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A. J. Accioly[1], H. Mukai[2], E. C. de Rey Neto[1] and H. S. Blas Achic[1]

[1] *Instituto de Física Teórica*
Universidade Estadual Paulista
Rua Pamplona 145
01405-900 - São Paulo, S.P.
Brazil

[2] *Departamento de Física*
Universidade Estadual de Maringá
Av. Colombo 5790
87020-900 Maringá, P.R.
Brazil

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Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145
01405-900 – São Paulo, S.P.
Brazil

Telephone: 55 (11) 251.5155
Telefax: 55 (11) 288.8224
Telex: 55 (11) 31870 UJMFBR
Electronic Address: LIBRARY@AXP.IFT.UNESP.BR
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Polarization in Fermion Scattering

A. J. Accioly[(1)], H. Mukai[(2)], E. C. de Rey Neto[(1)] and H. S. Blas Achic[(1)]

[1] *Instituto de Física Teórica, Universidade Estadual Paulista,*

Rua Pamplona 145, 01405-900 São Paulo, SP, Brazil

[2] *Departamento de Física, Fundação Universidade Estadual de Maringá,*

Av. Colombo 5790, 87020-900 Maringá, Pr, Brazil

Abstract.

The behaviour of the helicity of an initially left-handed beam of massive fermions first interacting with a Coulomb field and then with a charged scalar particle via a photon exchange is analysed. It is found that in both cases the massive fermions have their helicity flipped, while massless fermions seeem to be unaffected by the electromagnetic field as far as their helicity is concerned.

Currently there is no doubt that quantum electrodynamics is the best theory we have. In addition to being important on its own right, it is also the prototype for quantum chromodynamics. Moreover, it predicted - to name only one of its successes - the anomolous magnetic moment of the electron correctly to six decimal places.

Quantum electrodynamics is the theory which describes how point-like charged particles (usually with spin 1/2) interact with the electromagnetic field. As such, it is the foundation of the subject of atomic physics and of fundamental importance to condensed matter, nuclear, and particle physics. Accordingly, it would be interesting to analyse the behaviour of the helicity of a polarized beam of massive spin - 1/2 Dirac particles in the electromagnetic field, an important issue by its own which as far as we know has only been carried out for electrons scattered from a fixed Coulomb Potential.

To begin with we work out a version of the usual Coulomb scattering problem. Consider for this purpose the scattering of a negative charged fermion with mass m_F by the Coulomb field of a heavy nucleus, treated as a point charge. In the Coulomb gauge, the potential is given by

$$A^{\mu}_{ext}(\vec{x}) = \left(\frac{Ze}{4\pi\,|\vec{x}|} \,,\; 0 \,,\; 0 \,, 0 \right) \quad ,$$

with the momentum space potential

$$A^{\mu}_{ext}(\vec{k}) = \left(\frac{Ze}{\left|\vec{k}\right|^2} \,,\; 0 \,,\; 0 \,,\; 0 \right) \quad .$$

Here the charge of the electron is $-e < 0$.

The Lagrange function of the system is

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m_F)\,\psi + e\bar{\psi} A\!\!\!/_{ext.}\psi \quad .$$

The corresponding fermion-external-field vertex is shown in Fig. 1.

Suppose then that in the initial state we have a beam of fermions with momentum $p = (E, \vec{p})$ in the spin state $u(\vec{p}, S_L(\vec{p}))$. In the other words, the incident fermions are left-handed (L). The spinors u and $\bar{u}$ are such that[1-4]

$$\frac{(1+\gamma_5 \not{S}_i\ )}{2}\, u\,(\vec{p},S_i) = u\,(\vec{p},S_i) \ ,$$

$$\frac{(1+\gamma_5 \not{S}_i\ )}{2}\, u\,(\vec{p},-S_i) = 0 \ ,$$

$$u_\alpha\,(\vec{p},S_i(\vec{p}))\,\bar{u}_\beta\,(\vec{p},S_i(\vec{p})) = \left[\frac{\not{p}+m_F}{2m_F}\,\frac{1+\gamma_5 \not{S}_i(\vec{p})}{2}\right]_{\alpha\beta} \ ,$$

$$i = L,\ R \ ,$$

whereupon the polarization vectors are given by[5]

$$S^\alpha_{\rm R}(\vec{p}) = \left(\frac{|\vec{p}|}{m_{\rm F}}, \frac{p_o\hat{p}}{m_{\rm F}}\right) \ ,$$

$$S^\alpha_{\rm L}(\vec{p}) = \left(\frac{-|\vec{p}|}{m_{\rm F}}, \frac{-p_o\hat{p}}{m_{\rm F}}\right) \ ,$$

where R stands for right-handed fermions and $\hat{p} \equiv \frac{\vec{p}}{|\vec{p}|}$.

