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Tunneling time through a barrier using the tempus operator

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Tunneling Time through a Barrier using the *Tempus* Operator

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Abstract

The time a particle spends in a classically forbidden region of a potential barrier is expressed as an expectation value of the *tempus* operator in that region. Classically, *tempus* is canonically conjugate to the energy and is equal to the time for a conservative system. The tunneling time is calculated by this approach for a rectangular barrier, which gives a complex time. The imaginary part of the time is nonnegative, so it is interpreted as a tunneling time. The real part gives a negative value for some values of the parameters, and is therefore rejected because it violates causality. This tunneling time is compared with other tunneling times that have been suggested by also calculating them for the rectangular barrier.

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1. Introduction

Quantum mechanics has answered many questions about the microscopic world, but some questions still remain. One such question, addressed in this paper, is how long does a quantum particle spend inside a classically forbidden region. Two related questions are how long does it take for a quantum particle to traverse a classically forbidden region, and how long does it take for a quantum particle to be reflected from a classically forbidden region. Although there has been intense interest in answering these questions because of the need to design nanoscale electronic devices using semiconductor heterostructures, no consensus has yet emerged [1]-[6]. There are several different approaches to the above questions, which in general give quite different answers. They include the following. (1) The Feynman path integral is used to obtain an average time the particle traverses the forbidden region [7]-[13]. This *Feynman time* is a complex number, and a criterion is needed to interpret the real and imaginary parts. (2) The *Pollak-Miller time*, also complex, is the time average of the quantal microcanonical flux-flux correlation function [14, 15]. (3) The *dwell time* time is the time a particle spends in the classically forbidden region [16]-[19]. It is obtained by dividing the probability of finding the particle in the barrier by the incident probability flux. (4) The *bounce time* is the time required for a particle to traverse the barrier when its speed is equal to the magnitude of its imaginary momentum inside the barrier divided by its mass [18, 20, 21, 22]. (5) The *Martin-Landauer time* uses a method in which the barrier is modulated by a small oscillatory perturbation [18, 20]. It is extended to arbitrary potentials, regardless of whether free wave propagation or tunneling is involved [23]. (6) The *Larmor time* [24, 25, 16] is obtained by using a neutral particle with a magnetic moment and measuring its precession when a weak magnetic field is applied in the classically forbidden region. (7) The *Büttiker-Landauer time* is based on the energy modulation of the beam of incoming particles [20, 26]. (8) The *phase delay time* is obtained by differentiating with respect to the energy the phase shift due to the barrier [27]-[31]. (9) The *Bohm time* uses the quantum potential method and interpretation of quantum mechanics of David Bohm [32]-[37]. (10) Numerical simulations [38, 39, 40] have elucidated the dynamics of the wave function in barrier problems and have given insight into analytical methods. (11) The method of Olkhovsky and Recami [41]-[43] makes use of the expectation value with respect to the energy of their time operator. (12) The projection oper-

ator method of Muga, *et al.* [44]-[49] gives agreement with many previous results and also gives new results. (13) The *stay time* [50] is based on the Feynman path integral and gives a real time. It gives insight into the complex time of method (1). In this paper we give a new approach to the above question of how long a particle spends inside a classically forbidden region. We calculate an expectation value of the *tempus* operator [51] in the classically forbidden region. The *tempus* operator is the operator canonically conjugate to the energy operator. The time of evolution is still a parameter. Classically a canonical transformation can be made to a new set of canonical variables, in which the new canonical momentum is the energy and the new canonical coordinate conjugate to it is called *tempus* [52, 53]. For a conservative system, *tempus* is numerically equal to the time of evolution, but is conceptually different from it. Since the Poisson bracket is a canonical invariant [54], the Poisson bracket of *tempus* and energy is equal to one. When the system is quantized the commutator of the *tempus* operator and the energy operator is equal to $i\hbar$. The argument of Pauli [55] that a time operator in quantum mechanics does not exist does not apply, since the domain of definition of the *tempus* operator is restricted [56, 57, 43, 42, 51]. The time that a quantum particle spends in a classically forbidden region is calculated by taking an "expectation value" of the *tempus* operator using the wave function in the classically forbidden region. Which of the large number of tunneling times proposed is correct can only be decided by experiment. Different types of experiments may give different results for the tunneling time. The tunneling times based on the use of some kind of clock raises the question as to whether or not the result is independent of the type of clock used. If it is independent, then a universal intrinsic tunneling time exists. Very few experiments haven been performed up to now, and so the issue is undecided [58]. In this paper the new approach to the tunneling time question is given in Sec. 2. This new approach is applied in Sec. 3 to a rectangular barrier and the real and imaginary parts of the time are obtained. In Sec. 4 this method is compared with other methods for the rectangular barrier. The conclusion is given in Sec. 5.

