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IFT–P.001/97

On the long standing issue of an $R\phi^2$ coupling between gravity and a massless free scalar field *

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*To appear in Progress of Theoretical Physics

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ON THE LONG STANDING ISSUE OF AN $R\phi^2$ COUPLING BETWEEN GRAVITY AND A MASSLESS FREE SCALAR FIELD

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Abstract

Using 2 body trees on a flat space background, it is shown that the actions $A[g,\phi] = \int d^4x\ \sqrt{-g}\ \left[\frac{R}{2\kappa} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda R\phi^2\right)\right]$ and $\bar{A}[\bar{g},\bar{\phi}] = \int d^4x\ \sqrt{-\bar{g}}\ \left[\frac{\bar{R}}{2\kappa} + \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi}\right]$ describe the same theory at the tree-level in this case. We also demonstrate the quantum equivalence (at one-loop) of the barred and unbarred systems for $\lambda = -1/6$ (conformal coupling).

PACS numbers: 04.60.-m; 04.50.+h; 04.20.Cv

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The reasons for adding to Einstein's action for gravitation a nonminimal functional of a scalar field are manifold[1]. It is widely believed, moreover, that the action functional which describes the nonminimal interaction between the gravitational field $g_{\mu\nu}$ and a free, massless scalar field ϕ, is given by[2]

$$A_S = \int d^4x\ \sqrt{-g}\ \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\ \partial_\nu\phi + \lambda R\phi^2\right)\ ,$$

where λ is an adimensional coupling constant which in principle may assume whatever value one wishes. Here the Ricci tensor is defined by $R_{\mu\nu} = -\partial_\alpha\Gamma^\alpha{}_{\mu\nu} + \ldots$, and the metric convention is $g_{\mu\nu} = diag(1,-1,-1,-1)$. Accordingly, let us focus our attention on the gravitational theories described by the action functional

$$A\left[g,\phi\right] = \int d^4x\ \sqrt{-g}\ \left[\frac{R}{2\kappa} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \lambda R\phi^2\right)\right]\ , \tag{1}$$

with $\kappa = 8\pi G$ in natural units.

As it has previously been observed, the $R\phi^2$ coupling between gravity and the ϕ-field can be eliminated from (1) by local field redefinitions[3]. They are

$$\bar{g}_{\mu\nu} = \left(1 + \kappa\lambda\phi^2\right) g_{\mu\nu}\ , \tag{2a}$$

$$\bar{\phi} = \int d\phi\ \frac{\sqrt{1 + \kappa\lambda\left(1 + 6\lambda\right)\phi^2}}{1 + \kappa\lambda\phi^2}\ , \tag{2b}$$

where $1 + \kappa\lambda\phi^2$ is supposed to be positive. Under this transformation the A action of (1) becomes

$$\bar{A}\left[\bar{g},\bar{\phi}\right] = \int d^4x\ \sqrt{-\bar{g}}\ \left[\frac{\bar{R}}{2\kappa} + \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\ \partial_\nu\bar{\phi}\right]\ . \tag{3}$$

Thus, at the classical level, actions A and $\bar{A}$ prescribre the same theory; furthermore, for a single massless scalar field, the $R\phi^2$ coupling may be transformed away completly. (We are assuming that $1 + \kappa\lambda\phi^2 > 0$.)

On the other hand, according to a well-established custom among quantum field theorists, the S-matrix is supposed to be invariant under field redefinitions[4]. If this is so, then A and

$\tilde{A}$ describe the same quantum field theory. Of course, actions A and $\tilde{A}$ will lead to different off-mass-shell Green's functions in this case, but as long as we ask only physical questions, namely, on-shell S-matrix elements, the results will be the same. Moreover, if the S-matrix is indeed unchanged under the above field redefinitons, it must be that scattering amplitudes show no dependence upon the strenght of the $R\phi^2$ coupling. So, a formal argument that supports the equivalence based on the field redefinition is the explicit computation of on-shell scattering amplitudes. Consequently, we show from the start that to lowest order in two body scattering on a flat space background the scattering amplitudes do not depend on the parameter λ at all. To keep things simple, we firstly carry out the computation in conformally flat spaces.

