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LATTICE DATA AND GRIBOV-STINGL-ZWANZIGER GLUON PROPAGATOR

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Abstract

Several lattice data for the gluon propagator in the Landau gauge are compared with the predictions from the continuum form of the Gribov-Stingl-Zwanziger gluon propagator containing the renormalization group equation improvement factor

$$D_{\mu\nu}(p) = \frac{p^2}{p^4 + (c\nu)^4} \left(\ln\frac{\mu^4 + \nu^4}{\Lambda^4} / \ln\frac{p^4 + \nu^4}{\Lambda^4} \right)^{1/2} \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right),$$

where $\nu^4 \equiv <| \frac{\alpha_s}{\pi} : G^a_{\mu\nu} G^{\mu\nu}_a :|>$ is the gluon condensate with the Shifman-Veinshtein-Zakharov value for $\nu = 331$ MeV, c is a gauge-dependent constant with the value very close to one (slightly larger than 1) determined from the self-consistency demand that the above value of the gluon condensate is obtained from the vacuum to vacuum transition by closing up the line of the gluon propagator what is equivalent to taking the limit in the position- space at the same space-time point.

α_s is the running coupling constant with its validity extended to an arbitrary real momentum and given explicitly by the expression $\alpha_s = \frac{8\pi}{\beta_0} / \ln\frac{p^4+\nu^4}{\Lambda^4}$, $\beta_0 = 11 - \frac{2}{3} n_f$, Λ is the QCD mass scale determined at large momenta, n_f is the number of flavours, μ is an arbitrary (larger than a few GeV) mass scale used as the renormalization point in the $\overline{MS}$ renormalization scheme and the anomalous exponent 1/2 corresponds to the Landau gauge.

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The best comparison with the lattice data can be made for the zero three-momentum gluon propagator as the function of the Euclidean lattice time variable. This means the lowest possible value of the three-momentum and the position-space variation in the time variable. In contrast to that is the comparison of the lattice data for the full momentum-space dependence of the gluon propagagator which in the lattice calculations requires the evaluation of the Fourier transform from the whole position-space including the very small distances for which the lattice calulations have very limited validity. An ideal set of the lattice data for our comparison is provided by Fig.12 of the paper by Gupta et al[1].

To make our considerations analytical we simplify the above form of the gluon propagator and for the following calculations we remove the ratio of the logarithmic factors raised to the anomalous exponent 1/2. In this simplified case we also set equal to one the constant c which is only slightly larger than 1. Then, the zero three-momentum transverse gluon propagator has the following pole expansion in the complex energy-plane

$$p^2(p^4+\nu^4)^{-1}\,|_{\vec{p}=0}=\frac{1}{2}\left[\frac{1}{(p^0)^2-i\nu^2}+\frac{1}{(p^0)^2+i\nu^2}\right]=\frac{1}{4\nu}$$

$$\left[e^{-i\pi/4}\left(\frac{1}{p^0-\nu e^{i\pi/4}}-\frac{1}{p^0+\nu e^{i\pi/4}}\right)+e^{i\pi/4}\left(\frac{1}{p^0-\nu e^{-i\pi/4}}-\frac{1}{p^0+\nu e^{-i\pi/4}}\right)\right]. \tag{1}$$

This is an even function of p^0 and if we go from the Minkowski p^0 to the Euclidean p_4 by the relation $p^0=ip_4$ then we only pick up the overall minus sign while the poles in the complex variable p_4 remain at the same positions as they are in the complex variable p^0.

The Fourier transform from p^0 to the Minkowski time t we perform by integrating the above expression with the weighting factor $exp(-ip^0t)$, and for the Fourier transform from the Euclidean energy-variable p_4 to the Euclidean time-variable τ we use the weighting factor equal to $exp(-ip_4\tau)$ with the Euclidean time-variable τ equal to $it=\tau$. Altogether, we have the following relations between the Minkowski and the Euclidean energy and time variables and the exponential weighting factors in the Fourier transforms

$$p^0 = ip^4, \qquad \tau = it, \qquad exp(-ip_4\tau) = exp(-ip^0t)\,|_{\,t=-i\tau,\ \ p^0=ip_4}\,. \tag{2}$$

Up to the overall sign the result for the Fourier transform is the same independently wheather we do it in the Minkowski or in the Euclidean space.

If either t or τ are positive we close up the contour of integration in the lower half-plane of the respective complex-planes p^0 or p_4, while for the negative value of either t or τ we close up the contours in the upper half-planes of the complex variables p^0 or p_4, respectively. Including both positive and negative values of the time-variable τ the result written for the Fourier transform in the Euclidean case is

$$\text{Fourier transform}[p^2(p^4+\nu^4)^{-1}\,|_{\vec{p}=0}]=\frac{-1}{2\nu}\,e^{-\nu|\tau|/\sqrt{2}}\sin\left(\frac{\pi}{4}-\frac{\nu\,|\,\tau\,|}{\sqrt{2}}\right)=$$

$$=\frac{-1}{2\sqrt{2}\nu}\,e^{-\nu|\tau|/\sqrt{2}}\left[\cos\left(\frac{\nu\tau}{\sqrt{2}}\right)-\sin\left(\frac{\nu\,|\,\tau\,|}{\sqrt{2}}\right)\right]. \tag{3}$$

