IFT-P.009-94
Fermilab Library
0 1160 0033237 3

IFT

Instituto de Física Teórica
Universidade Estadual Paulista

MAR 29 1994

February/94

IFT-P.009/94

Flattening of the resonance spectrum of hadrons from κ-deformed Poincaré algebra

Jishnu Dey*, Paulo Leal Ferreira, Lauro Tomio

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 - São Paulo, S.P.
Brazil

and

Rajkumar Roy Choudhury

Indian Statistical Institute
203 B. T. Road
Calcutta 700 035
India

RETURN TO

*Supported by FAPESP, S.P., Brazil and in part by DST grant no. SP/S2/04/93, Govt. of India, permanent address : 1/10 Prince Golam Md. Road, Calcutta 700 026, India.

Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 251-5155
Telefax: 55 (11) 288-8224
Telex: 55 (11) 31870 UJMFBR
Electronic Address: LIBRARY@IFT.UESP.ANSP.BR
47553::LIBRARY

Flattening of the Resonance Spectrum of Hadrons from κ-deformed Poincaré Algebra

Jishnu Dey*, Paulo Leal Ferreira, Lauro Tomio,
Instituto de Física Teórica - UNESP, São Paulo, 01405-900, Brasil
and
Rajkumar Roy Choudhury
Indian Statistical Institute, 203 B. T. Road, Calcutta 700 035, India

March 1, 1994

Abstract : Recently Lukierski et al. [1] defined a κ-deformed Poincaré algebra which is characterized by having the energy-momentum and angular momentum sub-algebras not deformed. Further Biedenharn et al.[2] showed that on gauging the κ -deformed electron with the electromagnetic field, one can set a limit on the allowed value of the deformation parameter $\epsilon \equiv 1/\kappa < 1$ fm. We show that one gets Regge like angular excitations, J, of the mesons, non-strange and strange baryons, with a value of $\epsilon \sim 0.082$ fm and predict a flattening with J of the corresponding trajectories. The Regge fit improves on including deformation, particularly for the baryon spectrum.

*Supported by FAPESP, S.P., Brasil and in part by DST grant no. SP/S2/K04/93, Govt. of India, permanent address : 1/10 Prince Golam Md. Road, Calcutta 700 026, India

Q-deformed algebras are new symmetry structures recently introduced into physics [3]. For the Poincaré group the deformation structure is not unique, a particular form due to [1] is considered for this paper. They showed that the three dimensional rotation and the translation subgroups are not deformed, the algebra of Lorentz boosts is modified, both for bosons and fermions. The relevant q-deformation parameter, in this case is called κ, and when this goes to infinity we recover the undeformed algebra [1]. This κ-deformed version has two Casimir invariants C_1 and C_2 [1]. The κ-deformed κ-Dirac equation has recently been found [4], and it factorizes the second Casimir invariant, quite unlike the usual case which factorizes the first one.

We will use the notation of Biedenharn, Mueller and Tarlini [2], who for example use ϵ, the inverse of κ, to avoid confusion with the Dirac quantum number. To obtain the Dirac-Coulomb equation they gauged the deformed Dirac equation, applied it to the quantum relativistic hydrogen atom, with the aim of finding any induced shift in the ground state energy level with the recent precision quantum optics measurements. They obtained the remarkable result that the first order effect vanishes identically, while from the logarithmic cut-off of the second order perturbation they find ϵ to be less than 1 fm. As they comment this self consistent estimate of the Poincaré length is both speculative and not very accurate. But it raises the important question that this may then affect particle and nuclear physics phenomenology since the relevant length scale there is indeed the fermi.

Can one specify ϵ, by *assuming* κ-deformation and by looking at the Regge type angular excitations of the particle spectrum ? We answer this question in the affirmative. The present analysis adopts a simplified picture, associated to a single parameter, the Regge slope α'. Once α' is given by an acceptable theory, the parameter ϵ can be obtained by minimizing the sum of the average deviations (between theory and experimental results) we obtain for each particle trajectory. This slope we fix from calculations with strings ([5], [6]), that gives as a reasonable value $\alpha' = 0.85\ GeV^{-2}$. Given this α', the deformation parameter is obtained by the best fitting of the Poincaré deformed values of the energies of the states $E(J)$ to the experimental baryon spectrum. From the pion to the baryons, the spectrum of all particles is fitted with a uniform value of the deformation parameter $\epsilon = 0.082$ fm, given the Regge slope as $\alpha' = 0.85\ GeV^{-2}$. The results, displayed on Tables 1-7, already shows that our fits are better than the fits given by the Regge trajectories, using this set of parameters. Now, if we release this constraint from string theories, we observe that a better fit can be achieved with a smaller value of α'. We vary α' within a large range, $\alpha' < 0.9\ GeV^{-2}$, and find that with κ-deformation we always fit the experimental trajectories better than the Regge formula. Our best fit, in the κ-deformed case, gives $\alpha' = 0.72\ GeV^{-2}$ and $\epsilon = 0.14$ fm, as we show through the minimum of the total deviation given in Fig.1. Also we show in Fig.2 that such a best fit presents a significant curvature for some states which are well fitted. In Fig.1 we clearly show that