2. New Approach

In this section we discuss a new approach to the time a particle spends in a classically forbidden region based on the concept of the *tempus* operator. The classical concept of *tempus* is defined and the quantum *tempus* operator is given. An expectation value of the *tempus* operator in the classically forbidden region is defined.

In classical mechanics a canonical transformation can be made from the old canonical variables (q, p) to new canonical variables (q', p'), where q (q') is the old (new) generalized coordinate and p (p') is the old (new) canonical momentum conjugate to it. If we now choose p' to be the energy E, the canonical generalized coordinate q' conjugate to it is called *tempus*, and denoted by T [52, 53]. Because energy and *tempus* are canonically conjugate and the Poisson bracket is a canonical invariant [54], energy and *tempus* satisfy the same Poisson brackets as q and p satisfy,

$$\{T, E\} = 1. \tag{2.1}$$

In quantum mechanics variables are replaced by operators and Poisson brackets are replaced by commutators. Therefore the energy E is replaced by an energy operator $\widehat{E}$, and *tempus* T is replaced by a *tempus* operator $\widehat{T}$. Together the two operators satisfy the standard commutation relations of quantum mechanics,

$$[\widehat{T}, \widehat{E}] = i\hbar, \tag{2.2}$$

corresponding to the classical Poisson bracket in Eq.(2.1). A realization of the energy operator $\widehat{E} = E$ and the *tempus* operator $\widehat{T} = i\hbar\partial/\partial E$ satisfies Eq.(2.2).

The time-independent Schrödinger equation for a particle of mass m and wave function $\psi(x)$ in one dimension is

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x) + V(x)\psi(x) = E\psi(x), \tag{2.3}$$

where $V(x)$ is the potential energy and E is the energy eigenvalue. The wave function depends in general on the energy.

We take an "expectation value" of the *tempus* operator to be the tunneling time. A new type of "expectation value" for the *tempus* operator in the barrier region B is defined as

$$t \equiv < \widehat{T} > = N^{-1}\int_{\mathsf{B}} dx\psi^*\widehat{T}\psi, \tag{2.4}$$

where the normalization integral N is defined as

$$N = \int_{\mathsf{B}} dx\psi^*\psi. \tag{2.5}$$

The *tempus* operator $\widehat{T} = i\hbar\partial/\partial E$ acts on the energy parameter in the wave function and thus is not a Hermitian operator in the Hilbert space. Therefore

Eq.(2.4) for the tunneling time is complex $\tau = \tau_1 + i\,\tau_2$, where the real part is τ_1 and the imaginary part is

$$\tau_2 = \Im m < \widehat{T} >= \frac{\hbar}{2}\frac{\partial \ln N}{\partial E}. \tag{2.6}$$

A criterion is needed to interpret the real and imaginary parts of τ as "tunneling times".

3. Tunneling through a Rectangular Barrier

In this section the new approach is applied to the case of a rectangular barrier. The Schrödinger equation is solved exactly for this case and an expectation value of the *tempus* operator is calculated. The real and imaginary parts of an expectation value of the *tempus* operator are plotted for different energies and thicknesses.

For a rectangular barrier the potential energy is

$$V(x) = \begin{cases} V_0 & \text{if } -a/2 \le x \le a/2 \\ 0 & \text{otherwise} \end{cases}, \tag{3.1}$$

where the barrier region B is $(-a/2, a/2)$ in Eqs.(2.4) and (2.5). The solution of the Schrödinger equation in Eq.(2.3) for this potential is the wave function

$$\psi = \begin{cases} e^{ikx} + Ae^{-ikx} & \text{for } x < -a/2 \\ Be^{\kappa x} + Ce^{-\kappa x} & \text{for } -a/2 \le x \le a/2 \\ De^{ikx} & \text{for } x > a/2 \end{cases}, \tag{3.2}$$

where the wave number k is

$$k = (2mE)^{1/2}/\hbar, \tag{3.3}$$

$$\kappa = [2m(V_0 - E)]^{1/2}/\hbar, \tag{3.4}$$

and the energy $E \le V_0$ is equal to or less than the barrier height.

For the wave function and its first derivative to be continuous at $x = -a/2$ and at $x = a/2$ it is necessary for the complex coefficients A, B, and C to satisfy

$$A = -i(k_0^2/2\kappa k)\sinh(\kappa a)\, D, \tag{3.5}$$

$$B = \exp[(ik - \kappa)a/2]\ \{(\kappa + ik)/2\kappa\}\ D, \tag{3.6}$$

and

$$C = \exp[(ik + \kappa)a/2]\ \{(\kappa - ik)/2\kappa\}\ D, \tag{3.7}$$

in terms of the transmission amplitude D, where the characteristic wave number k_0 is defined as

$$k_0 = (2mV_0)^{1/2}/\hbar \tag{3.8}$$

From Eqs.(3.3), (3.4) and (3.8) we have $k_0^2 = \kappa^2 + k^2$. The complex transmission amplitude D can be written as

$$D = \exp(i\phi)\mathsf{T}^{1/2}, \tag{3.9}$$

where the phase ϕ is

$$\phi = -ka + \arctan\left[\frac{(k^2 - \kappa^2)}{2\kappa k}\tanh(\kappa a)\right], \tag{3.10}$$

and the transmission coefficient $\mathsf{T} = |D|^2$ is

$$\mathsf{T} = \frac{(2\kappa k/k_0^2)^2}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2}. \tag{3.11}$$

