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"Vacuum Instability" and Tachyon Poles in Non-Asymptotically Free Field Theories: a Quantitative Connection

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Abstract

One-fermion-loop corrections are known to destabilize the translationally invariant vacuum in all renormalizable theories. The inclusion of one-boson-loop corrections can only eliminate the problem in asymptotically free field theories. One of the manifestations of this instability is the presence of tachyon poles in the boson propagators. In the present work we derive a quantitative relation between the tachyonic pole positions and the vacuum fluctuations corrections at the one loop order for such theories. An application to the Linear Sigma Model is performed.

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Chiral field theories without asymptotic freedom are a current research tool in the area of low energy hadron physics[1]. It is hoped that the low energy properties of such phenomenological theories will not be sensitive to their high energy behavior. Soni[2] has noted that the inclusion of one-fermion-loop corrections to the energy of chiral solitons can result in a total energy which is negative. Cohen, Banerjee and Ren[3] further extended the previous work by including one boson corrections and considering all renormalizable theories which do not contain derivative coupling. They have shown that the negative energy solitons exist in all such theories. Later Perry[4] has shown that the vacuum instability is indicated by tachyon poles in boson propagators and divergence of the effective coupling constant. The analysis performed relates the vacuum instability to well known problem with Yukawa theories, leading to the simple interpretation that the loop expansion diverges at high momentum in non-asymptotically free field theories.

In this letter we provide for a quantitative relation between the discussed tachyon poles and the vacuum energy. The present results can be used to answer the question as to how much the high energy behavior of the theory affects its low energy behavior in terms of the vacuum fluctuation corrections. We show that the deeper the tachyonic pole is located, the smaller the vacuum fluctuations (they decrease as $|s_p|^{-1/2}$, where s_p is the ghost pole position).

Let us take the simple Lagrangeana

$$\mathcal{L}(x) = \bar{\psi}\,(i\,\not{\partial} - m - G\varphi)\,\psi + \frac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\,M^2\varphi^2 - \frac{1}{3!}\,b\,\varphi^3 - \frac{1}{4!}\,c\,\varphi^4 + C.T. \qquad (1)$$

where ψ stands for a fermionic field and φ is a scalar field with a Yukawa coupling to the fermionic fields plus self interaction terms. M (m) is the mass of the bosonic (fermionic) field quantum, b and c are constants and C.T. stand for counterterms.

Within the framework of the effective action formalism, once the one fermion and one boson loop corrections are included we get for the effective action in the present case

$$\begin{aligned}\Gamma(\varphi) \;=\; & \int d^4x\left[\frac{1}{2}\,\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\,M^2\varphi^2 - \frac{1}{3!}\,b\,\varphi^3 - \frac{1}{4!}\,c\,\varphi^4 + C.T.\right] + \\ & -i\,\hbar\,Tr\left[ln(i\,\not{\partial} - m - G\varphi)\right] + \frac{1}{2}i\,\hbar Tr\left[ln(-\Box - M^2 - b\,\varphi - \frac{1}{2}c\,\varphi^2)\right] + \mathcal{O}(\hbar^2)\end{aligned} \qquad (2)$$

where Tr stands for the trace operation. The parameters can be adjusted so that the expectation value of the scalar field is zero in the vacuum state $\langle\varphi\rangle_o = 0$.

Next consider small amplitude vibrations of the scalar field around the vacuum. We get, in the momentum space

$$\Gamma(\varphi) = \int \frac{d^4p}{(2\pi)^4}\left[\frac{1}{2}\,\varphi(p)\; iD^{-1}(p)\;\varphi(-p) + O(\hbar^2,\varphi^3)\right] \tag{3}$$

where D^{-1} stands for the one loop approximation to the inverse scalar field propagator.

$$\begin{aligned} iD^{-1}(p) &= zp^2 - M_o^2 + i\,\hbar\alpha G^2\, tr\int \frac{d^4k}{(2\pi)^4}\frac{1}{(\not k + \frac{1}{2}\not p - m)(\not k - \frac{1}{2}\not p - m)} + \\ &\quad - \frac{1}{2} i\,\hbar b^2 \int \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+\frac{1}{2}p)^2 - M^2][(k-\frac{1}{2}p)^2 - M^2]} \end{aligned} \tag{4}$$

where α stands for the trace over the internal fermionic degrees of freedon (isospin, flavor and color) and tr stands for the trace over Lorentz indices.

For time independent fields $\varphi(\mathbf{x},t) = \varphi(\mathbf{x})$ we get the one loop energy correction

$$E_1 = -\frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left|\int d^3x\varphi(\mathbf{x})e^{i\,\mathbf{x}\cdot\mathbf{p}}\right|^2 iD^{-1}(-\mathbf{p}^2) \tag{5}$$

Then, the $s = -\mathbf{p}^2 < 0$ is the relevant region in the momentum space. For these space-like momentum one can show that

$$iD^{-1}(s) = z's - M_o'^2 - 2\alpha\,\hbar G^2\left[(s-4m^2)I_m(s) - \frac{s}{24\pi^2}\right] - \frac{1}{2}\,\hbar b^2\left[I_M(s) + \frac{s}{96\pi^2M^2}\right] \tag{6}$$

with

$$I_m(s) = \frac{1}{8\pi^2 s}\left\{-\sqrt{-s(4m^2-s)}\;ln\left[\frac{\sqrt{-s}+\sqrt{4m^2-s}}{2m}\right] - s\right\} \tag{7}$$

The finite constants z' and $M_o'^2$ are fixed by the requeriment that D manifest the scalar boson.