In the particular case of those spacetimes the metric takes on the form[5]

$$g_{\mu\nu}(x) = \Omega^2(x)\eta_{\mu\nu} \quad ,$$

where $\Omega(x)$ is a position-dependent function and $\eta_{\mu\nu}$ is the flat space Lorentz metric. Let us then find what Einstein's constant for gravitational theories in conformally flat spaces is. To do that we compute the effective nonrelativistic potential for the gravitational interaction of two identical massive bosons of zero spin (see Fig. 1). The aforementioned nonrelativistic potential is given by[6]

$$U(\vec{r}) = \frac{1}{4m^2}\frac{1}{(2\pi)^3}\int d^3\vec{k}\,\mathcal{F}(NR)e^{i\vec{k}\cdot\vec{r}} \quad ,$$

whereupon $\mathcal{F}(NR)$ is the nonrelativistic limit of $i\mathcal{M}$, where $\mathcal{M}$ is the invariant amplitude for the process shown in Fig. 1.

Now, in conformally flat spaces, the action for the interaction of gravity with a free, massive scalar field ϕ, i.e.,

$$A_{g\phi} = \int d^4x\,\sqrt{-g}\left[\frac{R}{2\kappa} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - m^2\phi^2\right)\right] \quad ,$$

turns out to be

$$\tilde{A}_{g\phi} = \int d^4x\,\left[\frac{\Omega\Box\Omega}{-2B^2} + \frac{\Omega^2}{2}\left(\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - m^2\phi^2\right)\right] \quad , \tag{4}$$

whereupon $\Box \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$ and $K = -6B^2$, where K is the constant we are looking for. Note that K was chosen to be negative from the very start in order to avoid the presence of an abnormal scalar field. Of course, this choice of the sign for the constant in hand is not mandatory.

In the weak field approximation Ω^2 can be written as

$$\Omega^2 \equiv \exp\left[2Bf(x)\right] \quad , \tag{5}$$

where $|2Bf| \ll 1$. For physical reasons, the position-dependent function $f(x)$ is assumed to have the properties

$$\lim_{x\to\pm\infty} f(x) \to 0 \quad \forall\, x^\mu$$
$$\lim_{x\to\pm\infty} \partial_\mu f(x) \to 0 \quad \forall x^\mu \quad .$$

Substituting (5) into (4) we obtain

$$\tilde{A}_{gf} = \int d^4x\,\frac{f\Box f}{-2} + \int d^4x\,Bf\,\left[\partial_\mu\phi\,\partial^\mu\phi - 2m^2\phi^2\right] \quad . \tag{6}$$

Note that the tensorial indices are raised (lowered) by $\eta^{\mu\nu}$ ($\eta_{\mu\nu}$). The action for the free scalar field ϕ was omitted from (6) for the sake of simplicity.

The free propagator for the gravitational field is fixed by the quadratic part of the above action. In momentum space we have

$$D = \frac{i}{k^2 + i\varepsilon} \quad .$$

On the other hand, the vertex function associated with the coupling $\phi(p) - \phi(p') - f(k)$ is given by

$$V(p,p') = -2iB\left[p\cdot p' + 2m^2\right] \quad .$$

In this work, all momenta are supposed to be incoming as far as the vertex functions are concerned.

In terms of the Mandelstam variable $t = k^2$, the invariant amplitude for the process depicted in Fig. 1 becomes

$$\mathcal{M} = -4iB^2\left[\frac{m^4}{t} + m^2 + \frac{t}{4}\right] \quad .$$

Thus, the nonrelativistic potential can be expressed as

$$U(\vec{r}) = \frac{B^2 m^2}{4\pi r} + \ldots$$

One will therefore get agreement with Newtonian theory if $B^2 \equiv 4\pi G$, which shows that Einstein's constant in conformally flat spaces is given by $K = -24\pi G$.

Since the $2 \to 2$ processes concerning the action

$$A_{gR\phi} = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + \frac{1}{2} \left(g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + \lambda R \phi^2 - m^2 \phi \right) \right] \quad , \tag{7}$$

for which λ will very likely enter the invariant amplitudes are only the scalar-scalar and graviton-scalar scattering, we shall calculate in the following the lowest-order invariant amplitudes for these processes. Of course, there is no need to consider the annihilation of scalars into gravitons since, as is well known, the invariant amplitude for this process can be promptly inferred from the one concerning the graviton-scalar scattering.

