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PHYSICAL RUNNING COUPLING CONSTANT

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Abstract

The gluon condensate is incorporated in the definition of the dimenssionless scaling variable which parametrizes the characteristics, used in solving the partial differential equation of the renormalization group equation. We find the physical running coupling constant which is: positive for all real momenta, finite, and small for an arbitrary virtuality: space-like, time-like, and null. Our physical running coupling constant $\overline{\alpha}$ freezes out below 0.5 GeV, and at the null virtuality, i.e. at $p^2 = 0$, it takes the maximum value, which after dividing by π is either 0.3, or 0.2, depending on the mass scale of the gluon condensate: $\nu = 331$ MeV, or $\nu = 0.5$ GeV, respectively.

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The renormalization group equations for the one-particle irreducible vertex functions are usually solved by the method of characteristics[1]. This is the standard method to solve the linear partial differential equations. One ingredient of this method is the ordinary differential equation for the running coupling constant $\overline{\alpha}(t)$, where $\alpha \equiv g^2/(4\pi)$, and the variable t is the parameter along the characteristics. The ordinary differential equation for $\overline{\alpha}(t)$, and the initial condition for t= 0, which is necessary to define the solution of the differential equation, are

$$\frac{d\overline{\alpha}(t)}{dt} = \overline{\alpha}(t)\beta(\overline{\alpha}(t)), \qquad \overline{\alpha}(t=0) = \frac{g^2(\mu^2/\Lambda^2_{\rm QCD})}{4\pi} . \tag{1}$$

The mass scale μ is an arbitrary mass, with the only restriction, that it has to be sufficiently large, so that $g^2(\mu^2/\Lambda^2_{\rm QCD})/(4\pi^2) << 1$. In contrast to μ, the mass scale $\Lambda_{\rm QCD}$ is a very well defined mass, in any fixed renormalization scheme. $\Lambda_{\rm QCD}$ provides a necessary "unit", to give a definite numerical meaning to the ratio $\mu^2/\Lambda^2_{\rm QCD}$, for an arbitrarly chosen value of μ. We must know the numerical value of the ratio $\mu^2/\Lambda^2_{\rm QCD}$, to verify wheather the demand $g^2(\mu^2/\Lambda^2_{\rm QCD})/(4\pi^2) << 1$ is fulfilled. This is the necessary demand for using the perturbative calculations.

Here, we concentrate on the modified, minimal subtraction scheme $\overline{\rm MS}$, in which the numerical value of $\Lambda_{\rm QCD}$, as determined from experiment[2], is 0.15 GeV. The $\overline{\rm MS}$ scheme is such, that the β-function, appearing on the right-hand-side of Eq.(1), is independent of the gauge parameter ξ, and of mass. The coefficients β_1, and β_2, of the perurbative expansion of $\beta(\alpha)$ in powers of α, are renormalization group invariants, and are given below,

$$\beta(\alpha) = \beta_1 \frac{\alpha}{\pi} + \beta_2 \left(\frac{\alpha}{\pi}\right)^2 + \ldots ,$$

$$\beta_1 \equiv -\frac{1}{2}\beta_0, \qquad \beta_0 \equiv 11 - \frac{2}{3} n_f, \qquad \beta_2 \equiv \frac{1}{4}\left(\frac{19}{3} n_f - 51\right), \tag{2}$$

where n_f denotes the number of flavours, and the number of colours is set equal to 3.

The numerical value of the physical mass scale $\Lambda_{\rm QCD}$ does depend on the chosen scheme, and also an identity with respect to μ, which $\Lambda_{\rm QCD}$ obeys,

does depend on the specific convention, which is used in the given scheme to write down the explicit solution of the differential equation for $\overline{\alpha}(t)$. We show below an example of such identity in the case of the one-loop approximation of the β -function, and discuss the case of the two-loop approximation. Always, $\Lambda_{\rm QCD}$ has a fixed value, which has a definite physical meaning. In contrast to that, the numerical value of the mass scale μ is arbitrary, and must be left arbitrary. The value of μ has no physical relevance whatsoever, and this is true in QCD in any scheme. The only condition which μ must obey is, that $g^2(\mu^2/\Lambda^2_{\rm QCD})/(4\pi^2) << 1$, since this is necessary for the perturbative calculations.