When Eq.(3.2) for the wave function is substituted into Eq.(2.5) for the normalization integral, and the result is substituted into Eq.(2.6), the imaginary part of the complex tunneling time is

$$\begin{aligned} \frac{\tau_2}{\tau_0} = & \left(\frac{k_0}{k}\right)^2 + \left[\frac{(ak_0)^2(2\kappa a)^{-1}\sinh(2\kappa a) + 4(k^2 - \kappa^2)/k_0^2}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2}\right] \\ & - \frac{1}{2(\kappa/k_0)^2}\left[\frac{\cosh(2\kappa a) - (2\kappa a)^{-1}\sinh(2\kappa a) + (2\kappa/k_0)^2}{(2\kappa a)^{-1}\sinh(2\kappa a) + (\kappa^2 - k^2)/k_0^2}\right], \end{aligned} \tag{3.12}$$

in terms of the characteristic time

$$\tau_0 = \hbar/2V_0, \tag{3.13}$$

where k, κ and k_0 are given by Eqs.(3.3), (3.4) and (3.8), respectively. Equation (3.12) is plotted in Fig. 3.1 as a function of the dimensionless energy $\varepsilon = E/V_0$ and the dimensionless width ak_0. For all values of the parameters the imaginary part τ_2 is nonnegative, so that it satisfies causality in the sense that the particle does not emerge from the far side of the barrier before it arrives at the near side.

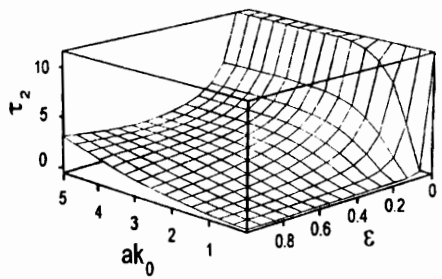

FIG. 3.1: Imaginary part of the tunneling time τ_2 in Eq.(3.12), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

The real part τ_1 of $\tau = < \widehat{T} >$ in Eq.(2.6) is obtained by substituting Eq.(3.2) into it and taking the real part, which gives

$$\frac{\tau_1}{\tau_0} = \frac{ak_0}{k/k_0}\left\{\frac{\sinh^2(\kappa a) - 2(2\kappa a)^{-1}\sinh(2\kappa a) + (k/k_0)^2}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2}\right. \tag{3.14}$$

$$\left. - \frac{(2\kappa a)^{-1}\sinh(2\kappa a) - 2(\kappa a)^{-2}\sinh^2(\kappa a) + 1}{2[(2\kappa a)^{-1}\sinh(2\kappa a) + (\kappa^2 - k^2)/k_0^2]}\right\}.$$

This expression is plotted in Fig. 3.2 as a function of $\varepsilon = E/V_0$ and ak_0. For some values of the parameters, τ_1 becomes negative, which violates causality. This violation of causality for some values of the parameters is sufficient reason to reject this expression as a legitimate tunneling time.

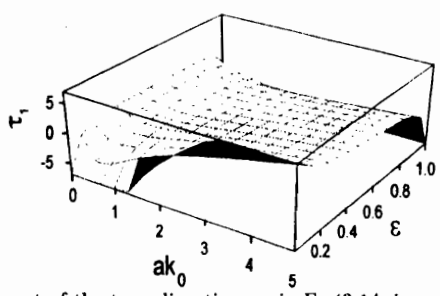

FIG. 3.2: Real part of the tunneling time τ_1 in Eq.(3.14, in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4. Other Approaches to Tunneling Times

In this section some other methods that have been proposed for the tunneling time are surveyed and applied to a rectangular barrier.

4.1. Feynman Path Integral Approach

The Feynman path integral has been another approach to tunneling times [7]-[13]. For a rectangular barrier of height V_0, the Feynman time τ_F for tunneling is complex [1]

$$\tau_F = -\hbar\frac{\partial\phi}{\partial V_0} + i\frac{\hbar}{2}\frac{\partial \ln \mathsf{T}}{\partial V_0}. \tag{4.1}$$

The phase ϕ is given in Eq.(3.10) and the transmission coefficient T is given in Eq.(3.11).