Combining the action for pure gravity

$$A_G = \int d^4x \left[\frac{\Omega \Box \Omega}{-2B^2} \right] \quad ,$$

with Eq. (5) yields

$$\tilde{A}_G = \int d^4x \left[-\frac{f \Box f}{2} - \frac{B}{2} f^2 \Box f + \ldots \right] \quad .$$

The trilinear graviton vertex is then given by

- $f(k_1) - f(k_2) - f(k_3)$

$$V_{fff} = -i2B \left[k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 \right] \quad .$$

Repeating the reasoning that led previously to Eq. (6) we now find that the action for the $R\phi^2$ coupling between gravity and a massive, free scalar field, assumes the form

$$\tilde{A}_{gR\phi} = \int d^4x \left\{ Bf \left[(1+6\lambda) \partial_\mu \phi \, \partial^\mu \phi - 2m^2 \phi^2 + 6\lambda \phi \Box \phi \right] \right.$$
$$\left. + B^2 \left(f^2 \partial_\mu \phi \, \partial^\mu \phi - 4 f^2 m^2 \phi^2 + 6\lambda \phi^2 f \Box f + 3\lambda \phi^2 \partial_\mu f \, \partial^\mu f \right) \right\} \quad ,$$

from which we obtain the vertex functions for the trilinear and quartic couplings:

- $f(k) - \phi(p_1) - \phi(p_2)$

$$V_{f\phi\phi} = -i2B \left[p_1 \cdot p_2 + 3\lambda \left(p_1 + p_2 \right)^2 + 2m^2 \right]$$

- $f(k_1) - f(k_2) - \phi(p_1) - \phi(p_2)$

$$V_{ff\phi\phi} = -i4B^2 \left[p_1 \cdot p_2 + 4m^2 + 3\lambda \left(k_1^2 + k_2^2 + k_1 \cdot k_2 \right) \right] \quad .$$

Firstly we present the gravitational scattering of massive scalars with arbitrary $R\phi^2$ coupling. The Feynman diagrams for the process $\phi(p_1)\phi(p_2) \to \phi(p_3)\phi(p_4)$ in lowest order are displayed in Fig. 2. Introducing the usual Mandelstam variables

$$s = (p_1 + p_2)^2 \ , \ t = (p_1 - p_3)^2 \ , \text{and} \ u = (p_1 - p_4)^2 \ ,$$

the invariant amplitude becomes

$$\mathcal{M}_{\phi\phi \to \phi\phi} = -i4B^2 m^2 \left[m^2 \left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u} \right) + 2 \ (2+3\lambda)(1+6\lambda) \right] \ ;$$

showing that for massless scalars, which is the case for the action (1), $\mathcal{M}_{\phi\phi \to \phi\phi}$ is independent of λ.

For the graviton-scalar scattering $f\phi \to f\phi$, the Feynman diagrams are shown in Fig. 3. The corresponding invariant amplitude is

$$\mathcal{M}_{f\phi \to f\phi} = -i2B^2 m^2 \frac{[su - m^2(t+m^2)]}{(s-m^2)(u-m^2)} \quad ,$$

which is independent of λ even for massive scalars. The preceding results show clearly that in the massless limit the two body scattering amplitudes become totally independent of the strenght of the $R\phi^2$ coupling. However, they just vanish! To convince the reader at least partially of the correctness of our reasoning we thus need some nonzero λ-independent amplitudes. To this purpose we extend our previous computation to arbitrary Riemannian spaces. Expanding now the action (7) around flat space through the expression $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\sqrt{\kappa} h_{\mu\nu}(x)$, where $h_{\mu\nu}(x)$ is the graviton field, and proceeding in the same way as we have done in the case of conformally flat spaces, we arrive, using much algebra, at the following expression for the invariant amplitude concerning the gravitational scattering of identical massive scalars with arbitrary $R\phi^2$ coupling

$$|\mathcal{M}_{\phi\phi\to\phi\phi}|^2 = \frac{\kappa^2}{4}\left[\frac{t^2+u^2}{s}+\frac{s^2+u^2}{t}+\frac{s^2+t^2}{u}\right.$$
$$\left. - 12m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right)+8m^2-8\lambda m^2(5+6\lambda)\right]^2 \quad . \tag{8}$$

It is clear that in the massless limit which is the case for the action (1), the invariant amplitude becomes totally independent of the parameter λ. Eq. (8) with $\lambda = 0$ agrees with the result of Peet[7].