The parameter t along the characteristics is t=ln(λ), where the standard expression[1] for the scaling factor λ, is

$$\lambda = \sqrt{\frac{-p^2}{\mu^2}} , \tag{3}$$

for the space like momentum p, with $-p^2 > \mu^2$.

Instead of the standard scaling factor λ we propose to parametrize the characteristics with a new scaling factor $\lambda_{\rm cond}$, which explicitly includes the nonzero value of the gluon condensate $\nu^4 \equiv <| \frac{\overline{\alpha}}{\pi} : G^a_{\mu\nu} G^{\mu\nu}_a :|>$, where the state $|>$ is the full, nonperturbative, vacuum state, $G^a_{\mu\nu}$ is the gluon field strength tensor, $\overline{\alpha}/\pi$ is the one-loop approximation of $-\beta/2g$, and the normal ordered product : : refers to the perturbative vacuum, and amounts to an appropriate subraction. Our scaling variable is defined as follows

$$\lambda^4_{\rm cond} \equiv \frac{p^4 + \nu^4}{\mu^4 + \nu^4} . \tag{4}$$

This is our assumption, that it is possible to change the parametrization of the characteristics using the new parameter t$\equiv$ln($\lambda_{\rm cond}$) $\equiv \frac{1}{4}$ ln(($p^4 + \nu^4$) / ($\mu^4 + \nu^4$)), and keeping the same numerical values of the constant coefficients β_1, and β_2, of the perurbative expansion of $\beta(\alpha)$ in powers of α, which are renormalization group invariants.

We are here taking for granted, that the values of β_1, and β_2, which in the perturbative calculation are renormalization group equation invariants, remain unchanged. Our only, a posteriori, verification of that is, that indeed for all values of the virtuality p^2, including the null virtuality, as well as for

all values of the arbitrary mass scale μ, including $\mu = 0$, the numerical value of $\overline{\alpha}/\pi$ is small.

In the temporary notation:

$$t_c \equiv t_{\rm cond} \equiv \ln(\lambda_{\rm cond}) \equiv \frac{1}{4}\ln\left(\frac{p^4+\nu^4}{\mu^4+\nu^4}\right), \quad t \equiv \ln(\lambda) \equiv \frac{1}{2}\ln\left(\frac{-p^2}{\mu^2}\right),$$

we find that the Jacobian dt_c/dt, which is necessary to pass from the differentiation d/dt, to d/dt_c, is

$$\frac{dt_c}{dt} = \left\{1 - \left[(1+\mu^4/\nu^4)\exp(4t_c) - 1\right]^{-1}\right\}^{-1}.$$

For large values of the new parameter t_c this Jacobian is practically equal to 1. However, for small values of t_c, in particular for $t_c = 0$, we have $dt_c/dt = (1-\nu^4/\mu^4)^{-1}$, which for small values of μ is a very small negative number. Therefore, for small values of t_c, and small values of μ, i.e. in the nonperturbative, soft domain, our modification of the change of the parametrization along the characteristics is a nontrivial one.

In the following, we shall come back to denoting by t the new parameter along the characteristics t_c, and another justification, for our choice of the new parametrization of the characteristics, is shown at the very end of this paper, where we calculate the integral over the low energy region, from 0 to 1 GeV, of the couplant $\overline{\alpha}/\pi$, and get rather small value ≈ 0.2 GeV, in a very good agreement with the phenomenological estimates. Moreover, staying with the standard parametrization of the characteristics, we could neither use the perturbative framework, nor we could get any sensible result for the integral over the couplant, anywhere near 0.2 GeV.

A priori, we may replace ν^4 in the above definition for $\lambda_{\rm cond}$ by the whole expression for the negative of the vacuum expectation value of the divergence of the Belinfante dilatation current[1,3,4]

$$\nu^4 \to <|\frac{\overline{\alpha}}{\pi} : G^a_{\mu\nu}G^{\mu\nu}_a :|> -(1+\gamma_m)\Sigma_f m_f <|: \overline{\psi_f}\psi_f :|>, \qquad (5)$$

where γ_m is the mass anomalous dimension, and Σ_f is the sum over all flavours. However, the above terms added to ν^4, play numerically an insignificant role. Aesthetically, the right-hand-side of Eq.(5) is more appealing than ν^4 itself, because there exists a proof[3,4], that Eq.(5) is invariant under renormalization group, on the one-loop level.

