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The Green's Function in the Bloch-Nordsieck Model for QED_3

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Abstract

The electron Green's function is obtained in the Bloch-Nordsieck approximation of three-dimensional QED. Dimensional regularization is used in the intermediate stages of calculation.

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The infrared problem in QED has been object of investigation since the work of Bloch and Nordsieck[1], who have pointed out the origin of infrared divergences in the radiative corrections to the Mott scattering cross-section. While some authors[2] have shown that these divergences cancel out if bremsstrahlung is taken into account, others[3] considered the charged asymptotic states together with their respective Coulomb fields, so that finite S-matrix elements can be defined. Alternative approaches to the problem have also been proposed[4], even though all such treatments are still far to exaust the subject.

Lately, quantum electrodynamics in three dimensions (QED_3) has attracted much interest, as a model for the quantum Hall effect[5] and also for the high-Tc superconductivity. Despite of being a super-renormalizable theory, QED_3 suffers seriously from perturbative infrared divergences. A

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number of nonperturbative schemes have been suggested to avoid these divergences; they essentialy consist in softening the low-energy singularity of the photon propagator through the inclusion of polarization effects due to zero-mass intermediate states[6]. Another possibility is the use of the gauge technique[7]. In this work we shall study the infrared behavior of QED_3, calculating the electron Green's function $G(x,y)$, as a functional average of the electron Green's function in a given external field, $G(x,y|A)$, in the Bloch-Nordsieck (BN) model[8].

The BN zero order approximation is equivalent to replacing the Dirac γ matrices by c-numbers u_α, with $u^2 = 1$. It turns out that the Green's function G_0 for a free electron is purely retarded. Therefore, matrix elements corresponding to closed fermion loops will contain at least one function G_0 equal to zero. Physically, this corresponds to the fact that there are no antiparticles in the BN model and, in consequence, no pairs can be created. Thus, there are no radiative corrections to the photon Green's function in the model. In QED_3, where dynamical mass generation occurs[9], this means that the photon remains massless in the limit of low frequencies, so that a Chern-Simons counterterm is absent of the renormalized Lagrangian.

The electron Green's function in an external field satisfies

$$\{u^\alpha[i\partial_\alpha + eA_\alpha(x)] - m\}G(x,y|A) = -\delta(x-y) \ . \tag{1}$$

Using the Fock's proper time method, we may write the solution of Eq.(1) as

$$G(x,y|A) = i\int_0^\infty d\nu\, U(\nu) \ , \tag{2}$$

where $U(\nu)$ have the form

$$U(\nu) = \int \frac{d^3p}{(2\pi)^3} \exp\left[iK(x;\nu) + i\nu(u^\alpha p_\alpha - m + i\epsilon) - ip(x-y)\right] \ . \tag{3}$$

The function K satisfies the linear equation

$$\frac{\partial K}{\partial \nu} = -u^\alpha \partial_\alpha K(x;\nu) + eu^\alpha A_\alpha(x) \ , \tag{4}$$

with the boundary condition

$$K(x;0) = 0 \ . \tag{5}$$

Solving Eq.(4) by means of a Fourier transformation, yields

$$K(\nu|A) = \frac{e}{(2\pi)^{3/2}} \int d^3k\,(u.A(k))e^{-ikx} \int_0^\nu d\nu'\, e^{i\nu'(u.k)} \,, \tag{6}$$

while, substituting Eq.(3) into Eq.(2), gives

$$G(x,y|A) = \frac{i}{(2\pi)^3} \int_0^\infty d\nu \;\exp\left[-ip(x-y) - i\nu(m - u.p - i\epsilon) + iK(\nu|A)\right] . \tag{7}$$

The electron Green's function is the functional average

$$G(x,y) = \frac{\int G(x,y|A)\, S_0(A)\, \delta A}{\int S_0(A)\, \delta A} \,, \tag{8}$$

where $S_0(A)$ is the expectation value of the S-matrix in a Fermi vacuum. In this case it is equal to unity, since the BN model does not include the vacuum polarization. So, Eq.(8) simplifies to

$$G(x-y) = \frac{1}{(2\pi)^{n/2}} \int d^n p\, G(p)\, e^{-ip(x-y)} \,, \tag{9}$$

$$G(p) = i \int_0^\infty d\nu e^{-i\nu(m-u.p-i\epsilon)} \int e^{iK(\nu|A)}\, \delta A \,. \tag{10}$$

In the present case, of course, $n = 3$.