the best Regge trajectory results without deformation (corresponding to $\epsilon = 0$ in the deformed case) are near $\alpha = 0.9\ GeV^{-2}$, and that such result is considerably improved by introducing deformation and a smaller value of α.

As is well known, the model of a rotating string predicts a linear relation between the square of the mass of a state and its orbital angular momentum [7]. In the simplest variant, the quarks at the ends of the rotating string are massless, so the model contains only a single parameter - the string tension. In more refined versions, strings with masses at the ends - quarks or di-quarks, are introduced to treat mesons [5] and baryons (see [6] and references therein).

We start with the commutation relations [1] between the components of energy-momentum P_ν (ν = 0,1,2,3), angular momentum M_i and Lorentz boosts L_i:

$$
\begin{gathered}
[P_i, P_j] = 0, \quad [P_i, P_0] = 0, \quad [M_i, P_j] = i\varepsilon_{ijk}P_k, \\
[M_i, P_0] = 0, \quad [L_i, P_0] = iP_i, \quad [L_i, P_j] = i\epsilon^{-1}\delta_{ij}sinh(\epsilon P_0), \\
[M_i, M_j] = i\varepsilon_{ijk}M_k, \quad [M_i, L_j] = i\varepsilon_{ijk}L_k, \\
[L_i, L_j] = -i\varepsilon_{ijk}\left(M_k cosh(\epsilon P_0) - \frac{\epsilon}{4}P_k P_l M_l\right),
\end{gathered}
\tag{1}
$$

where i, j, k = 1, 2, 3. Following is the first Casimir operator, C_1, which commutes with all the generators :

$$C_1 = \left[\frac{2}{\epsilon}sinh\left(\frac{\epsilon P_0}{2}\right)\right]^2 - P_i P_i. \tag{2}$$

In the rest frame ($P_i = 0$) :

$$C_1 = \left[\frac{2}{\epsilon}sinh\left(\frac{m\epsilon}{2}\right)\right]^2 \tag{3}$$

and for a general frame $(E, \mathbf{P})$ $(E \equiv P_0)$

$$\left[\frac{2}{\epsilon}sinh\left(\frac{\epsilon E}{2}\right)\right]^2 = \mathbf{P}^2 + \left[\frac{2}{\epsilon}sinh\left(\frac{m\epsilon}{2}\right)\right]^2. \tag{4}$$

This gives

$$E = \frac{2}{\epsilon}sinh^{-1}\left[\left(\frac{\epsilon}{2}\right)^2\frac{J}{\alpha'} + sinh^2\left(\frac{m\epsilon}{2}\right)\right]^{1/2} \tag{5}$$

where we have put, in the spirit of the Regge phenomenology :

$$E^2 = m^2 + \frac{J}{\alpha'} \tag{6}$$

with J = 0, 1, 2, ... for mesons and J = 1/2, 3/2, ... for baryons. In eq.(6), m^2 is fixed by the first physical state of the trajectory and α' gives the Regge slope which, in string theories, is related to the string tension σ through $\alpha' = (2\pi\sigma)^{-1}$ and, therefore, with the dimension of the system. This m is sometimes unphysical for example for the ρ meson where the first state starts from J = 1, as is well known to users of Regge theory [5], [6]. In the limit ϵ going to zero, we get the above equation from eq.(5). Observe that we can rewrite the eq.(5) such that

$$E = \frac{1}{\epsilon}cosh^{-1}\left[\left(\frac{\epsilon^2(J - j_o)}{2\alpha'}\right) + cosh\left(\epsilon m_o\right)\right], \tag{7}$$

where m_o and j_o are the mass and the angular momentum of the first state.

We note that the present phenomenology is solely based on the first deformed Casimir operator C_1 of the κ-deformed Poincaré algebra. It is clear however, that a more refined analysis may require the explicit introduction of the 2nd. deformed Casimir operator C_2, as well.