To defend the particular choice of $\lambda_{\rm cond}$ we offer five reasons:

1. For large space-like momenta, in practice for $-p^2 > 2$ GeV^2, $\lambda_{\rm cond}$ coincides numerically with the standard λ, and $-p^2 = \mu^2$ corresponds to $\lambda_{\rm cond} = 1$, i.e. to t=0.

2. For any real momentum the physical running coupling constant is positive, and we have a trivial continuation from the space-like to the time-like momenta, without causing any nonzero absorptive part in the result for the QCD physical running coupling constant. The QCD nonzero value of the hadronic absorptive part of the current-current correlation is provided by the nonzero value of the absorptive part of the nonperturbative photon-quark vertex function. It is neither given by the zero absorptive part of the QCD physical running coupling constant, nor by the zero absorptive part of the cut of the confined quark-antiquark pair. Physical cross section $\sigma_{e^+e^-}$ has only MULTIHADRONIC thresholds.

3. For small space-like, or small time-like momenta, and even for the null momentum, the numerical value of the physical running coupling constant, divided by π, is small, and we can allow ourselves to start even with $\mu = 0$, and still remain within the perturbative regime, as far as the condition $g^2\left((\mu^4+\nu^4)/\Lambda^2_{\rm QCD}\right)/(4\pi^2) << 1$ is concerned.

4. An analog of the QED Landau singularity[5], in the QCD physical running coupling constant, is totally removed from the real momentum axis, and for QCD it is shifted to the complex plane, at four points: $p = \pm(\nu^4 - \Lambda^4_{\rm QCD})^{1/4}\exp(\pm i\pi/4)$.

5. For the null momentum, which can be also associated with the vacuum state, our expression for $t = \ln(\lambda_{\rm cond})$, coincides with that given by Pagels and Tomboulis [6], in the one-loop approximation, when $-\beta/(2g) \approx \alpha/\pi$.

It should be noticed, that it is necessary, for all of the above reasons to work, that we take the sum $p^4+\nu^4$, and not any other function of momentum and the gluon condensate ν^4. A priori, the combination $ap^4+b\nu^4$, with a and b different from 1 is allowed, but it is an unnecessairly complicated variant of the above definition of $\lambda_{\rm cond}$. Seting a=b=1, we specify our convention,

and coincide with Pagels and Tomboulis[6] for the null momentum. We are also perfectly tuned up to the standard convention for the large momenta.

Another convention is needed[1], when we want to define the integration constant in the integral version of Eq.(1). To write down the solution for the running coupling constant, for any explicit form of the β-function, we must make a specific choice of the integration constant. Therefore, for the one-loop approximation of the β-function, we make some convention of the integration constant, and if we then consider the β-function which contains terms up to the two-loop order, then we can take another convention, in comparison with the former one.

All these conventions must be explicitly defined, for writing down the unique expression for the physical running coupling constant. The final result for the physical running coupling constant, written in terms of the kinematical momentum, and the physical mass scale $\Lambda_{\rm QCD}$, which has a particular (numerical) value, should refer to the order of approximation of the β-function.

Taking the β-function in the one-loop approximation, we specify the commonly used convention[1] by stating, that the prime function of the integral $\int dz/[z\beta(z)]$, evaluated for the zero value of the parameter t along the characteristics, is equal to $4\pi\left[\beta_0 \ln\left(\mu^2/\Lambda^2_{\rm QCD}\right)\right]^{-1}$. This corresponds to the standard $\overline{\rm MS}$-convention[1] of fixing the integration constant in the one-loop approximation for the β-function: $\overline{\alpha}(t=0) \equiv \overline{\alpha}(\ln\lambda = 0) \equiv g^2\left(\mu^2/\Lambda^2_{\rm QCD}\right)/(4\pi) = 4\pi/\left[\beta_0 \ln\left(\mu^2/\Lambda^2_{\rm QCD}\right)\right]$.