The polarization of the scattered fermions is measured by

$$P = 1 - \frac{2N_{\rm R}}{N_{\rm L}+N_{\rm R}} \ ,$$

where $N_{\rm L}(N_{\rm R})$ denotes the number of fermions emerging with negative(positive) helicity. Note that $P=1$ indicates no deporalization of the initial fermions, while $P=-1$ says that all the left-handed initial fermions have their helicity flipped.

For the process in hand,

$$N_R = \left|\bar{u}\,(\vec{p}\,',S_R(\vec{p}\,'))\ \not{A}_{\rm ext.}\,(\vec{k})\ u(\vec{p},S_L(\vec{p}))\right|^2$$

and

$$N_L = \left|\bar{u}\,(\vec{p}\,',S_L(\vec{p}\,'))\ \not{A}_{\rm ext.}\,(\vec{k})\ u(\vec{p},S_L(\vec{p}))\right|^2 \ ,$$

where p' is the momentum of the emerging fermion.

Some more algebra and we arrive at the following expressions for $N_{\rm R}$ and $N_{\rm L}$:

$$N_{\rm R} = \frac{1}{4m_{\rm F}^2}\Big\{\left(2A\cdot p'\ A\cdot p - A^2\ p'\cdot p + A^2 m_{\rm F}^2\right)(1 - S_{\rm R}(\vec{p}\,')\cdot S_{\rm L}(\vec{p}))$$
$$+ 2\left(m_{\rm F}^2 - p\cdot p'\right) A\cdot S_{\rm R}(\vec{p}\,')\ A\cdot S_{\rm L}(\vec{p}) + 2p'\cdot S_{\rm L}(\vec{p})\ S_{\rm R}(\vec{p}\,')\cdot A\ p\cdot A$$
$$+ \left(2p'\cdot A\ A\cdot S_{\rm L}(\vec{p}) - A^2\ p'\cdot S_{\rm L}(\vec{p})\right) S_{\rm R}(\vec{p}\,')\cdot p\Big\} \ ,$$

$$N_{\rm L} = \frac{1}{4m_{\rm F}^2}\Big\{\left(2A\cdot p'\ A\cdot p - A^2 p'\cdot p + m_{\rm F}^2 A^2\right)(1 - S_{\rm L}(\vec{p}\,')\cdot S_{\rm L}(\vec{p}))$$
$$+ 2\left(m_{\rm F}^2 - p\cdot p'\right) A\cdot S_{\rm L}(\vec{p})\ S_{\rm L}(\vec{p}\,')\cdot A + 2A\cdot S_{\rm L}(\vec{p}\,')\ p'\cdot S_{\rm L}(p)\ A\cdot p$$
$$+ \left(2A\cdot p'\ A\cdot S_{\rm L}(\vec{p}) - A^2 p'\cdot S_{\rm L}(\vec{p})\right) S_{\rm L}(\vec{p}\,')\cdot p\Big\} \ ,$$

where $A \equiv A^\mu_{\rm ext.}(\vec{k})$ and $A^2 \equiv A\cdot A$.

Therefore, the polarization of the scattered fermions is given by:

$$P = 1 - \left[\frac{2m_{\rm F}^2\ tg^2\theta/2}{E^2 + m_{\rm F}^2\ tg^2\theta/2}\right] \ , \qquad (1)$$

where θ denotes the scattering angle.

In Fig. 2 it is shown the behavior of P as a function of both θ and $m_{\rm F}$, for fixed E. As may be seen, the larger the scattering angle, the larger the helicity flip. Fig. 2 tells us also that $P \to 1$ as $m_{\rm F} \to 0$, as it was expected.

Consider now the reaction

$$F^- + S^+ \to F^- + S^+ \ ,$$

where F^- denotes a negative charged fermion and S^+ stands for a positive charged scalar particle (see Fig. 3).

The corresponding lagrangian in the Lorentz gauge is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\lambda}\left(\partial_\mu A^\mu\right)^2 + \bar{\psi}\left(i\not{\partial} - m_{\rm F}\right)\psi + e\bar{\psi}\not{A}\psi + \partial_\alpha\phi^*\ \partial^\alpha\phi$$
$$- m_{\rm S}^2\phi^*\phi + ie\left(A^\mu\phi\partial_\mu\phi^* - A^\mu\phi^*\partial_\mu\phi\right) + e^2 A_\mu\phi^* A^\mu\phi \ ,$$

where $m_{\rm F}$ and $m_{\rm S}$ are the masses of the fermion and scalar particle, respectively. The pertinent Feynman rules for the process shown in Fig. 3 are displayed in Fig. 4.