From the expressions (5) and (6) it becomes clear that the vacuum is unstable. Let us consider a static field distribution given by

$$\varphi(\mathbf{x}) = \sigma\left(\frac{\mathbf{x}}{R}\right) \tag{8}$$

where σ is an arbitrary functional form and R is a distance scale which we will take as a variational parameter. Inserting (8) in (5) gives

$$E_1 = -\frac{1}{2}\,R^3\int \frac{d^3q}{(2\pi)^3}|\sigma(\mathbf{q})|^2\; iD^{-1}(-\frac{\mathbf{q}^2}{R^2}) \tag{9}$$

where $\mathbf{q} = R\mathbf{p}$. In the limit of small values of R we get

$$E_1 = A\;R\;ln(mR) + B\;R \tag{10}$$

where

$$\begin{aligned} A &= \frac{\hbar\alpha}{8\pi^2}\,G^2\,I_1 \\ B &= \frac{\hbar}{16\pi^2}\left(\frac{8\pi^2z'}{\hbar} - \frac{b^2}{24M^2} + \frac{8\alpha G^2}{3}\right) I_1 - \frac{\hbar\alpha}{16\pi^2}\,G^2\,I_2 \\ I_1 &= \int \frac{d^3q}{(2\pi)^3}|\sigma(\mathbf{q})|^2\,\mathbf{q}^2 \\ I_2 &= \int \frac{d^3q}{(2\pi)^3}|\sigma(\mathbf{q})|^2\,\mathbf{q}^2\;ln\;\mathbf{q}^2 \end{aligned}$$

Now, minimizing with respect to R

$$\left.\frac{\partial E_1}{\partial R}\right|_{R=R_o} = 0 \tag{11}$$

gives

$$R_o = \frac{1}{m}\,\exp\left\{-\frac{1}{2}\left[\left(\frac{8\pi^2z'}{\hbar} - \frac{b^2}{24M^2}\right)\frac{1}{\alpha G^2} + \frac{8}{3} + I\right]\right\} \tag{12}$$

where $I \equiv 2 - I_2/I_1$. The value of E_1 at this point is

$$E_{min} = -\;A\;R_o \tag{13}$$

As pointed out by Perry[4], the fact that the inverse bosonic propagator starts out negative for $p^2 = 0$ and turns positive in the deep euclidean region indicates that it must go through zero for some space-like momentum. Thus the bosonic propagator presents a tachyonic pole for some $s \doteq -\mathbf{p}^2 < 0$. In what follows we show the explicit relationship between the tachyonic pole and the energy fluctuation just derived.

From eq.(5) and the condition

$$\lim_{s\to -\infty} \left[iD^{-1}(s) \right] \Big|_{s=s_p} = 0 \tag{14}$$

we get

$$s_p = -m^2 \exp\left\{ \left[\frac{8\pi^2 z'}{\hbar} - \frac{b^2}{21M^2} \right] \frac{1}{\alpha G^2} + \frac{8}{3} \right\} \tag{15}$$

The corresponding residue can be obtained as

$$\lim_{s\to -\infty} \left[\frac{d}{ds} iD^{-1}(s) \right] \Big|_{s=s_p} \equiv Res^{(-)} \tag{16}$$

with

$$Res^{(-)} = -\frac{\hbar\alpha G^2}{8\pi^2} \tag{17}$$

The above relations allow for an analytic expressions which relate R_o and E_{min} with s_p and $Res^{(-)}$. We find

$$R_o = \frac{1}{\sqrt{|s_p|}} e^{-1/2} \tag{18}$$

and

$$E_{min} = \frac{Res^{(-)}}{\sqrt{|s_p|}} I_1 e^{-1/2} \tag{19}$$

This result is quite general and should apply to all renormalizable theories without asymptotic freedom. Therefore we restricted ourselves to the simplest possible case. We have also applied the results to the well known Linear Sigma Model[5] which follows in the category of field theories discussed and moreover finds applications in the description of low energy hadronic properties[6,7]. Due to lack of space we only quote the results: in its renormalized

versions[8] both the pion and the sigma propagators exhibit tachyonic poles[2]. In the case of the pion this pole is found at $\sqrt{|s_-|} \approx 1.6\, GeV$. Its location is rather independent of the free renormalization parameter $\lambda_{\pi\pi}$. On the other hand, for the sigma propagator the position of the ghost pole strongly depends on $\lambda_{\pi\pi}$, but also for a large range of values of this parameters the location of the pole is found for smaller values of energy, i.e., $\sqrt{|s_-|} \lesssim 1\, GeV$. However, if we permit lower values of the coupling strenghth G, due to chiral symmetry the position of both poles coincide in the asymptotic region. We thus looked for analytical asymptotic expressions for s_p we find

$$\begin{aligned} s_p &= -m^2 \exp\left\{ \frac{4\pi^2}{G^2}(1+a) \right\} \\ Res^{(-)} &= -\frac{G^2}{4\pi^2} \end{aligned} \tag{20}$$

with $a = 4\lambda_{\pi\pi}^2 F^2\, I'_{M\mu}(0)$. We numerically checked that the above expressions show a remarkable agreement with numerical values for $G \lesssim 3$. In this case, our results suggest that this model results will be rather stable and independent of the high energy behavior of the theory. Considerable disagreement (larger than 50%) is found for π–N coupling constant values ($G \simeq 10$).

* * *

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[2]From here on we use the expressions and notation of Ref. 8.

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