The marked difference between the equations eq.(5) and eq.(6) is for high J, the first one showing flattening of the spectrum, while E^2 is linear in J in the second one. The kind of result that we get is indicated by considering any hadron, with J = 1000 say, where eq.(5) gives $\sim 12.8\ GeV$ for $\alpha' = 0.85\ GeV^{-2}$ and $\sim 9.0\ GeV$ for $\alpha' = 0.72\ GeV^{-2}$, when eq.(6) gives 34.3 GeV. To our knowledge, the first reference to a possible flattening of the hadronic trajectories is due to Hendry [8].

We calculate several hadronic trajectories, 27 in all, 13 mesonic and 14 baryonic. When the parameter α' is taken to be 0.85 GeV^{-2}, and ϵ is fitted to be 0.415 GeV^{-1}, we get results that are displayed in Tables 1-7, both for eq.(6) and eq.(5), or equivalently eq.(7). We see that there is good fit to experiment [9]. We define the dimensionless root mean square deviation as

$$\delta = \sqrt{\left[\frac{1}{n}\sum_i^n\left(1 - \frac{E_{th}}{E_{exp}}\right)_i^2\right]}, \tag{8}$$

where n is the number of states for which E_{exp} are known. We determine ϵ by requiring that the sum of δ-s be minimum for the chosen fit of mesons and baryons. Eq.(6) gives a sum of δ-s = 1.188 (for 27 trajectories), whereas the κ deformed case gives a value 0.990. As we have already mentioned the baryon fit looks better than the meson fit for the κ deformed case.

As we have stated before, we have also varied α' and in each case find the ϵ that gives the minimum total deviation for the 27 trajectories chosen by us. These are shown in Fig. 1 where the total deviation for the Regge case, and for the minimum ϵ found in κ deformed case, are also shown. We find that the latter show a global minimum for a value of $\alpha' = 0.72\ GeV^{-2}$. At this value the meson trajectory f_0-ω and the baryon Σ^*

are plotted in fig. 2, since they have observed high angular momentum excitations. The graphs show significant curvature.

Recently there has been some excitement about hadronic scale string theories [11], [12], [13]. This started from the work of Kutasov and Seiberg [14] who observed that destabilizing tachyons in a string theory severely constrains the difference of the asymptotic densities of bosons and fermions in that theory. Their result shows that tachyon elimination does not require full fledged supersymmetry. Cancellation between the boson and fermion density of states is all that is needed. These results were translated for hadronic scale string theory [11] where indeed the experimental and some model density of states of mesons and baryons [15] are seen to be equal over a large energy range. In [12] these results are analyzed further, and it was shown that the boson/fermion ratio of the theory fluctuates around unity, gradually approaching unity. In particular, it is obvious that at low energy the meson density is higher than that of baryons, and then the latter take over. The pattern continues for more involved boson and fermion configurations. This is reviewed in [16]. In the more recent paper Dienes and Cudell [13] claim that conformal invariance and modular invariance is indeed reflected in the hadron spectrum. It could be that a complete treatment of κ deformed Poincaré algebra with both could also yield information about the density of states and allow comparison with their results. This would be fascinating. At this point the only comment we can make is that we take α' as suggested in [13] for our tables and so our good fit supports their limiting temperature of $T_H = 300$ MeV.

A simple model of rotator based on the κ-Poincaré energy mass relation describes the rotational levels of even nuclei [17]. Since the velocities here are typically a thousandth part of that of light, the appropriate from of the angular momentum is taken to be $J(J+1)$ by these authors.

To summarize, the present work is based on the κ-deformed Poincaré algebra of [1] which encompasses and extends, in a natural way, the Wigner pioneering work on the inhomogeneous Lorentz group[18]. We find that the form of the κ-deformed Poincaré algebra combined with a Regge-like behaviour for the angular momentum hadronic excitations, provides an universal fitting of the experimental data with a value of $\epsilon = 0.082$ fm. In addition, by means of eq.(5), we predict a flattening of the energy spectrum as a function of J. This would imply that many higher angular momentum states are densely packed in an energy region of say 13 to 20 GeV. There will be experimental consequences.

We are grateful to Dr. Keith Dienes for providing us with a fax of ref. [13] and to Dr. Marco Tarlini for pointing out the reference [17] and encouraging comments. LT would like to thank CNPq of Brasil for partial support.