In the case of our scaling factor $\lambda_{\rm cond}$, we take the following convention for the integration constant

$$\overline{\alpha}(\ln(\lambda_{\rm cond}) = 0) \equiv \frac{g^2\left((\mu^4+\nu^4)/\Lambda^4_{\rm QCD}\right)}{4\pi} = 8\pi\left[\beta_0 \ln\frac{\mu^4+\nu^4}{\Lambda^4_{\rm QCD}}\right]^{-1}. \quad (6)$$

Corresponding to this integration constant convention, the one-loop solution of the differential equation (1) for the running coupling constant, with the initial condition stated in Eq.(1), and adapted to the convention written in Eq.(6), is

$$\overline{\alpha}\left(t = \frac{1}{4}\ln\left(\frac{p^4+\nu^4}{\mu^4+\nu^4}\right)\right) = \frac{\overline{\alpha}(t=0)}{1+\frac{1}{2\pi}\beta_0 t\overline{\alpha}(t=0)}. \quad (7)$$

The integration constant convention, written in Eq.(6), can be understood as the formal definition of $\Lambda_{\rm QCD}$. It corresponds to $\lambda_{\rm cond} = 1.0$, i.e. t=0, and it means, that any extra integration constant is set equal to zero in the one-loop level of the approximation of the β-function, i.e. we just evaluate the prime function at t=0. The form of Eq.(6), which is ready to use in Eq.(7), is

$$\overline{\alpha}(t=0) = 8\pi\left[\beta_0 \ln\left(\frac{\mu^4+\nu^4}{\Lambda^4_{\rm QCD}}\right)\right]^{-1}. \quad (8)$$

In turn, the last equation can be "inverted", i.e. we can write it as the expression for $\Lambda_{\rm QCD}$, in terms of μ, and $g^2\left((\mu^4+\nu^4)/\Lambda^4_{\rm QCD}\right)$. The result of this inversion is in fact an IDENTITY, with respect to the arbitrary mass scale μ, and it takes the following form

$$\Lambda^4_{\rm QCD} = (\mu^4+\nu^4)\exp\left[-\frac{32\pi^2}{\beta_0 g^2\left((\mu^4+\nu^4)/\Lambda^4_{\rm QCD}\right)}\right]. \quad (9)$$

If Eq.(8) is used as the expression for $g^2\left((\mu^4+\nu^4)/\Lambda^4_{\rm QCD}\right)/(4\pi)$, and we substitute it in Eq.(9), then we indeed find the trivial idendentity $\Lambda^4_{\rm QCD} \equiv \Lambda^4_{\rm QCD}$. This is the identity with respect to the arbitrary value of the unphysical mass scale μ, which must be considered as the arbitrary normalization point.

Finally, we can use the initial condition stated in Eq.(8), and substitute it in the solution for the running coupling constant, written in Eq.(7). As the result of this substitution we completely eliminate the arbitrary mass scale μ, and find the one-loop expression for the physical running coupling constant, expressed in terms of the kinematical momentum p, and the physical mass scale $\Lambda_{\rm QCD}$. As the argument of the physical running coupling constant we shall write now not the variable t, but rather the argument which appears explicitly under the logarithm.

Therefore, we have the following form of the physical running coupling constant, corresponding to the one-loop level expansion of the β -function,

$$\overline{\alpha}\left(\frac{p^4+\nu^4}{\Lambda^4_{\rm QCD}}\right) = 8\pi\left[\beta_0 \ln\left(\frac{p^4+\nu^4}{\Lambda^4_{\rm QCD}}\right)\right]^{-1}. \quad (10)$$

In the formal case, when we set $\nu = 0$, we recover the standard form of the physical running coupling constant, written in terms of the momentum p, and the physical mass scale Λ_{QCD}. It should be noted, that we use the terminology the "running coupling constant" for any solution of Eq.(1), in particular that given in Eq.(7), but we name the "physical running coupling constant", the particular solution of Eq.(1), written in Eq.(10), in which the arbitrary normalization mass scale μ is totally eliminated.

It is appropriate to emphasize the essential difference between Eq.'s (7) and (10). The former one is the solution of the renormalization group equation for the running coupling constant, but it has the arbitrary mass scale μ explicitly appearing in it. Therefore, it is of no use for physics, since it contains the arbitrary mass scale μ. It is completely incorrect to try to fix a particular value for the mass scale μ, since μ is arbitrary, as corresponding to the normalization point $p^2 = -\mu^2$, and it must be left arbitrary, from the "first principles", as the arbitrary normalization point.

In contrast to that, the expression for the physical running coupling constant written in Eq.(10), is also the solution of the renormalization group equation, but has the arbitrary mass scale μ totally eliminated. It provides a physical, and nonperturbative, information, which is expressed in terms of the kinematical momentum p, and the physical mass scale Λ_{QCD}. The numerical value of p^2 is at our disposal in Eq.(10), and in contrast to Eq.(7), we do not have to demand that in a calculation of a physical process we must allow ourselves an arbitrary variation of p^2, what is necessairly demanded for the arbitrary values of the nonphysical μ^2.