The real part of the Feynman tunneling time is obtained by substituting Eq.(3.10) for the phase and Eq.(3.4) into Eq.(4.1). When the real part is taken we obtain

$$\frac{\Re e\,\tau_F}{\tau_0} = 2ka\left[\frac{(\kappa^2 - k^2)/k_0^2 + (2\kappa a)^{-1}\sinh(2\kappa a)}{(2\kappa k/k_0^2)^2 + \sinh^2(\kappa a)}\right], \tag{4.2}$$

where the characteristic time τ_0 is defined in Eq.(3.13). This tunneling time is plotted as a function of the energy $\varepsilon = E/V_0$ and the width ak_0 in Fig. 4.1. The real part of the Feynman tunneling time is nonnegative, so it can be taken as a legitimate tunneling time.

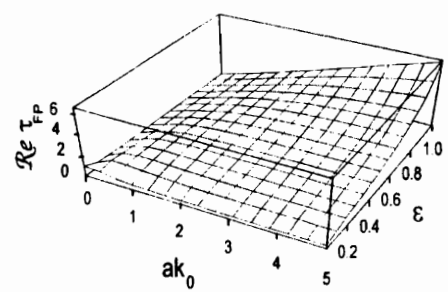

FIG. 4.1: Real part of Feynman time τ_F for tunneling in Eq.(4.2), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

The imaginary part of the Feynman time is obtained by substituting Eq.(3.11) for the transmission coefficient and Eq.(3.4) into Eq.(4.1). When the imaginary part is taken, we get

$$\frac{\Im m\, \tau_F}{\tau_0} = (ak_0)^2 \left[\frac{(\kappa a)^{-2} \sinh^2(\kappa a) - (2\kappa a)^{-1} \sinh(2\kappa a)}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2} \right] \tag{4.3}$$

This time is plotted as a function of ε and ak_0 in Fig. 4.2.

The negative times in Fig. 4.2 for some parameters violate causality, which provides a sufficient reason for rejecting the imaginary part of the Feynman time as a tunneling time.

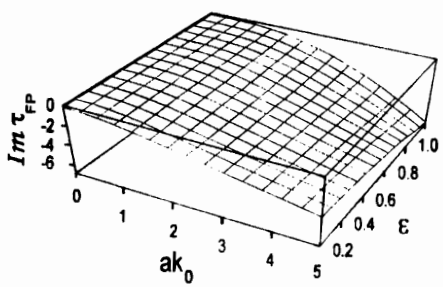

FIG. 4.2: Imaginary part of the Feynman time τ_F for tunneling in Eq.(4.3), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4.2. Pollak and Miller Approach

Pollak and Miller [14, 15] applied the quantal flux-flux correlation function to barriers to obtain a complex tunneling time, in analogy with the use of classical flux-flux correlation functions to obtain characteristic times in chemical systems. The imaginary part of their tunneling time is [1]

$$\tau_{PM} = \Im m \left\{ -i\hbar \frac{\partial \ln D}{\partial E} \right\} = -\frac{\hbar}{2} \frac{\partial \ln T}{\partial E}, \tag{4.4}$$

when Eq.(3.9) is used. This expression is similar to the imaginary part of Eq.(4.1), except for a minus sign and in the derivative the total energy E is used instead of the height of the barrier V_0.

When the transmission coefficient in Eq.(3.11) is substituted into Eq.(4.4), the result for the rectangular barrier is

$$\frac{\tau_{PM}}{\tau_0} = \frac{(ak_0)^2}{2} \left[\frac{(1 - \kappa^2/k^2)(\kappa a)^{-2} \sinh^2(\kappa a) - (2\kappa a)^{-1} \sinh(2\kappa a)}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2} \right]. \tag{4.5}$$

This expression is plotted in Fig. 4.3 as a function of the energy ε and the width ak_0 and is nonnegative for all parameters.

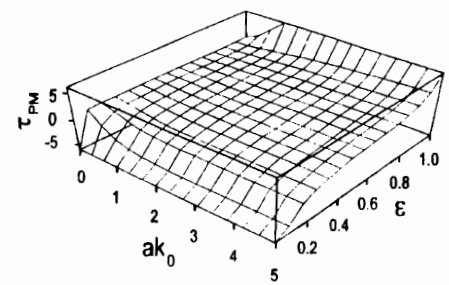

FIG. 4.3: The imaginary part of the Pollak and Miller tunneling time τ_{PM} in Eq.(4.5), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

The real part of the expression in braces in Eq.(4.4) gives the real part of the Pollak-Miller time

$$\tau'_{PM} = \Re e \left\{ -i\hbar \frac{\partial \ln D}{\partial E} \right\} = \hbar \frac{\partial \phi}{\partial E}, \tag{4.6}$$

when Eq.(3.9) is used. This expression is similar to the real part of Eq.(4.1), except for the minus sign and in the derivative the total energy E is used instead of the height of the barrier V_0.