Suppose then that in the initial state we have a scalar particle with momentum q and a left-handed fermion with momentum $p = (E,\vec{p})$.

For the aforementioned process,

$$N_{\rm R} = \left|\bar{u}\,(\vec{p}\,',S_{\rm R}(\vec{p}\,'))\ V_1^\alpha\ u\,(\vec{p},S_{\rm L}(\vec{p}))\ \Delta_{\alpha\beta}(k)\ V_2^\beta(q,q')\right|^2$$

and

$$N_{\rm L} = \left|\bar{u}\,(\vec{p}\,', S_{\rm L}(\vec{p}\,'))\ V_1^{\alpha}\ u\,(\vec{p}, S_{\rm L}(\vec{p}))\ \Delta_{\alpha\beta}(k)\ V_2^{\beta}(q,q')\right|^2 \quad ,$$

whereupon

$$\Delta_{\mu\nu}(k) = -\frac{i}{k^2 + i\epsilon}\left[\eta_{\mu\nu} + \frac{\lambda - 1}{k^2}k_\mu k_\nu\right]$$

is the photon propagator in momentum space, where $p'(q')$ is the momentum of the emerging fermion (scalar particle).

Using much algebra we arrive at the following expressions for $N_{\rm R}$ and $N_{\rm L}$:

$$\begin{aligned} N_R = \frac{e^4}{4m_{\rm F}^2 t^2}\Bigg\{&[1 - S_L(\vec{p})\cdot S_R(\vec{p}\,')]\left[\frac{(s-u)^2}{2} + \frac{t}{2}\left(4m_{\rm S}^2 - t\right)\right] + t\,(S_R(\vec{p}\,')\cdot Q)\,(S_L(\vec{p})\cdot Q) \\ &+ (s-u)\,[(S_L(\vec{p})\cdot p')\,(S_R(\vec{p}\,')\cdot Q) + (S_L(\vec{p})\cdot Q)\,(S_R(\vec{p}\,')\cdot p)] \\ &- \left(4m_{\rm S}^2 - t\right)(S_L(\vec{p})\cdot p')\,(S_R(\vec{p}\,')\cdot p)\Bigg\} \quad . \end{aligned}$$

$$\begin{aligned} N_L = \frac{e^4}{4m_{\rm F}^2 t^2}\Bigg\{&[1 + S_L(\vec{p})\cdot S_R(\vec{p}\,')]\left[\frac{(s-u)^2}{2} + \frac{t}{2}\left(4m_{\rm S}^2 - t\right)\right] - t\,(S_R(\vec{p}\,')\cdot Q)\,(S_L(\vec{p})\cdot Q) \\ &- (s-u)\,[(S_R(\vec{p}\,')\cdot p)\,(S_L(\vec{p})\cdot Q) + (S_R(\vec{p}\,')\cdot Q)\,(S_L(\vec{p})\cdot p')] \\ &+ \left(4m_{\rm S}^2 - t\right)(S_L({\rm p})\cdot {\rm p}')\,(S_R(\vec{p}\,')\cdot p)\Bigg\} \quad . \end{aligned}$$

where $Q \equiv q' + q$ and $s = (p+q)^2$, $t = (p-p')^2$, and $u = (p-q')^2$ are the usual Mandelstam variables.

Next, we evaluate the polarization P in the scalar particle rest system $\left[q^\mu = \left(m_{\rm S}, \vec{0}\right)\right]$ (see Fig. 5).

In this frame

$$s = m_{\rm F}^2 + m_{\rm S}^2 + 2Em_{\rm S} \quad ,$$

$$t = 2m_{\rm S}\,(E' - E) \quad ,$$

$$u = 2(m_{\rm F}^2 + m_{\rm S}^2) - s - t \quad ,$$

whereupon

$$E' = \frac{(m_{\rm S}E + m_{\rm F}^2)(E + m_{\rm S}) + (E^2 - m_{\rm F}^2)\cos\theta\sqrt{m_{\rm S}^2 - m_{\rm F}^2\sin^2\theta}}{(E + m_{\rm S})^2 - (E^2 - m_{\rm F}^2)\cos^2\theta} \quad ,$$

where $E(E')$ is the initial(final) energy of the fermion.

