<r^2>^{1/2}$	BE2	Q_{el}
0	—	2.99	—	—	—	2.99	—	—
2	1.37	2.99	55.40	-2.64	1.37	2.99	55.23	-2.56
4	4.54	2.98	83.00	-12.87	4.54	2.98	82.96	-12.87

Table IV

Prolate Basis

	EXACT				VACUUM			
L	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}
0	—	3.41	—	—	—	3.40	—	—
2	1.04	3.41	232.85	-31.00	1.04	3.41	231.39	-30.89
4	3.46	3.43	339.03	-40.03	3.47	3.43	336.47	-39.85
	2-PHONON				3-PHONON			
L	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}
0	—	3.41	—	—	—	3.41	—	—
2	1.03	3.41	232.40	-30.97	1.04	3.41	233.07	-31.01
4	3.45	3.43	338.24	-40.00	3.46	3.43	339.21	-40.03
	4-PHONON				5-PHONON			
L	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}	E.E.	$< r^2 >^{1/2}$	BE2	Q_{el}
0	—	3.41	—	—	—	3.41	—	—
2	1.04	3.41	232.91	-31.00	1.04	3.41	232.88	-31.00
4	3.46	3.43	339.07	-40.03	3.46	3.43	339.05	-40.03

Table V



FIGURE−1

$Q_{el}(2_1^+)(e.fm^2)$

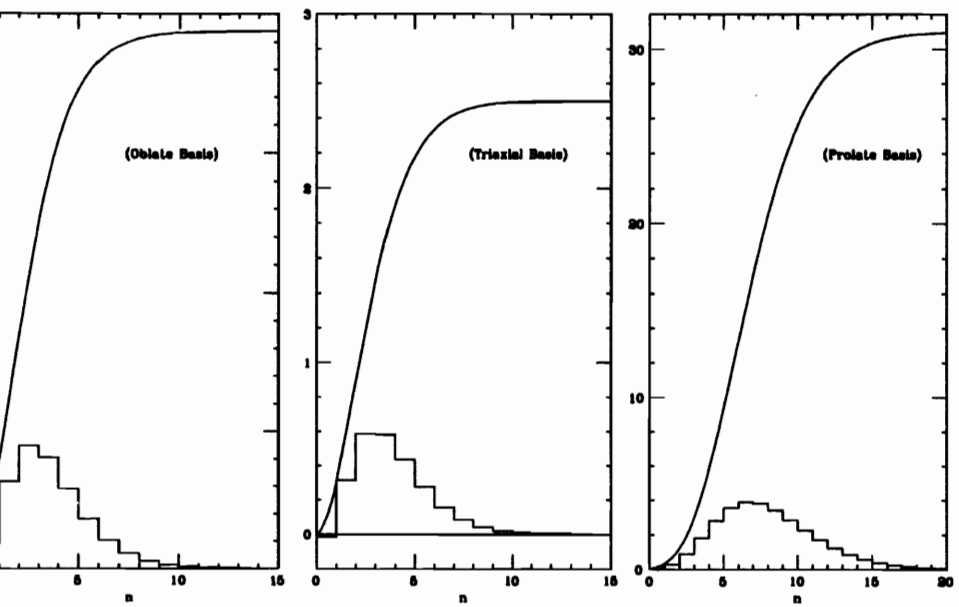


FIGURE-2

$B(E2;2_1^+ \rightarrow 0_1^+)\ (e^2.fm^4)$

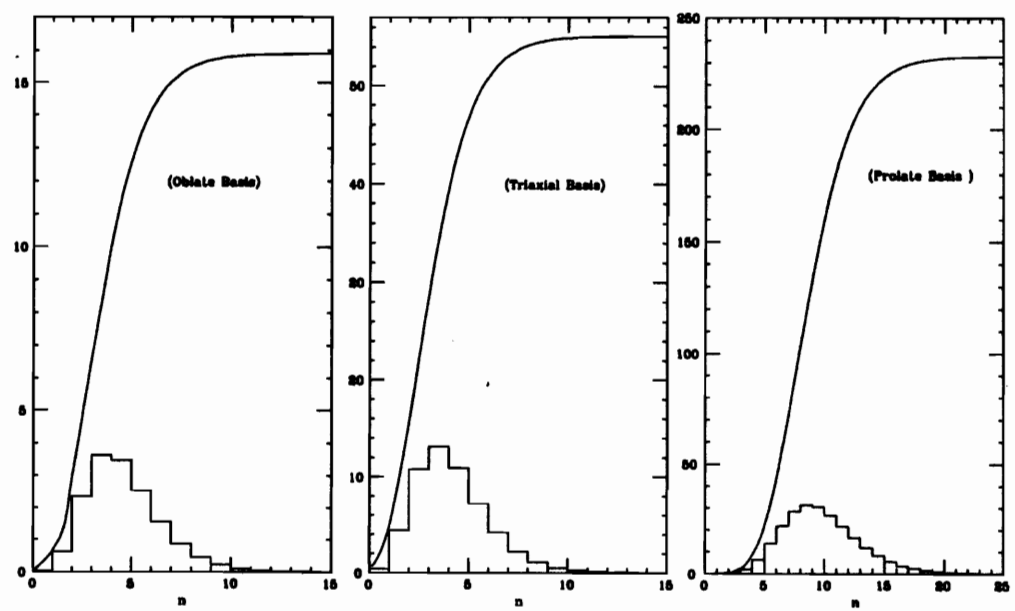
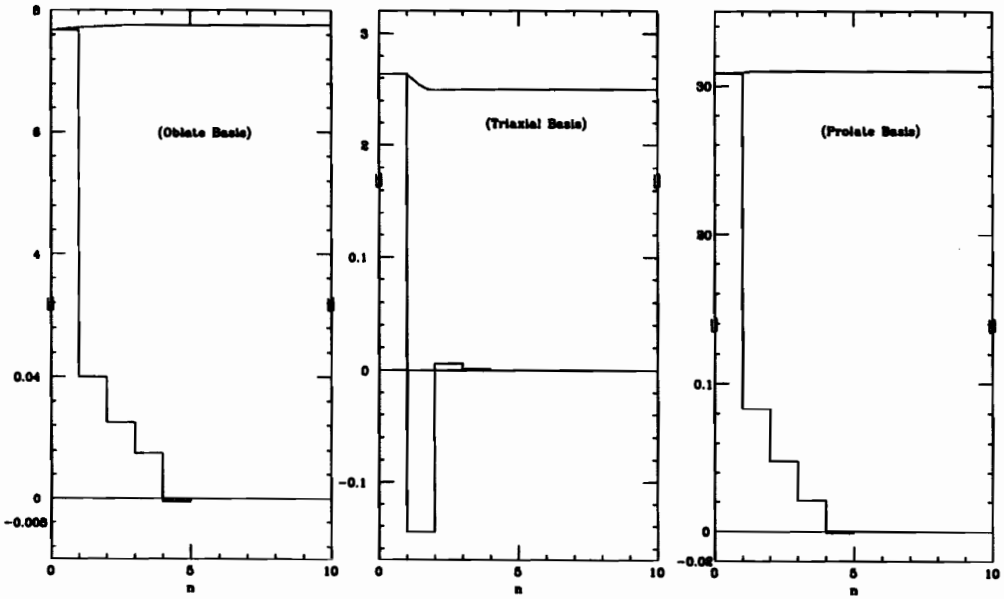


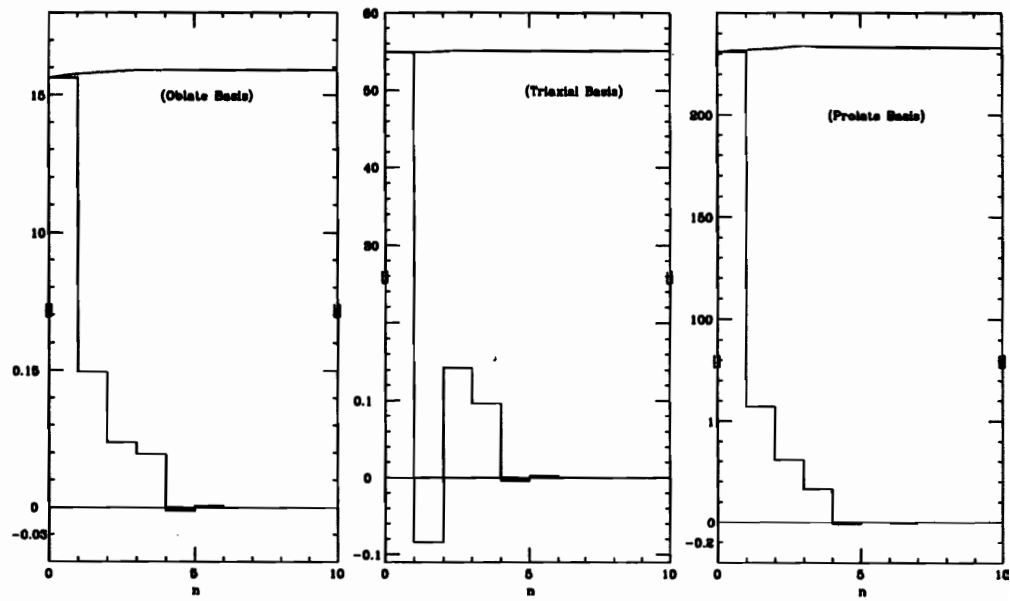
FIGURE-3

$Q_{el}(2_1^+)$ (Deformed Basis)($e.fm^2$)



FIGURE−4

$B(E2;2_1^+\rightarrow 0_1^+)$ (Deformed Basis)($e^2.fm^4$)



FIGURE−5