When Eq.(3.10) is substituted into Eq.(4.6), we obtain for the real part of the Pollak-Miller time

$$\frac{\tau'_{PM}}{\tau_0} = -\frac{ak_0}{2(k/k_0)} + \frac{ak_0}{(k/k_0)} \frac{[k^2(\kappa^2 - k^2)/k_0^2 + (2\kappa a)^{-1} \sinh(2\kappa a)]}{[(2\kappa k/k_0^2)^2 + \sinh^2(\kappa a)]} \tag{4.7}$$

The first term on the right-hand side of this equation is due to the contribution of $-ka$ in the phase ϕ in Eq.(3.10). The second term on the right-hand side is the phase delay time discussed in Sec. 4.8. Equation (4.7) is plotted in Fig. 4.4 as a function of the energy ε and the width ak_0.

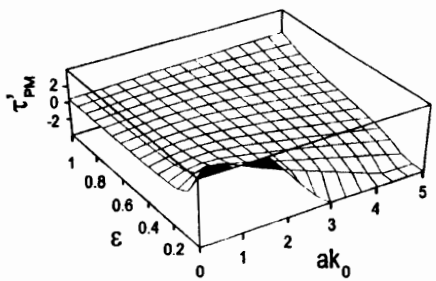

FIG. 4.4: The real part of the Pollak and Miller tunneling time τ'_{PM} in Eq.(4.7), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4.3. Dwell Time

The dwell time [16]-[19] is defined as the relative probability for a particle to be in the barrier B divided by the probability current density J of the incident particle,

$$\tau_D = \frac{\int_{\mathsf{B}} dx \psi^* \psi}{J}. \tag{4.8}$$

The probability current density J for an incident particle with a wave function $\psi_{inc} = \exp(ikx)$ as in Eq.(3.2) is

$$J = \Re e\ \psi^*_{inc}(p_x/m)\psi_{inc} = \hbar k/m, \tag{4.9}$$

where $p_x = -i\hbar\partial/\partial x$ is the linear momentum operator. For a rectangular barrier, the barrier region B is the region from $(-a/2, a/2)$. When Eq.(2.5) is evaluated for a rectangular barrier using Eq.(3.2) and is substituted into Eq.(4.8) along with Eq.(4.9), the result is

$$\frac{\tau_D}{\tau_0} = (2ak)\left[\frac{(2\kappa a)^{-1}\sinh(2\kappa a) + (\kappa^2 - k^2)/k_0^2}{\sinh^2(\kappa a) + (2\kappa k/k_0^2)^2}\right]. \tag{4.10}$$

This expression in plotted in Fig. 4.5 as a function of the energy and the width of the barrier. The dwell time is a measure of how long the particle stays in the barrier, regardless if it is ultimately reflected or transmitted. It is positive for all parameters.

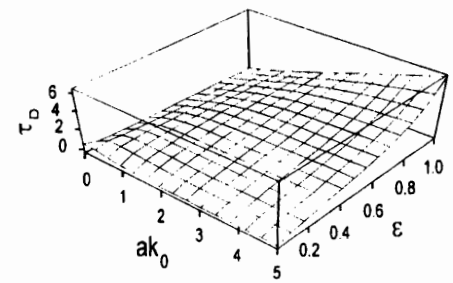

FIG. 4.5: Dwell time τ_D in Eq.(4.10), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4.4. Bounce Time

The bounce time [18, 20, 21, 22] is the time that it takes a particle in a barrier B with a velocity which is the magnitude of its imaginary momentum $\hbar\kappa(x) = [E - V(x)]^{1/2}$ divided by its mass m to travel through the barrier

$$\tau_B = \int_{\mathsf{B}} dx \frac{m}{\hbar\kappa(x)}. \tag{4.11}$$

For a rectangular barrier of width a this bounce time reduces to

$$\frac{\tau_B}{\tau_0} = \frac{ak_0}{\kappa/k_0}, \tag{4.12}$$

when written in dimensionless form.

The addition of a small oscillatory perturbation to the potential gives an adiabatic regime for low frequency and a high frequency regime [18, 20]. The two regimes are separated by a frequency corresponding to the bounce time.

Figure 4.6 shows the bounce time as a function of the energy ε and the dimensionless width ak_0. It is nonnegative for all parameters.

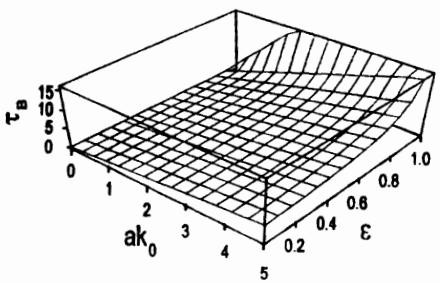

FIG. 4.6: The bounce time τ_B in Eq.(4.12), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the width ak_0.

4.5. Martin and Landauer Time

In 1993 Martin and Landauer [23] reconsidered a method used by Büttiker and Landauer [18, 20] in which the barrier is modulated by a small oscillatory perturbation. For an arbitrary potential barrier preceded by a region of free propagation, they find that the interaction time is

$$\tau_{ML} = \hbar \left| \frac{\partial \ln(\partial D/\partial E)}{\partial E} \right|, \tag{4.13}$$

where D is the transmission amplitude. They determine the interaction time by observing how the transmission of the barrier with an oscillatory modulation superimposed upon it begins to deviate from adiabatic behavior.