Moreover, if p denotes the gluon momentum in a loop, which undergoes an integration, then it is impossible to demand that $\mid p^2 \mid >> \mu^2$, since the normalization mass scale μ must be left arbitrary, and μ can be always larger than the magnitude of any momentum in the loop, for any fixed point of the integrand. It is only possible to use a momentum, which is the momentum external to the loop, to split up the domain of integration in the loop. However, such an external momentum must not be identified with the arbitrary mass μ, which appears in Eq.(7) through the parameter t along the characteristics, and through the initial condition for $\overline{\alpha}(t = 0)$.

An immediate consequence of the above remarks is the result, that it makes no sense whatsoever to try to fix a particular value of the arbitrary mass scale μ, by demanding, for example, that either the derivative with

respect to μ of α, or any function of α, is equal to zero, or that the derivative with respect to μ of any physical quantity, e.g. Λ_{QCD}, is equal to zero.

There are two reasons for that:

1. the value of μ can not, and must not be fixed, since it is a priori an arbitrary normalization mass scale,

2. α, or any function of α, expanded perturbatively in the powers of α/π, can only depend on μ superficially through the right-hand-side of Eq.(7), but in fact the dependence on μ is only seeming, and it totally goes away if we use Eq.(8). The last point is reiterated by realizing, that Eq.(9) is indeed an identity with respect to the arbitrary mass variable μ, and demanding that the derivative with respect to μ of Λ_{QCD}, or any other physical quantity, expanded perturbatively in the powers of α/π, is equal to zero, is an empty statement, since all of these quantities are identically independent of μ. Also, it is impossible to take Eq.(9), which is an identity with respect to μ, and try to determine from it the arbitrary mass scale μ, using as an input the experimental value for Λ_{QCD}, or vice versa.

For the two-loop approximation of the β-function all of the above arguments can be repeated, but algebraically, we find a transcendental equation, instead of Eq.'s (7), and (8). Therefore, i) only an approximate algebraic solution can be written down, and ii) the inversion of the analog of Eq.(8), into an analog of Eq.(9), is only possible on the approximate algebraic level. Thus, due to these algebraic approximations, it is hard to see, in the approximate expressions, that the independence of the physical running coupling constant of the unphysical mass scale μ does take place algebraically. Also the identity with respect to μ, such as is seen from Eq.'s (8) and (9), taken together, is harder to verify, due to the transcendental character of the appropriate algebraic equations.

Keeping however in mind, that the dependence on the unphysical mass scale μ is only seeming, we can use the approximate algebraic results, and write down the result for the physical running coupling constant, in the case of the β-function given by the formula with the two-loop terms included in the calculation. In writing the integral version of the differential Eq.(1),

we are free to specify a new convention for the integration constant, since the β-function is a new one, in comparison with the one-loop case. The standard $\overline{MS}$ choice[1], for the two-loop approximation of the β-function, is: $\overline{\alpha}(t=0) \equiv g^2\left(\mu^2/\Lambda^2_{QCD}\right)/(4\pi) = 4\pi\left[\beta_0 \ln\left(\mu^2/\Lambda^2_{QCD}\right)\right]^{-1} - \frac{\beta_2}{\beta_1^2}\ln\left(-\frac{\beta_1}{\pi}\right)$.

In our case, when the gluon condensate ν^4 is recognized to be not equal to zero, we use the following convention in defining the integration constant, for the case of the two-loop approximation of the β-function,

$$\overline{\alpha}(t=0) \equiv \frac{g^2\left((\mu^4+\nu^4)/\Lambda^4_{QCD}\right)}{4\pi} = 8\pi\left[\beta_0 \ln\frac{\mu^4+\nu^4}{\Lambda^4_{QCD}}\right]^{-1} - \frac{\beta_2}{\beta_1^2}\ln\left(-\frac{\beta_1}{\pi}\right). \tag{11}$$