The functional integral in the above expression can be converted into an integral of the Gaussian type with respect to A_α. We thus obtain

$$G(p) = i \int_0^\infty d\nu \exp -i\nu(m - u.p - i\epsilon) + f(\nu) \tag{11}$$

where

$$f(\nu) \equiv -\frac{ie^2}{2(2\pi)^{3/2}} \int d^3k[u^\alpha D_{\alpha\beta}(k)u^\beta] \int_0^\nu d\nu' e^{i\nu'(u.k)} \int_0^\nu d\nu'' e^{-i\nu''(u.k)} \,. \tag{12}$$

In the last equation, $D_{\alpha\beta}$ stands for the free photon propagator,

$$D_{\alpha\beta} = \frac{1}{k^2 + i\epsilon}(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2 + i\epsilon}) + a\frac{1}{k^2 + i\epsilon}\frac{k_\alpha k_\beta}{k^2 + i\epsilon} \,, \tag{13}$$

where a is the gauge parameter. So, substituting Eq.(13) into Eq.(12),

$$f(\nu) = -\frac{ie^2}{(2\pi)^{3/2}} \int_0^\nu d\nu' \int_0^{\nu'} d\nu'' \int d^3k e^{i(u.k)(\nu'')} F(k) \,, \tag{14}$$

$$F(k) \equiv \frac{1}{k^2 + i\epsilon} + (a-1)\frac{(u.k)^2}{(k^2 + i\epsilon)^2} \,. \tag{15}$$

It is convenient for our purposes to rewrite $f(\nu)$ in the form

$$f(\nu) = -ie^2 \int_0^\nu d\nu' \int_0^{\nu'} d\nu''\{F^{(1)}(\nu'') + (1-a)\frac{d^2}{d\nu''^2}F^{(2)}(\nu'')\} \,, \tag{16}$$

where

$$F^{(1)}(\nu'') \equiv \frac{1}{(2\pi)^{n/2}} \int d^n k \frac{e^{i\nu''(u.k)}}{k^2 + i\epsilon} \,, \tag{17}$$

$$F^{(2)}(\nu'') \equiv \frac{1}{(2\pi)^{n/2}} \int d^n k \frac{e^{i\nu''(u.k)}}{(k^2 + i\epsilon)^2} \,. \tag{18}$$

We have left open the dimension in Eqs.(9), (17) and (18), to antecipate that we are going to use dimensional regularization for the integrals $F^{(1)}$ and $F^{(2)}$ above. One of the advantages of using dimensional as well as analytic regularization is the fact that, in this particular case, it is not necessary to attribute a ficticious mass to the photon, in order to take care of possible infrared divergences, in contrast with the Pauli-Villars regularization. For a given n we find

$$F^{(1)}(\nu'') = \frac{(-i)^{n-1}}{2^{(2-n/2)}}\Gamma(n/2-1)(\nu'' - i\epsilon)^{2-n} \,, \tag{19}$$

$$F^{(2)}(\nu'') = \frac{(-i)^{n-1}}{2^{(4-n/2)}}\Gamma(n/2-2)(\nu'' - i\epsilon)^{4-n} \,, \tag{20}$$

so that

$$f(\nu) = e^2 \frac{(-i)^n}{2^{(3-n/2)}} \frac{\Gamma(n/2-1)}{(3-n)}[2 + (1-a)(n-3)]P(\nu) \,, \tag{21}$$

with

$$P(\nu) \equiv \frac{1}{(4-n)}[(\nu - i\epsilon)^{4-n} - (-i\epsilon)^{4-n}] - \nu(-i\epsilon)^{3-n} \,. \tag{22}$$

Making the replacement $n = 3 - \eta$ and expanding around $\eta = 0$, we finally obtain

$$f(\nu) = -ie^2\sqrt{\frac{\pi}{2}}\nu + ie^2\sqrt{\frac{\pi}{2}}\nu \log(i\nu/\epsilon) \ . \tag{23}$$

Looking at the expression (23) we note that this result is gauge independent. We also realize that the electron Green's function has an infrared divergence, controled by the small parameter ϵ, which can be removed through mass renormalization[8]. If we had employed the Pauli-Villars regularization, we would have obtained

$$f(\nu) = -ie^2\sqrt{\frac{\pi}{2}}\frac{(a+1)}{2}\nu + ie^2\sqrt{\frac{\pi}{2}}\nu\Gamma(0, i\mu\nu) \ , \tag{24}$$

where the parameter μ corresponds to a small photon mass. In the vicinity of the singularity the incomplete gamma function has a logarithm behavior, Thus, expressions (23) and (24) coincide in the Feynman gauge.

Other authors have performed these calculations in a specific gauge [10], using the Pauli-Villars regularization. However, they start with a mass term for the electromagnetic field in the Lagrangian density, spoiling the gauge invariance of the theory. In addition to keeping the photon massless, dimensional regularization enables us to extend the previous analisis to other dimensions than three. To conclude, we would like to mention that in four dimensions we get the same result of reference [8] using dimensional regularization. The parameter ϵ in this case controls the ultraviolet divergences and plays the same role of the auxiliary mass introduced by the Pauli-Villars regularization.

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