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ON THE DERIVATION OF CAUSAL PROPAGATORS FOR ALGEBRAIC GAUGES FROM THE PRINCIPLE OF ANALYTIC EXTENSION

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Abstract

The principle of analytic continuation can be used to derive causal distributions for covariant propagators. We apply this principle as a basis for deriving analyticaly continued causal distributions for algebraic non-covariant propagators.

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Over a quarter of a century ago Bollini, Giambiagi, and Domínguez[1,2] considered causal distributions in the context of the Fourier transform of radial functions, $f(Q^2)$, where $Q^2 \equiv k_0^2 - \vec{k}^2 = k_0^2 - k_1^2 - k_2^2 - k_3^2$. By introducing a positive parameter α such that

$$(f_\alpha, \tau) = (f(\alpha^2 k_0^2 - \vec{k}^2), \tau(k_0, k_1, k_2, k_3)) \equiv \frac{1}{\alpha}(f(Q^2), \tau(\frac{k_0}{\alpha}, k_1, k_2, k_3)), \quad (1)$$

where τ is a test function, one defines that f_α is analytic in α if for any τ, the functional (f_α, τ) is also analytic in α.

Now, when f_α is analytically continued to the whole of the upper half plane of α, then a causal distribution is defined through the following extension $k_0 \to \alpha k_0$, i.e.,

$$f(k^2 + i\epsilon) = \lim_{\alpha \to 1+i\epsilon} f(\alpha^2 k_0^2 - \vec{k}^2), \quad \epsilon \to 0^+. \quad (2)$$

From this reasoning of analytic continuation as a postulate, one can derive the covariant Feynman propagator in momentum space as follows

$$\frac{1}{k^2} \to \lim_{\alpha \to 1+i\epsilon} \frac{1}{\alpha^2 k_0^2 - \vec{k}^2} = \frac{1}{k^2 + 2i\epsilon k_0^2}, \quad \epsilon \to 0^+. \quad (3)$$

Since $k_0^2 > 0$, one has the usual prescription for handling covariant poles, namely,

$$\frac{1}{k^2} \to \lim_{\epsilon \to 0^+} \frac{1}{k^2 + i\varepsilon}, \quad \varepsilon \equiv 2\epsilon k_0^2 \to 0^+. \quad (4)$$

Recently, Pimentel and Suzuki[3] have proposed a causal prescription for the light-cone gauge starting from the premise that the propagator as a whole must be causal. In this sequel, we propose that within the framework of analytic continuation as discussed above, we can arrive at the causal prescription for the light-cone gauge.

To begin with, consider the product $(k^2 k\cdot n)^{-1}$ with $n^\mu \equiv (n^0, 0, 0, n^3)$[1] being an external, arbitrary vector which determines the choice of a gauge of the algebraic or non-covariant type. The factor $(k^2 k\cdot n)^{-1}$ upon the hypothesis of analytic continuation becomes

$$\frac{1}{k^2 k\cdot n} \rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k\cdot n + i\epsilon k^0 n^0)}. \tag{5}$$

As long as the external vector n is quite arbitrary, we can choose it so that $n^0 > 0$, and since ϵ is strictly positive, equation (5) may be rewritten as[2]

$$\begin{aligned} \frac{1}{k^2 k\cdot n} &\rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k\cdot n + i\epsilon \mid k^0 \mid n^0)}, \quad for\ k^0 > 0 \\ \frac{1}{k^2 k\cdot n} &\rightarrow \frac{1}{(k^2 + 2i\epsilon k_0^2)(k\cdot n - i\epsilon \mid k^0 \mid n^0)}, \quad for\ k^0 < 0 \end{aligned} \tag{6}$$

[1] for convenience we have chosen components $n^1 = n^2 = 0$

[2] recall that we continue analytically to the whole of the *upper* half plane of α

or, using the Heaviside Θ-function,

$$\frac{1}{k^2 k\cdot n} \rightarrow \frac{1}{k^2 + i\varepsilon}\left\{\frac{\Theta(-k^0)}{k\cdot n - i\xi} + \frac{\Theta(k^0)}{k\cdot n + i\xi}\right\}, \qquad \begin{aligned} \varepsilon &\equiv 2\epsilon k_0^2 \rightarrow 0^+ \\ \xi &\equiv \epsilon \mid k^0 \mid n^0 \rightarrow 0^+ \end{aligned}, \tag{7}$$

which is exactly the causal prescription considered in reference [3].

We can, of course, generalize for higher order poles of $(k\cdot n)$ as well as for non-covariant gauge choices other than the light-cone one. We shall only consider the double pole case and briefly discuss the pure homogeneous axial gauge ($n^0 = 0$) and the pure homogeneous temporal gauge ($n^3 = 0$) choices for n^μ.

First of all, let us consider the simple pole cases. From equation (5) we note that the analytic continuation of $k\cdot n$ entails a sign dependence of the imaginary part coming from k^0 and n^0.

The pure temporal case[3] is such that there will be a violation of causality if one employs the principal-value (PV) prescription to treat the pole $(k\cdot n)^{-1}$, in the same manner as it breaks causality in the light-cone case [3]. Indeed, evaluation of the Wilson loop to the fourth order carried out by Caracciolo *et al*[4] has shown that in the temporal gauge the PV prescription used to treat the gauge-dependent poles leads to results which do not

[3] we stick to the case $n^0 > 0$

agree[4] with the ones obtained in the Feynman and Coulomb gauges.

On the other hand, one has a very different situation for the pure axial gauge for which $n^\mu = (0,0,0,n^3)$. Here, nothing whatsoever can be said *a priori* whether a given prescription to treat the gauge dependent pole $(k \cdot n)^{-1}$ will or will not violate causality, since analytic continuation for this peculiar case is ill-defined. In fact, preliminary results indicate that calculations in this particular gauge cannot be consistently carried out using the PV prescription, contrary to what has usually been claimed so far in the pertinent literature.

Secondly, let us consider the double pole $(k \cdot n)^{-2}$ cases. In the light-cone gauge, the appearance or no of the double pole factor $(k \cdot n)^{-2}$ in the Feynman amplitudes depends upon whether one uses the four-component uneliminated formalism or the eliminated two-component formalism. In the latter case, it arises, for instance, in the evaluation of the one-loop gluon self-energy. A sample calculation of a typical integral of this type has been presented in reference [3], where use of the causal prescription (or any other prescription which preserves causality) is *mandatory*.

Finally, a word on the axial gauge when one chooses for the external

[4] not surprisingly since causality has been broken by the PV prescription

vector $n^\mu = (n^0,0,0,n^3)$ such that $(n^0)^2 < (n^3)^2$. In this case, one ***has to*** use the causal prescription (or any other prescription preserving causality) in order to circumvent the gauge dependent poles.

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