Now, we can take the similar approximations, as they are done in the standard case[1], but for $\nu \neq 0$ they are legitimate for any momentum, while for the case when we formally set $\nu = 0$ there are some lower bounds on $\mid p^2 \mid$. The approximate expression for the physical running coupling constant, for the two-loop approximation of the β-function, is

$$\overline{\alpha}^{\text{two-loop}}\left(\frac{p^4+\nu^4}{\Lambda^4_{QCD}}\right) \approx \overline{\alpha}^{\text{one-loop}}\left(\frac{p^4+\nu^4}{\Lambda^4_{QCD}}\right)$$

$$\left[1 - \frac{\beta_2}{\beta_1\pi}\,\overline{\alpha}^{\text{one-loop}}\left(\frac{p^4+\nu^4}{\Lambda^4_{QCD}}\right)\ln\left(\frac{1}{4}\ln\left(\frac{p^4+\nu^4}{\Lambda^4_{QCD}}\right)\right)\right], \tag{12}$$

where $\overline{\alpha}^{\text{one-loop}}\left((p^4+\nu^4)/\Lambda^4_{QCD}\right)$ is given by Eq.(10).

For the null momentum $p^2 = 0$, and $n_f = 3$, the above two-loop correction is positive, and it is on the level of 8%, while at $\mid p \mid = \Lambda_{QCD}[e^4 - (\nu/\Lambda_{QCD})^4]^{1/4} \approx 2.4\Lambda_{QCD}$ the two-loop correction becomes a small negative term, added to 1. Of course, our physical running coupling constant, given either by Eq.(10), or Eq.(12), is never negative, for any real momentum p.

If in Eq.(10) we would turn off the gluon condensate, seting formally $\nu = 0$, then for $\mid p^2 \mid < \Lambda^2_{QCD}$, we would find a negative value for $\overline{\alpha}$. This, of course, has no physical meaning, since we put ourselves beyond the domain of the applicability of the perturbative calculations. However, this illustrates

the point, that in the standard treatment, if we would take the momentum p, below the analog of the Landau[5] singularity, i.e. below $p = \Lambda_{QCD}$, then $\overline{\alpha}$ would become negative.

The change of sign of $\overline{\alpha}$(t) seems to be, a priori, an unavoidable consequence of Eq.(7), since for the negative values of the parameter t along the characteristics, the result seems to change the sign, if $\mid t \mid$ is sufficiently large. Moreover, if we take the normalization mass scale μ sufficiently large, in comparison with Λ_{QCD}, then it is perfectly legitimate to go to smaller momenta than μ, and remain in the perturbative regime, having negative values of t. This paradox is resolved by realizing, that in Eq.(7) we have only a formal solution of Eq.(1), in which the unphysical mass scale μ has not yet been eliminated.

Therefore, we can not make a sensible statement about the sign of $\overline{\alpha}$, refering to the parameter t, which contains the unphysical mass μ. Only after we eliminate μ, i.e. when we use Eq.(10) for the physical running coupling constant, instead of Eq.(7), then it is possible to discuss the sign of $\overline{\alpha}$, in terms of the physical quantities such as the momentum p, and the QCD mass scale Λ_{QCD}. Unfortunately, in the recent discussions[7] of the infrared, and ultraviolet renormalons, Eq.(7) is used, with an incorrect identification of the arbitrary mass scale μ with the external momentum of the loop, carried by the electromagnetic current.

For the nonperturbative calculations, in the domain of soft momenta, we need the expression for the physical running coupling constant for any value of the real momentum p. We have the physical, light-cone singularities in the momentum p of the gluon leg in the nonperturbative gluon-quark vertex, and also in the nonperturbative triple-gluon vertex, at $p^2 = 0$. Therefore, we must be able to reach the domain of the null virtuality, $p^2 = 0$.

If two of the gluon-quark vertices are joined by the nonperturbative, transverse gluon propagator, which vanishes on the light-cone, then the Coulomb type interaction shows up in the colour-singlet quark-antiquark system, with the strength, at the pole $p^2 = 0$, equal to

$$\frac{4}{3}\left(\frac{g}{2\pi}\right)^2 = \frac{4}{3}\,\frac{\overline{\alpha}(p^2=0)}{\pi}. \tag{13}$$

Therefore, it is desirable to find a finite value of the physical running coupling constant for the null virtuality.

In accord with that, Dokshitzer[8] found rather small values for the estimates of the maximum of the physical running coupling constant, divided by π,

$$\left(\frac{\overline{\alpha}}{\pi}\right)^{\max} = 0.22 \pm \begin{cases} 0.08, \\ 0.05. \end{cases} \tag{14}$$

The Dokshitzer estimates follow from a comparison of the measured heavy quark losses, with the expressions found from the generalized perturbation theory, assuming, that the physical running coupling constant is frozen at small momenta.