The evaluation of the invariant amplitude for scalar-graviton scattering is rather involved. To calculate the helicity amplitudes in this case, we have made use of the formalism of Refs. 8 and 9. The result is

$$\begin{aligned}
|\mathcal{M}_{\phi g\to\phi g}(++)|^2 &= |\mathcal{M}_{\phi g\to\phi g}(--)|^2 \\
&= \frac{\kappa^2}{4}\frac{(su-m^4)^4}{t^2(s-m^2)^2(u-m^2)^2} \quad , \\
|\mathcal{M}_{\phi g\to\phi g}(+-)|^2 &= |\mathcal{M}_{\phi g\to\phi g}(-+)|^2 \\
&= \frac{\kappa^2}{4}\frac{m^8t^2}{(s-m^2)^2(u-m^2)^2} \quad ,
\end{aligned}$$

where the $\pm$ signs refer to the initial and final graviton helicities. We can see that even for massive scalars the invariant amplitudes are λ-independent. Our result agrees with the one given by Berends and Gastmans[10], who have calculated the elastic scattering of gravitons on massive scalars for gravitational minimally coupled scalars ($\lambda = 0$).

Hence, we come to the conclusion that the tree-level invariant amplitudes for the $2 \to 2$ processes concerning action (1) ($\phi-\phi$ and $\phi-g$ scattering) are completely independent of the $R\phi^2$ coupling parameters, which directly implies that this parameters cannot be measured at tree-level scattering in this case.

Let us now turn our attention to on-shell divergences. In particular, we shall show that on mass-shell and for $\lambda = -1/6$ (conformal coupling) the one-loop divergences of actions (1) and (3) coincide.

Using the method of background fields and the Schwinger-DeWitt generalized technique, which had already become standard,[11,12] we obtain the following expression for the one-loop divergences of the theory (3)

$$W^{\rm div}_{1-\rm loop}\left[\bar g,\bar\phi\right] = \frac{1}{16\pi^2\varepsilon}\int d^4x\,\sqrt{-\bar g}\,\left[\frac{43}{60}\bar R^2_{\alpha\beta}\right.$$
$$\left. + \frac{\bar R^2}{40} + 5\kappa^2\left(\nabla_\alpha\bar\phi\,\nabla^\alpha\bar\phi\right)^2 + \frac{2}{3}\kappa\left(\nabla_\alpha\bar\phi\,\nabla^\alpha\bar\phi\right)\bar R\ \right]$$

where $\varepsilon \equiv n-4$.

On mass-shell we have

$$W^{\rm div}_{1-\rm loop}\left[\bar g,\bar\phi\right] = \frac{203}{640\pi^2\varepsilon}\int d^4x\,\sqrt{-\bar g}\,\bar R^2 \quad , \tag{9}$$

which agrees with the result of t'Hooft and Veltman[13].

On the other hand, on mass-shell and for $\lambda = -1/6$ the one-loop divergences of the theory (1) are given by

$$W^{\rm div}_{1-\rm loop}\left[g,\phi\right] = \frac{203\kappa^2}{640\pi^2\varepsilon}\int d^4x\,\sqrt{-g}\,\left(\nabla_\alpha\phi\,\nabla^\alpha\phi\right)^2\left(1-\frac{\kappa\phi^2}{6}\right)^{-4} \quad .$$

The steps used to obtain this equation were:

i) to transform the theory (1) from $(g_{\mu\nu},\phi)$ to new variables $\left(\bar g_{\mu\nu},\bar\phi\right)$ by means of (2);

ii) to compute the one-loop divergences using the Schwinger-De-Witt generalized technique;

iii) to rewrite the result in terms of the old field variables $g_{\mu\nu}$ and ϕ using (2).