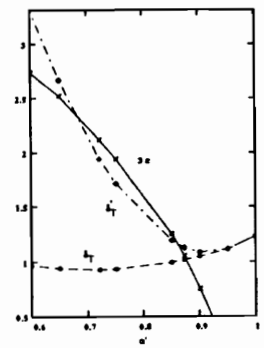

Figure 1: The deformation ϵ (full line) and the corresponding minimum total deviation, $\delta_T = \sum_{trajectory} \delta$, and the deformation parameter ϵ which gives this (dashed line), as a function of the slope parameter α' for the trajectories chosen. The ϵ is multiplied by 3 to put it in the same scale. The sum of deviations for the Regge case, δ'_T, is also given for comparison (dot-dashed line).

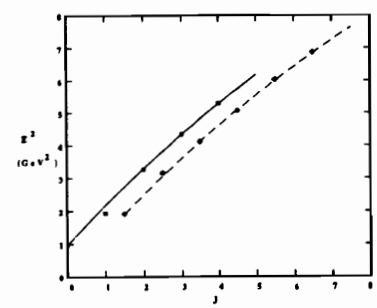

Figure 2: The f_0-ω (solid line) and the Σ^* (dashed-line) κ-deformed trajectories as a function of J, for the parameters $\alpha' = 0.72\ GeV^{-2}$ and $\epsilon = 0.705\ GeV^{-1}$, clearly showing curvature and good fits to the experimental observed states (boxes for f_0-ω and losangles for Σ^*).

References

[1] J. Lukierski, H. Ruegg, A. Nowicki and V. N. Tolstoy, Phys. Lett. B 264 (1991) 331. J. Lukierski, A. Nowicki and H. Ruegg, Phys.Lett. B 293 (1992) 344.

[2] L. C. Biedenharn, B. Mueller and M. Tarlini, Phys. Lett. B 318 (1993) 613.

[3] V. G. Drinfeld, Proc. of the ICM (Berkeley, Calif.), ed. A. M. Gleason (American Mathematical Society, Providence, RI, 1986) p. 798 ; S. Woronowicz, Publ. RIMS (Kyoto University) 23 (1987) 117, Commun. Math. Phys. 122 (1989) 125 ; L. Faddeev, N. Yu Reshetikhin and L. Takhtajian, *Braid Group, Knot Theory and Statistical Mechanics* ed. C. N. Yang and M. L. Ge (World Scientific, Singapore, 1989).

[4] A. Nowicki, E. Sorace and M. Tarlini, Phys. Lett. B 302 (1993) 419.

[5] F. Lizzi and C. Rosenzweig, Phys. Rev. D 31 (1985) 1685.

[6] I. Yu Kobzarev, L. A. Kondratyuk, B. V. Martem'yanov and M. G. Shchepkin, Sov. J. Nucl. Phys. 45 (1987) 330.

[7] Y. Nambu Phys. Rev. D 10, (1974) 4262.

[8] A. W. Hendry, talk given at Toronto Conference (1980), IV th. International Conference on Baryon Resonances, edited by N. Isgur, page 113.

[9] Particle Data Group, Phys. Rev. D 45 (1992).

[10] D. Kutasov and N. Seiberg, Nucl. Phys. B 358, (1991) 600.

[11] P. G. O. Freund and J. L. Rosner, Phys. Rev. Lett. 68, (1992) 765.

[12] J. R. Cudell and K. R. Dienes, Phys. Rev. Lett. 69, (1992) 1324.

[13] K. R. Dienes and J. R. Cudell, Phys. Rev. Lett. 72 (1994) 187.

[14] D. Kutasov and N. Seiberg, Nucl. Phys. B 358, (1991) 600.

[15] J. Dey, M. Dey and L. Tomio, Phys. Lett. B 288 (1992) 306. A. Ansari, M. G. Mustafa, J. Dey and M. Dey, Phys. Lett B 311 (1993) 277, Erratum B 314,(1993) 482-83.

[16] M. Dey and J. Dey, **Nuclear and Particle Physics**, *the Changing Interface*, Springer Verlag, 1993.

[17] E. Celeghini, R. Giachetti, E. Sorace and M. Tarlini, Phys. Lett. B 280 (1992) 180.

[18] E. P. Wigner, Ann. of Math. 40 (1939) 149; E. P. Wigner and V. Bargmann, Proc. Nat. Acad. Sci. 34 (1948) 211.

Table 1. Experiment and theory for two trajectories of π and two of $a-\rho$ states, obtained from eq.(7), given the mass m_o and the spin j_o of the first physical state of the trajectory. The second row for each J gives the Regge results, eq.(6). The percentage deviations are marked δ_κ for our fit, and δ_R in the case of Regge results.