For $\Lambda_{\rm QCD} = 0.15$ GeV, as found in ref.(2), Eq.(10) gives the following numerical results for $\overline{\alpha}(p^2 = 0)/\pi$, for two values of the mass scale ν,

$$\frac{\overline{\alpha}(p^2 = 0)}{\pi} = \begin{cases} 0.3, & \nu = 331 \text{ MeV}, \\ 0.2, & \nu = 500 \text{ MeV}. \end{cases} \tag{15}$$

These results remain numerically the same, if instead of ν^4 in Eq.(10), we put the negative value of the vacuum expectation value of the whole trace of the Belinfante energy-momentum tensor. We also note, that both of the values for $\overline{\alpha}(p^2 = 0)/\pi$, given in Eq.(15), are within the errors set up by Dokshitzer[8] in Eq.(14) for the maximum value of the physical running coupling constant, divided by π.

From Eq.(10) it is easy to see, that the physical running coupling constant freezes out for small momenta, roughly below $\mid p \mid \approx 0.5$ GeV. Another, interesting feature of Eq.(10) is, that for a given value of p^4, we find the same value of the physical running coupling constant, independently of the space-like, or time-like character of the virtuality p^2.

Our change of the parametrization along the characteristics, from Eq.(3) to Eq.(4), gives us the finite, and small results for $\overline{\alpha}\left((p^4 + \nu^4)/\Lambda^4_{\rm QCD}\right)/\pi$, if the mass scale ν of the gluon condensate is sufficiently larger than the QCD mass scale $\Lambda_{\rm QCD}$. The smaller value $\nu = 331$ MeV is the one, which is used by Shifman, Vainshtein and Zakharov[9] in their QCD sum rules. It is quoted as the "standard" value in ref.(10). It is also within 20-30 % limits set up by Eidelman, Kuradze and Vainshtein[11] from the e^+e^--annihilation into hadrons below 2 GeV.

The higher value of the gluon condensate mass scale $\nu \approx 0.5$ GeV, is proposed by Marrow and Shaw[12], on the basis of the comparison of the

experimental value of the sum rule ratio for the S-wave charmonium with the theoretical estimates. On the level of ν^4, these two values of ν differ by more than factor 5. However, for the numerical results of $\overline{\alpha}(p^2 = 0)/\pi$, the large difference between the values of ν^4, has rather small effect, since we have the logarithmic dependence on $\lambda_{\rm cond}$. Therefore, both values of ν give for $\overline{\alpha}(p^2 = 0)/\pi$ the results, which are within the errors set up by Dokshitzer[8] in Eq.(14).

Independently of the above reasons, we need the running coupling constant for an arbitrary value of virtuality if we want to check the self- consistency of the basic ansatz for the inverse of the gluon and quark nonperturbative propagators. That is within the framework of a decoupled subset of Dyson-Schwinger equations, solved on the one-loop (nonperturbative) level by the method of nonperturbative logarithms[13]. The expressions for the inverses of the gluon and quark nonperturbative propagators are given in terms of the gluon and quark condensates, respectively. The same values of these condensates, which we put for parametrizing the nonperturbative propagators, we have to retain from the loops, which are formed by closing up, in the position space, the begining and the end of the respective propagators.

Such loops represent themselves the vacuum-to-vacuum transitions and naturally give these condensates. In evaluating these loops we need to include the renormalization group equation improvement factors, which are logarithmic factors raised to the fractional, anomalous exponents. The presence of these factors insures us the finite, and stable results of the physical mass scales of the respective condensates, independently of an arbitrary variation of the large mass scale μ, used for normalization at the large space-like momentum $p^2 = -\mu^2$. The logarithmic factors in the improved propagators in these loops are the same one, as they appear in the expression for the running coupling constant. In these loops we must integrate over all virtualities, and we must be able to reach the null virtuality.

It is interesting to note, that Mattingly and Stevenson[14] found also the freezing of $\overline{\alpha}/\pi$ at low energies, with the value 0.26, from the third-order calculation of $R_{e^+e^-}$. However, their calculations crucially depend on fixing the arbitrary mass scale μ by the demand, that there is an infrared fixed point, where the β-function vanishes. Neither such demand is appropriate, since the normalization mass scale μ must remain arbitrary, nor it is meaningfull, since α, considered as a function of the mass scale μ, is only seemingly depending on μ. If we would calculate a derivative with respect to μ, then we would

find identically the zero result, from which, of course, we can not determine the value of μ.