But according to the transformation rule (2)

$$\begin{aligned}
\sqrt{-g}\,(\nabla_\alpha\phi\,\nabla^\alpha\phi)^2 &= \left(1-\kappa\frac{\phi^2}{6}\right)^{-2}\sqrt{-\bar g}\left(\frac{d\phi}{d\bar\phi}\right)^4\left(g^{\alpha\beta}\nabla_\alpha\bar\phi\,\nabla_\beta\bar\phi\right)^2 \\
&= \sqrt{-\bar g}\left(1-\frac{\kappa}{6}\phi^2\right)^4\left(\nabla_\alpha\bar\phi\,\nabla^\alpha\bar\phi\right)^2 \\
&= \sqrt{-\bar g}\left(1-\frac{\kappa}{6}\phi^2\right)^4\frac{\bar R^2}{\kappa^2}
\end{aligned}$$

on using the on-shell condition $\bar R = -\kappa\nabla^\alpha\bar\phi\,\nabla_\alpha\bar\phi$.

So,

$$W^{\rm div}_{1-\rm loop}[g,\phi] = \frac{203}{640\pi^2\varepsilon}\int d^4x\,\sqrt{-\bar g}\,\bar R^2 \quad . \tag{10}$$

A comparison of (9) and (10) therefore yields

$$W^{\rm div}_{1-{\rm loop}}\left[\bar{g},\bar{\phi}\right] = W^{\rm div}_{1-{\rm loop}}\left[g,\phi\right] \quad .$$

Note that we are assuming that $1-\frac{\kappa\phi^2}{6}>0$. Thus the preceding computation of on-shell counter-terms illustrates the quantum equivalence (to one-loop, at least) of the barred and unbarred systems for $\lambda=-\frac{1}{6}$. We shall not proceed any further the computation of on-shell divergences, instead we discuss the physical meaning of the constraint $1+\kappa\lambda\phi^2>0$. To do that we write the field equations obtained by variation of the action (1) in the suggestive form

$$\begin{aligned} G^{\mu\nu} = \mathcal{H}\Big[&\lambda\left(g^{\mu\nu}\Box-\nabla^\mu\nabla^\nu\right)\phi^2 \\ &+\frac{1}{2}g^{\mu\nu}\nabla_\alpha\phi\,\nabla^\alpha\phi-\nabla^\mu\phi\,\nabla^\nu\phi\Big] \quad , \qquad (11a)\\ &\Box\phi-\lambda R\phi=0 \quad , \qquad (11b)\end{aligned}$$

where

$$\mathcal{H}\equiv\frac{\kappa}{1+\lambda\kappa\phi^2}$$

is an effective Einstein constant which will be positive if and only if $1+\lambda\kappa\phi^2>0$. Thus, the adoption of the constraint in hand leads to Einstein's standard formula $G^{\mu\nu}=\mathcal{H}T^{\mu\nu}$, where $T^{\mu\nu}$ is given by the right-hand side of Eq. (11a) and $\mathcal{H}>0$. This result has of course important consequences for cosmology and black holes. To conclude we analyse the apparent anomaly concerning the classical and semi-classical results. The classical equivalence and consequently, the quantum one, are only valid for $1+\lambda\kappa\phi^2>0$. The semi-classical equivalence, in turn, seems to be completely independent of the parameter λ. The reason for that is very simple: the semi-classical results were obtained by expanding the metric around flat space, while to show that the barred and unbarred systems are equivalent at the classical(quantum) level we have made use of the full theory.

Acknowledgments

We are very grateful to the referees for their comments which helped to improve this work so much. RA, SFN and DS gratefully acknowledge financial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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Figures Captions

Fig. 1 Lowest-order graviton exchange force.

Fig. 2 Feynman diagrams for scalar-scalar scattering.

Fig. 3 Feynman diagrams for scalar-graviton scattering.