For a rectangular barrier, the transmission amplitude D is given in Eq.(3.9). When this expression is used in Eq.(4.13), the Martin-Landauer time τ_{ML} is obtained for a rectangular barrier. Figure 4.7 shows τ_{ML} for a rectangular barrier as a function of the dimensionless energy ε and the dimensionless width ak_0.

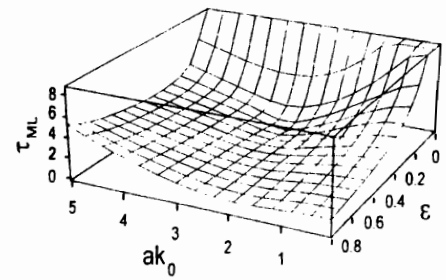

FIG. 4.7: The Martin-Landauer time τ_{ML} in Eq.(4.13) for a rectangular barrier, in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the width ak_0.

4.6. Larmor Time

The Larmor precession [24, 25] of a neutral particle with a magnetic moment in a weak magnetic field can be used as a clock. Büttiker [16] showed that if the spin of the particle were polarized in the y direction perpendicular to the direction of the magnetic field (z direction) and the direction of motion of the particle (x direction), the time associated with the precession of the spin is

$$\tau_L^y = \hbar \frac{\partial \phi}{\partial V_0}. \tag{4.14}$$

The time associated with the rotation of the spin out of the xy plane is

$$\tau_L^z = -\frac{\hbar}{2} \frac{\partial \ln T}{\partial V_0}. \tag{4.15}$$

This effect is due to the fact that the spin down state has a different energy than the spin up state, and consequently the two spin states are transmitted through the barrier differently.

As shown by Sokolovski and Baskin [7], the times in Eqs.(4.14) and (4.15) are the negative of the real and imaginary parts of the Feynman time in Eq.(4.1), so $\tau_L^y = -\Re e\, \tau_F$ and $\tau_L^z = -\Im m\, \tau_F$.

Büttiker [16] argued that a total Larmor tunneling time τ_L^T should be specified by taking the square root of the sum of the squares of the tunneling times in Eqs.(4.14) and (4.15), which gives a measure of the total rotation,

$$\tau_L^T = [(\tau_L^y)^2 + (\tau_L^z)^2]^{1/2} = |\tau_F|. \tag{4.16}$$

where τ_F is given in Eq.(4.1).

For the rectangular barrier the total Larmor tunneling time τ_L^T is obtained by substituting the negatives of Eqs.(4.2) and (4.3) for Eqs.(4.14) and (4.15), respectively, in Eq.(4.16). The result for the total Larmor tunneling time is plotted in Fig. 4.8.

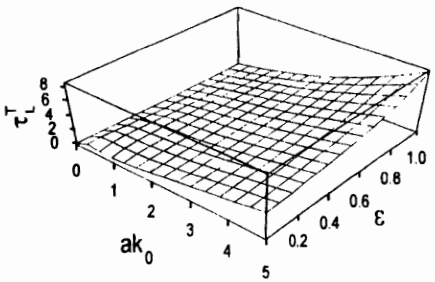

FIG. 4.8: Total Larmor tunneling time τ_L^T in Eq.(4.16), in units of $\tau_0 = \hbar/2V_0$, as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4.7. Büttiker and Landauer Time

In 1985 Büttiker and Landauer [20, 26] suggested using a modulated incident amplitude, which was investigated further by Martin and Landauer [23]. This method yields the Büttiker-Landauer tunneling time τ_{BL} defined as

$$\tau_{BL} = \hbar \left| \frac{\partial \ln D}{\partial E} \right|, \tag{4.17}$$

where D is the transmission amplitude. When the transmission amplitude D in Eq.(3.9) in terms of the phase ϕ and the transmission coefficient T is substituted into Eq.(4.17), we obtain

$$\tau_{BL} = \left[\left(\hbar \frac{\partial \phi}{\partial E} \right)^2 + \left(\frac{\hbar}{2} \frac{\partial \ln \mathrm{T}}{\partial E} \right)^2 \right]^{1/2}. \tag{4.18}$$

The Büttiker-Landauer time τ_{BL} is equal to the magnitude of the complex Pollak-Miller time in Eqs.(4.4) and (4.6), $\tau_{BL} = [\tau_{PM}^2 + \tau_{PM}'^2]^{1/2} = \left|\tau_{PM}' + i\tau_{PM}\right|$.

For a rectangular barrier Eqs.(4.7) and (4.5) can be used for the real and imaginary Pollak-Miller times in Eq.(4.18). The resulting Büttiker-Landauer time τ_{BL} obtained is plotted in Fig. 4.9.