The argument that the physical cross section is independent of μ, must be also understood as: "identically independent of μ", providing we include the hadronization part, which also contains the μ-dependence, if for evaluating the hadronic wave functions we would use Eq.(7) for the running coupling constant. However, instead of using Eq.(7), we should rather take Eq.(10) for the physical running coupling constant, and use it with p as the gluon momentum. Then, including all loops, which are closed up by hadrons, we can expect, that these loop integrals are dominated by the external momentum which is carried by lepton, for example. Then, we can find the best value of such an external momentum from the MEAN VALUE THEOREM for these integrals, by the fit to the experimental data.

The requirement, that the physical cross section should be fit in the best way, corresponds to the finding of the minimum of such integrals, and can be expressed by the vanishing of the derivative, with respect to this external momentum. Such derivative can be equal to zero, and it is not identically equal to zero. The vanishing of this derivative, at a physical, external momentum, can therefore determine the value of this external momentum, which corresponds to the best fit to the experimental data. It is in this way, that the numerical value of $\Lambda_{\rm QCD} = 0.15$ GeV, in the $\overline{\rm MS}$ is determined[2].

For the $\overline{\alpha}$, expressed in terms of such an external momentum to the quark-loop, the vanishing of the derivative at one point[14], i.e. the vanishing of the β-function at this point would, of course, not mean the freezing in the mathematical sense, since at other points the derivative is not equal to zero. It is rather a numerical observation, that for the small momenta, the physical running coupling constant, evaluated from Eq.(10), is practically constant for $|\,p\,| < 0.5$ GeV.

To see other consequences of our expression in Eq.(10) for the physical running coupling constant, we yet evaluate the integral $\int_0^{1\ \rm GeV} d\kappa\overline{\alpha}(\kappa^2)/\pi$, which was found as the model-independent prediction of Dokshitzer, Khoze and Troyan [15], from the heavy quark energy losses in e^+e^- annihilation. The long-distance contribution to the QCD field energy of a heavy quark is given in Table below, for various values of the mass scale ν of the gluon condensate, and also for cases when either only ν^4 is kept, or the whole right-hand-side of Eq.(5) is taken.

In Table below the case with ν^4, corresponds to the value of a parameter a=1.0, while the full right-hand-side of Eq.(5) to a=1.5. In the value a=1.5, gluons contribute with 1.0, the u and d quarks with 0.02, the s quark with 0.3, and the c and b quarks with 0.09, each. For these calculations we take for the current s-quark mass 300 MeV, and for the chiral s-quark condensate the value $-0.8(0.24\ \text{GeV})^3$.

The difference between the expression with ν^4, and with the negative of the vacuum expectation value of the trace of the energy-momentum tensor, replacing ν^4, is numerically the same as between the Shifman-Vainshein-Zakharov[9] value ν=331 MeV, and the Reinders-Rubinstein-Yazaki[16] value ν=360 MeV.

Table

$$\int_0^{1\ \rm GeV} d\kappa \frac{\overline{\alpha}\left(\frac{\kappa^4+\nu^4}{\Lambda^4}\right)}{\pi} = \frac{8}{11-2n_f/3}\int_0^{1\ \rm GeV} d\kappa/\ln\left(\frac{\kappa^4+a\nu^4}{\Lambda^4}\right)$$

ν	a=1.0	a=1.5
331 MeV	0.23 GeV	0.22 GeV
360 MeV	0.22 GeV	0.21 GeV
500 MeV	0.19 GeV	0.18 GeV

$n_f = 5.$

The model-independent prediction of Dokshitzer, Khoze and Troyan[15] for the value of the above integral is $\int_0^{1\ \rm GeV} d\kappa\alpha_s(\kappa^2)/\pi \approx 0.2$ GeV. Note, that with the conventional expression for the physical running coupling constant, which formally is obtained from Eq.(10) by switching off the gluon condensate ($\nu = 0$), it would be hard to get the above integral of the couplant from 0 to 1 GeV around 0.2 GeV, if the analog of the Landau[5] singularity would be present on the real momentum axis at $\kappa = \Lambda_{\rm QCD} = 0.15$ GeV.

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