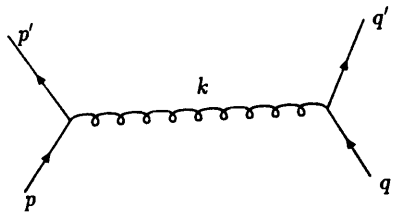

FIGURE 1

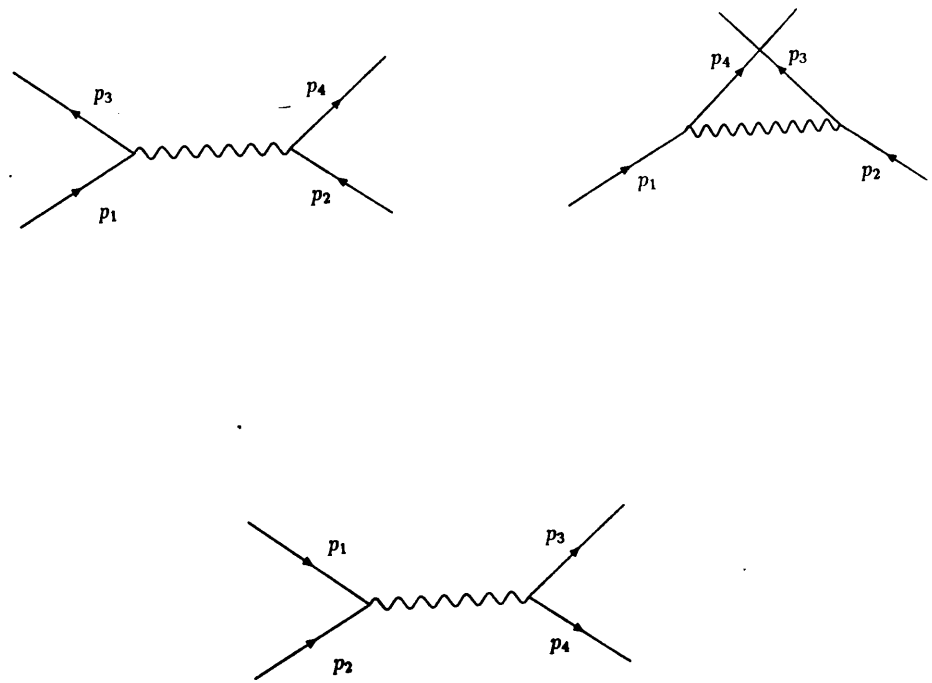

FIGURE 2

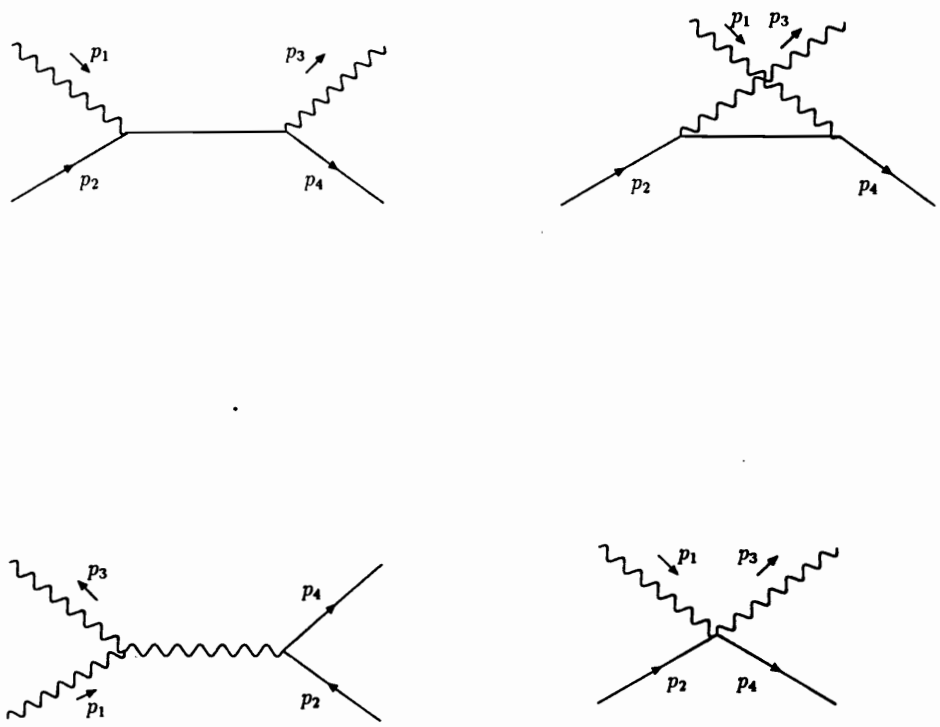

FIGURE 3