The final expression for P is then given by

$$\begin{aligned} P = 1 - 2\Bigg\{&\left[1 + \frac{\sqrt{E^2 - m_{\rm F}^2}\sqrt{E'^2 - m_{\rm F}^2} - EE'\cos\theta}{m_{\rm F}^2}\right]\left[\frac{(s-u)^2}{2} + \frac{t}{2}\left(4m_{\rm S}^2 - t\right)\right] \\ &- t\left[\frac{\sqrt{E'^2 - m_{\rm F}^2}(2m_{\rm S} + E) - E'\sqrt{E^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right] \\ &\times \left[\frac{\sqrt{E^2 - m_{\rm F}^2}(2m_{\rm S} - E') + E\sqrt{E'^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right] \\ &- (s-u)\Bigg[\left(\frac{E'\sqrt{E^2 - m_{\rm F}^2} - E\sqrt{E'^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right) \\ &\times \left(\frac{\sqrt{E'^2 - m_{\rm F}^2}(2m_{\rm S} + E) - E'\sqrt{E^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right) \\ &+ \left(\frac{\sqrt{E^2 - m_{\rm F}^2}(2m_{\rm S} - E') + E\sqrt{E'^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right) \\ &\times \left(\frac{E\sqrt{E'^2 - m_{\rm F}^2} - E'\sqrt{E^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right)\Bigg] \\ &+ (4m_{\rm S}^2 - t)\left(\frac{E'\sqrt{E^2 - m_{\rm F}^2} - E\sqrt{E'^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right) \\ &\times \left(\frac{E\sqrt{E'^2 - m_{\rm F}^2} - E'\sqrt{E^2 - m_{\rm F}^2}\cos\theta}{m_{\rm F}}\right)\Bigg\} \Bigg/ \\ &\left\{(s-u)^2 + t(4m_{\rm S}^2 - t)\right\} \quad . \end{aligned} \qquad (2)$$

In Fig. 6 it is displayed P as a function of both θ and $m_{\rm F}$, for fixed E and $m_{\rm S}$. As can be easily seen the result is the same as in (1), namely, the larger the scattering angle, the larger the helicity flip. $P \to 1$ as $m_{\rm F} \to 0$, as well.

In summary, we have shown that at the tree level all the massive spin - 1/2 Dirac particles have their helicity flipped on account of their interaction with the electromagnetic field. Massless fermions, in turn, seem to be unaffected by the electromagnetic field as long as their helicity is concerned.

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FIGURES CAPTIONS

Fig. 1. The fermion-external-field vertex. Here $\vec{k} = \vec{p}\,' - \vec{p}$ and $|\vec{p}| = |\vec{p}\,'|$.

Fig. 2. P as a function of both θ and m_F, for $E = 100MeV$.

Fig. 3. Lowest order contribution to the reaction $F^- S^+ \rightarrow F^- S^+$ (F^- and S^+ are defined as particles).

Fig. 4. The relevant Feynman rules for fermion-scalar-field interaction.

Fig. 5. The definition of the F^- scattering angle in $F^- S^+$ scattering.

Fig. 6. P as function of both θ and m_F, for $E = 100MeV$ and $m_S = 110MeV$.

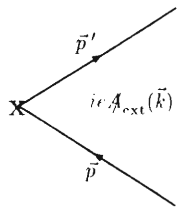
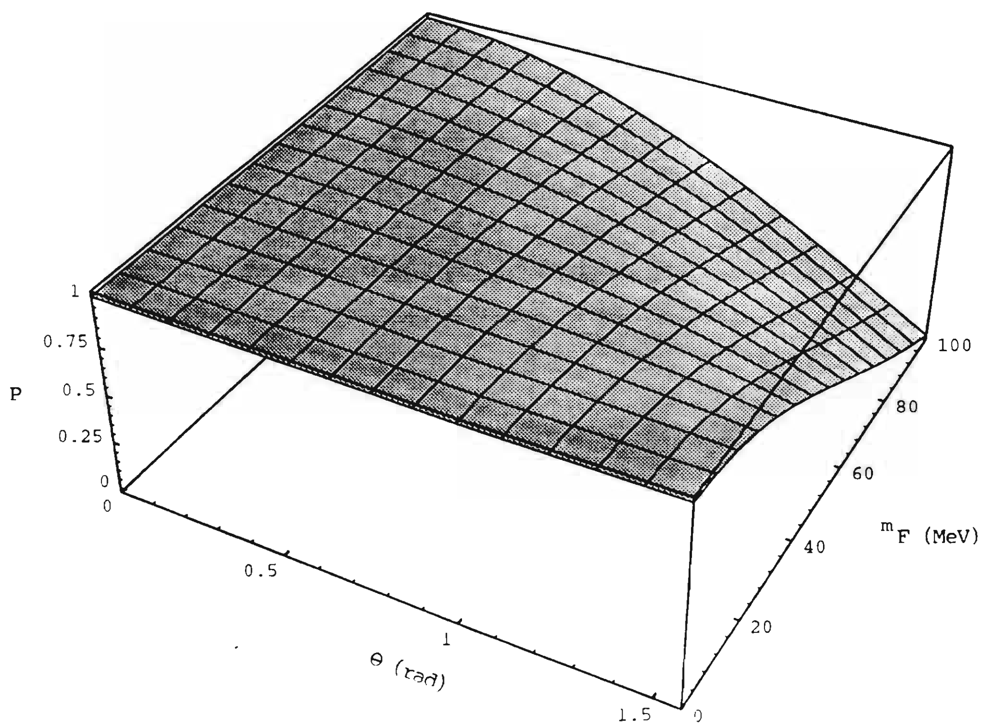
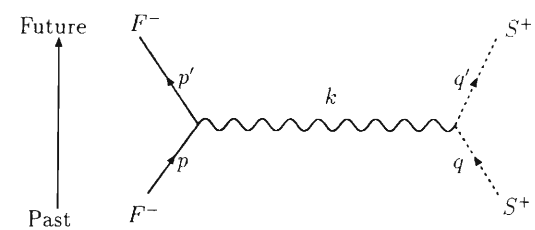