J	$\pi - b$	Exp.	π	Exp.	$\rho - a$	Exp.	$a - \rho$	Exp.
0	0.138	$\pi(.138)$	1.300	$\pi(1.30)$	-	-	.980	$a_0(.98)$
	0.138		1.300				.980	
1	1.084	$b_1(1.235)$	1.672	-	0.770	$\rho(0.77)$	1.445	$\rho(1.45)$
	1.093		1.693		0.770		1.462	
2	1.515	$\pi_2(1.670)$	1.966	$\pi_2(2.10)$	1.316	$a_2(1.32)$	1.783	-
	1.540		2.011		1.330		1.820	
3	1.839	-	2.215	-	1.683	$\rho_3(1.69)$	2.059	$\rho_3(2.25)$
	1.884		2.285		1.716		2.119	
4	-	-	-	-	1.976	$a_4(2.04)$	2.295	-
	-		-		2.030		2.380	
5	-	-	-	-	2.223	$\rho_5(2.35)$	2.504	-
	-		-		2.302		2.616	
6	-	-	-	-	-	-	2.692	$a_6(2.45)$
	-		-				2.832	
7	-	-	-	-	-	-	2.863	$a_6(2.45)$
	-		-				3.032	
δ_κ		9%		4.5%		1.6%		6.5%
δ_R		8%		3%		0.9%		9%

Table 2. The same as in Table 1 for three trajectories of K states.

J	K	Exp.	K	Exp.	K	Exp.
0	0.494	K (.494)	1.460	K (1.46)	1.830	K(1.83)
	0.494		1.460		1.83	
1	1.180	K_1(1.27)	1.795	K_1(1.65)	2.099	-
	1.192		1.819		2.127	
2	1.583	K_2(1.77)	2.069	-	2.330	K_2(2.25)
	1.612		2.118		2.388	
3	1.894	-	2.304	K_3(2.32)	2.535	-
	1.943		2.379		2.623	
4	-	-	2.512	K_4(2.50)	-	-
			2.615			
5	-	-	2.699	-	-	-
			2.831			
δ_κ		7.3%		4.4%		2.5%
δ_R		6.3%		5.8%		4.3%

Table 3. The same as in Table 1 for two trajectories of K^* and one of ϕ states.

J	K^*	Exp.	K^*	Exp.	ϕ	Exp.
1	0.892	K^* (0.892)	1.680	K^* (1.68)	1.020	ϕ (1.02)
	.892		1.680		1.020	
2	1.388	K_2^* (1.43)	1.973	K_2^* (1.98)	1.471	f_2' (1.525)
	1.404		2.000		1.489	
3	1.739	K_3^* (1.78)	2.221	-	1.804	ϕ_3(1.85)
	1.774		2.275		1.842	
4	2.022	K_4^* (2.045)	-	-	2.077	f_4 (2.22)
	2.080				2.138	
5	2.263	K_5^* (2.38)	-	-	2.311	-
	2.346				2.397	
6	2.476	-	-	-	-	-
	2.584					
δ_κ		2.8%		0.3%		3.9%
δ_R		1.3%		0.7%		2.2%

Table 4. The same as in Table 1 for three trajectories of $\omega - f$ states.

J	$\omega - f$	Exp.	$f - \omega$	Exp.	$f - \omega$	Exp.
0	-	-	0.975	f_0(.975)	1.240	f_0(1.24)
			0.975		1.240	
1	0.783	ω(0.783)	1.441	ω(1.39)	1.627	ω(1.60)
	0.783		1.459		1.647	
2	1.323	f_2 (1.27)	1.780	f_2(1.81)	1.929	f_2(2.01)
	1.338		1.818		1.972	
3	1.689	ω_3(1.67)	2.057	-	2.183	-
	1.722		2.117		2.251	
4	1.980	f_4 (2.05)	2.294	f_4(2.3)	-	-
	2.035		2.378			
5	2.227	-	2.503	-	-	-
	2.306		2.614			
6	2.444	f_6(2.51)	-	-	-	-
	2.549					
7	2.637	-	-	-	-	-
	2.770					
δ_κ		2.7%		2%		2.5%
δ_R		2.9%		3%		2%

Table 5. The same as in Table 1 for three trajectories of nucleon and Λ states. The third one, in each case, have the first state with negative parity.