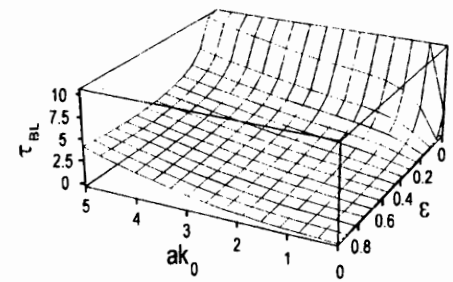

FIG. 4.9: The Büttiker-Landauer time τ_{BL} in Eq.(4.18), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

4.8. Phase Delay Time

The phase delay time τ_ϕ is defined as [27]-[31]

$$\tau_\phi = \hbar \frac{\partial \Delta\phi}{\partial E}, \tag{4.19}$$

where $\Delta\phi$ is the phase change due to the presence of the barrier.

For the rectangular barrier the phase change $\Delta\phi$ is defined as

$$\Delta\phi = ka + \phi = \arctan\left[\frac{(k^2 - \kappa^2)}{2\kappa k} \tanh(\kappa a) \right], \tag{4.20}$$

from Eq.(3.10). When Eq.(4.20) is differentiated with respect to the energy, and written in a dimensionless form, we find that for the rectangular barrier the phase delay time τ_ϕ is

$$\frac{\tau_\phi}{\tau_0} = \frac{(ak_0)}{(k/k_0)} \frac{[k^2(\kappa^2 - k^2)/k_0^4 + (2\kappa a)^{-1}\sinh(2\kappa a)]}{[\sinh^2(\kappa a) + (2k\kappa/k_0^2)^2]}, \tag{4.21}$$

where $\tau_0 = \hbar/2V_0$ is a characteristic time. Equation (4.21) for the phase delay time is plotted in Fig. 4.10.

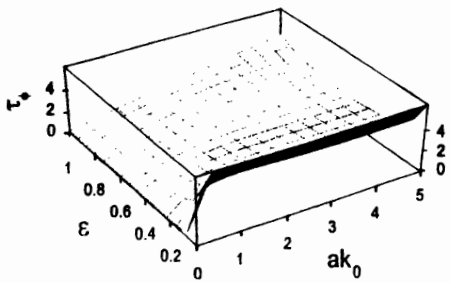

FIG. 4.10: The phase delay time τ_ϕ in Eq.(4.21), in units of $\tau_0 = \hbar/2V_0$, is plotted as a function of the energy $\varepsilon = E/V_0$ and the barrier width ak_0.

5. Conclusion

We give a new formulation of the tunneling time as an expectation value of the *tempus* operator in the barrier region. The *tempus* operator, conjugate to the energy operator, is not Hermitian and only acts on the energy as a parameter, so its "expectation value" is complex. For a rectangular barrier of width a and height V_0 we calculate the real and imaginary parts of this tunneling time, and plot them as functions of the dimensionless energy $\varepsilon = E/V_0$ and barrier thickness ak_0. The imaginary part of this time is nonnegative for the rectangular barrier, whereas the real part is negative for some values of the parameters. The tunneling time has to be nonnegative for causality not to be violated, so the real part of the calculated tunneling time is rejected. Only the imaginary part of the tunneling time, which of course is a real number, has physical significance. For the rectangular barrier we also calculate other tunneling times that have been proposed, and compare them with our results. The other tunneling times we calculate for the rectangular barrier are the Feynman time from the path integral method, the Pollak-Miller time, the dwell time, the bounce time, the Martin-Landauer time, the Larmor time, the Büttiker-Landauer time, and the phase delay time.

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References

[1] R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994).

[2] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989).

[3] M. Jonson, in *Quantum Transport in Semiconductors*, edited by D. K. Ferry and C. Jacobini (Plenum Press, NY, 1991), pp. 193-238.

[4] A. P. Jauho, in *Hot Carriers in Semiconductor Nanostructures: Physics and Applications*, edited by J. Shah (Academic Press, Boston, 1992), pp. 121-151.

[5] M. Kleber, Phys. Rep. **236**, 331 (1994).

[6] R. Landauer, Ber. Bunsenges. Phys. Chem. **95**, 404 (1991).

[7] D. Sokolovski and L. M. Baskin, Phys. Rev. A **36**, 4604 (1987).

[8] D. Sokolovski and J. N. L. Connor, Phys. Rev. A **42**, 6512 (1990).

[9] D. Sokolovski and J. N. L. Connor, Phys. Rev. A **44**, 1500 (1991).

[10] D. Sokolovski and J. N. L. Connor, Phys. Rev. A **47**, 4677 (1993).

[11] D. Sokolovski, S. Brouard, and J. N. L. Connor, Phys. Rev. A **50**, 1240 (1994).

[12] H. A. Fertig, Phys. Rev. Lett. **65**, 2321 (1990).

[13] H. A. Fertig, Phys. Rev. B **47**, 1346 (1993).