Fig.1


P
1
0.75
0.5
0.25
0
0
0.5
1
1.5
Θ (rad)
0
20
40
60
80
100
mF (MeV)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q
S⁺
S⁺

Fig.3

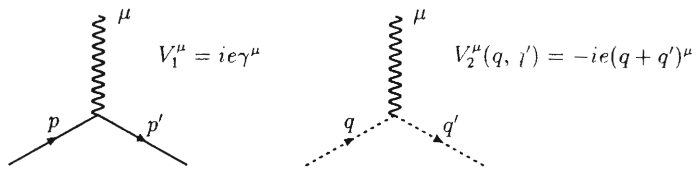


Fig. 4

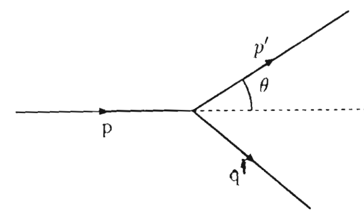
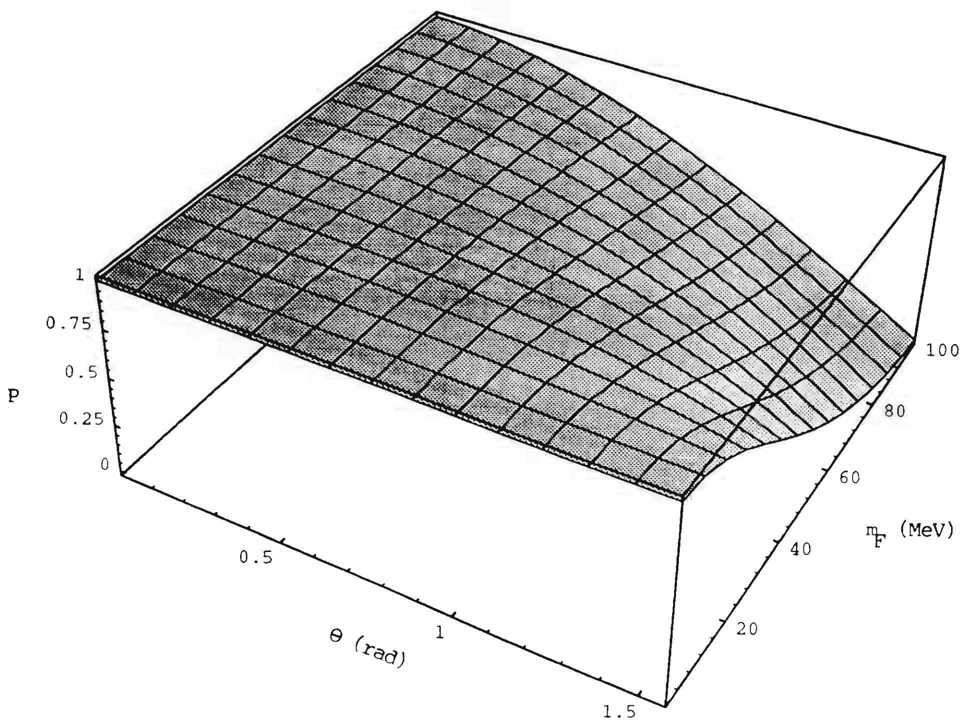


Fig.5



P
1
0.75
0.5
0.25
0
0.5
1
1.5
θ (rad)
20
40
60
80
100
m_F (MeV)