J	N	Exp.	N	Exp.	N	Exp.
1/2	0.939	(.939)	1.440	(1.44)	1.535	(1.535)
	0.939		1.440		1.535	
3/2	1.418	(1.52)	1.779	(1.70)	1.855	(1.72)
	1.435		1.803		1.880	
5/2	1.762	(1.68)	2.056	(2.00)	2.120	(2.2)
	1.799		2.104		2.170	
7/2	2.041	(2.19)	2.293	-	2.349	-
	2.100		2.367		2.426	
9/2	2.280	(2.22)	-	-	2.552	(2.25)
	2.364				2.658	
11/2	2.491	(2.60)	-	-	2.735	-
	2.601				2.870	
13/2	2.680	(2.70)	-	-	-	-
	2.818					
15/2	2.852	-	-	-	-	-
	3.019					
δ_κ		4.5%		3.1%		8%
δ_R		4.8%		4.6%		10.2%
J	Λ	Expt.	Λ	Expt.	Λ	Expt.
1/2	1.116	(1.116)	1.600	(1.6)	1.670	(1.67)
	1.116		1.600		1.670	
3/2	1.537	(1.52)	1.907	(1.69)	1.965	(1.89)
	1.556		1.933		1.991	
5/2	1.857	(1.82)	2.164	(2.11)	2.214	-
	1.897		2.217		2.268	
7/2	2.121	(2.1)	-	-	-	-
	2.185		2.468			
9/2	2.350	(2.35)	-	-	-	-
	2.440					
11/2	2.553	-	-	-	-	-
	2.670					
δ_κ		1.1%		7.6%		2.8%
δ_R		3.3%		8.8%		3.8%

Table 6. The same as in Table 1 for four trajectories of Σ. No spin was assigned for the (†) states in ref.[9]. We predict the spin on the basis of a small deviation δ_κ.

J	Σ	Expt.	Σ	Expt.	Σ	Expt.	Σ^*	Expt.
1/2	1.197	(1.197)	1.660	(1.66)	1.620	(1.62)	-	-
	1.197		1.660		1.620			
3/2	1.595	(1.58)	1.957	(1.94)	1.924	(2.08)	1.385	(1.385)
	1.615		1.983		1.950		1.385	
5/2	1.903	(1.915)	2.207	(2.07)	2.178	-	1.736	(1.775)
	1.946		2.260		2.231		1.795	
7/2	2.161	(2.1)	2.425	-	-	-	2.020	(2.03)
	2.228		2.507				2.067	
9/2	2.385	-	-	-	-	-	2.261	(2.25) (†)
	2.478						2.334	
11/2	-	-	-	-	-	-	2.474	(2.455) (†)
							2.574	
13/2	-	-	-	-	-	-	2.665	(2.62) (†)
							2.793	
15/2	-	-	-	-	-	-	2.838	-
							2.996	
δ_κ		1.6%		3.8%		5.3%		1.2%
δ_R		3.3%		5.5%		4.4%		3.8%

Table 7. The same as in Table 1 for four trajectories of Δ states. Isobar states marked $^{(1)}$ are from Hendry and $^{(2)}$ are from Hoehler et al., p. VIII-57 of ref.[9].

J	Δ	Expt.	Δ	Expt.	Δ	Expt.	Δ	Expt.
1/2	-	-	-	-	1.620	(1.62)	-	-
					1.620			
3/2	1.232	(1.232)	1.600	(1.6)	1.924	(1.92)	1.700	(1.7)
	1.232		1.600		1.950		1.700	
5/2	1.621	-	1.907	(1.93)	2.178	(2.35)	1.989	(1.905)
	1.641		1.933		2.231		2.017	
7/2	1.924	(1.95)	2.164	-	2.400	(2.39)	2.235	(2.2)
	1.967		2.217		2.481		2.290	
9/2	2.179	(2.40)	2.388	(2.4)	2.598	-	2.451	(2.3)
	2.247		2.468		2.707		2.534	
11/2	2.401	(2.42)	2.587	-	2.777	-	2.644	-
	2.495		2.696		2.917		2.756	
13/2	2.599	$(2.65)^{(1)}$	-	-	2.942	$(2.794)^{(2)}$	-	-
	2.720				3.112			
15/2	2.778	$(2.85)^{(1)}$	-	-	3.094	$(2.99)^{(2)}$	-	-
	2.929				3.295			
17/2	2.942	-	-	-	3.235	-	-	-
	3.123				3.469			
δ_κ		1.5%		0.7%		3.9%		3.9%
δ_R		2.2%		1.6%		6.8%		6.3%