[14] E. Pollak and W. H. Miller, Phys. Rev. Lett. **53**, 115 (1984).

[15] E. Pollak, J. Chem. Phys. **83**, 1111 (1985).

[16] M. Büttiker, Phys. Rev. B **27**, 6178 (1983).

[17] F. T. Smith, Phys. Rev. **118**, 349 (1960).

[18] M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).

[19] M. Büttiker, in *Electronic Properties of Multilayers and Low-Dimensional Semiconductor Structures*, edited by J. M. Chamberlain, L. Eaves, and J.-C. Portal (Plenum Press, NY, 1990), pp. 297-315.

[20] M. Büttiker and R. Landauer, Phys. Scr. **32**, 429 (1985).

[21] A. J. Leggett, Prog. Theor. Phys. Suppl. **69**, 80 (1980).

[22] D. Mugnai and A. Ranfagni, Europhys. Lett. **6**, 1 (1988).

[23] Th. Martin and R. Landauer, Phys. Rev. A **47**, 2023 (1993).

[24] A. I. Baz', Sov. J. Nucl. Phys. **4**, 229 (1967).

[25] V. F. Rybachenko, Sov. J. Nucl. Phys. **5**, 635 (1967).

[26] M. Büttiker and R. Landauer, IBM J. Res. Dev. **30**, 451 (1986).

[27] E. P. Wigner, Phys. Rev. **98**, 145 (1955).

[28] T. E. Hartman, J. Appl. Phys. **33**, 3427 (1962).

[29] S. Collins, D. Lowe, and J. R. Barker, J. Phys. C **20**, 6213 (1987).

[30] M. Büttiker and R. Landauer, J. Phys. C **21**, 6207 (1988).

[31] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951), (Dover, New York, 1989). pp. 260-261, 290-292.

[32] C. R. Leavens and G. C. Aers, in *Scanning Tunneling Microscopy III*, edited by R. Wiesendanger and H.-J. Günterodt (Springer, NY, 1993), p. 105.

[33] W. R. McKinnon and C. R. Leavens, Phys. Rev. A **51**, 2748 (1995).

[34] C. R. Leavens, Solid State Commun. **86**, 781 (1993).

[35] C. R. Leavens, Solid State Commun. **85**, 115 (1993).

[36] C. R. Leavens, Found. Phys. **25**, 229 (1995).

[37] C. R. Leavens, Phys. Lett. A **197**, 88 (1995).

[38] A. Goldberg, H. M. Schey, and J. L Schwartz, Am. J. Phys. **35**, 177 (1967).

[39] A.-P. Jauho, in *Quantum Transport in Semiconductors*, edited by D. K. Ferry and C. Jacobini (Plenum Press, NY, 1991), pp. 179-192.

[40] A. P. Jauho and M. Jonson, Superlattices and Microstructures **6**, 303 (1989).

[41] V. S. Olkhovsky and E. Recami, Phys. Rep. **214**, 339 (1992).

[42] V. S. Olkhovsky, E. Recami, and A. J. Gerasimchuk, Nuovo Cimento A **22**, 263 (1974).

[43] E. Recami, in *The Uncertainty Principle and Foundations of Quantum Mechanics*, edited by W. C. Price and S. S. Chissick (Wiley, New York, 1977), pp. 21-28.

[44] J. G. Muga, S. Brouard, and R. Sala, Phys. Lett. A **167**, 24 (1992).

[45] J. G. Muga, S. Brouard, and R. F. Snider, Phys. Rev. A **46**, 6075 (1992).

[46] J. G. Muga and R. F. Snider, Phys. Rev. A **45**, 2940 (1992).

[47] J. G. Muga, S. Brouard, and R. Sala, J. Phys. Condens. Matter **4**, L579 (1992).

[48] S. Brouard, R. Sala, and J. G. Muga, Phys. Rev. A **49**, 4312 (1994).

[49] J. G. Muga, S. Brouard, and D. Macías, Ann. Phys. (N.Y.) **240**, 351 (1995).

[50] G. Iannaccone and B. Pellegrini, Phys. Rev. B **49**, 16548 (1994).

[51] D. H. Kobe and V. C. Aguilera-Navarro, Phys. Rev. A **50**, 933 (1994).

[52] D. H. Kobe, Am. J. Phys. **61**, 1031 (1993).

[53] D. H. Kobe, Eur. J. Phys. **14**, 262 (1993).

[54] H. Goldstein, *Classical Mechanics*, 2nd ed. (Addison-Wesley, Reading, MA, 1980), p. 398.

[55] W. Pauli, *General Principles of Quantum Mechanics* (Springer-Verlag, Berlin, 1980), p. 63.

[56] E. Fick and F. Engelmann, Z. Phys. **175**, 271 (1963).

[57] F. Engelmann and E. Fick, Z. Phys. **178**, 551 (1964).

[58] R. Landauer, Nature **341**, 567 (1989).