This is a continuous function of τ with a non-zero value at $\tau=0$. However, its derivative with respect to τ is discontinuous at $\tau=0$. In the expression for the derivative of such function normalised to +1 at $\tau=0$ we obtain the step-function $\epsilon(\tau)$

$$\frac{d}{d\tau}\left[\sqrt{2}e^{-\nu|\tau|/\sqrt{2}}\sin\left(\frac{\pi}{4}-\frac{\nu\,|\,\tau\,|}{\sqrt{2}}\right)\right]=-\epsilon(\tau)\,\nu\sqrt{2}e^{-\nu|\tau|/\sqrt{2}}\cos\left(\frac{\nu\tau}{\sqrt{2}}\right), \tag{4}$$

where the step-function $\epsilon(\tau)$ has the value -1 for the negative values of τ and $+1$ for the positive values of τ. Note, that both for the positive and the negative value of τ the absolute value of this derivative <u>decreases</u> with the absolute value of τ.

The effect of the discontinuity of the derivative with respect to τ at $\tau=0$ is related to the too slow decrease of the integrand of the Fourier transform at large energies when we calculate this derivative by the Fourier transform. If instead of the function $-p_4^2/(p_4^4+\nu^4)$ we would consider for example the function $-\nu^2/(p_4^4+\nu^4)$ then its Fourier transform is again the function of $|\,\tau\,|$ but its derivative with respect to τ is continuous at $\tau=0$ and the value of this derivative is zero at $\tau=0$. The original gluon transverse propagator having in it the logarithmic renormalization group equation improvement factor is still too slowly decreasing at very large energies. However, if we multiply it by an extra factor $\alpha_s(p)$, i.e. by the first power of the running coupling constant mentioned in the Abstract, then such a product decreases sufficiently fast at large energies and gives a convergent integral in the expression for the derivative with respect to τ of the Fourier transform of the product of the gluon propagator and $\alpha_s(p)$.

There are still two analytical issues which we have to consider before making the comparison with the lattice data. First is the time derivative of the logarithm of the Fourier transform of the gluon propagator since usually the lattice data are presented in the plot with the logarithmic scale for the correlation function and with the linear scale for the time-variable. Second, is an interesting additional notion of the "effective gluon mass" introduced by Mandula and Ogilvie[2], denoted as $M(\tau)$, and defined in the region where the Fourier transform normalised to $+1$ at $\tau = 0$ is different from zero. $M(\tau)$ is defined by the following expression

$$e^{-M(\tau)\tau} \equiv \sqrt{2}e^{-\nu|\tau|/\sqrt{2}}\sin\left(\frac{\pi}{4} - \frac{\nu\,|\,\tau\,|}{\sqrt{2}}\right). \tag{5}$$

There are three interesting results concerning the analytical predictions.

1. The time derivative of the Fourier transform of the gluon propagator itself is negative for the positive values of the time variable τ and the modulus of this derivative decreases with time.

2. The time derivative of the logarithm of the Fourier transform of the gluon propagator is for the positive values of the time variable also negative but its modulus increases with time.

3. The "effective gluon mass" $M(\tau)$ also increases with the increasing value of time τ.

The explicite expressions for the time derivative of the logarithm of the Fourier transform of the gluon transverse propagator, and for the "effective gluon mass" $M(\tau)$ are, respectively,

$$\frac{d}{d\tau}\left\{\ln\left[\sqrt{2}e^{-\nu|\tau|/\sqrt{2}}\sin\left(\frac{\pi}{4} - \frac{\nu\,|\,\tau\,|}{\sqrt{2}}\right)\right]\right\} = \frac{-\nu\sqrt{2}}{1-\tan(\nu\tau/\sqrt{2})},$$

$$M(\tau) = \frac{\nu}{\sqrt{2}} - \frac{1}{\tau}\ln\left[\cos\left(\frac{\nu\tau}{\sqrt{2}}\right) - \sin\left(\frac{\nu\tau}{\sqrt{2}}\right)\right]. \tag{6}$$

In Table 1 below we show the absolute values of the decreasing derivative with respect to time of the Fourier transform of the gluon transverse propagator normalised to $+1$ at $\tau = 0$. The lattice time intervals shown

in the first row of Table 1 correspond to the following times in the lattice units: 0, 2a, 4a, 6a, 7a, and 8a. For $a^{-1} = 2.5$ GeV taken by Gupta et al these intervals are the following Euclidean times τ: 0, 0.16 fm, 0.32 fm, 0.47 fm, 0.55 fm, and 0.63 fm. In the second row of Table 1 there are given the numerical values of this derivative calculated from an enlarged copy of Fig.12 of Gupta et al.[1]. In the third row of Table 1 there are given the analytical predictions for the absolute values of the derivative of the gluon propagator for the Shifman- Veinshtein- Zakharov[3] (SVZ) value of $\nu = 0.331$ GeV. In the fourth row of Table 1 there are shown the analytical predictions for the Marrow and Shaw[4] (MS) value $\nu = 0.5$ GeV.

Table 1 $\left|\frac{d}{d\tau}\text{Fourier transform}\,[p^2(p^4+\nu^4)^{-1}\,|_{\vec{p}=0}]\,\right| = \nu\sqrt{2}e^{-\nu|\tau|/\sqrt{2}}\cos\left(\frac{\nu\tau}{\sqrt{2}}\right)$

τ	0	0.16 fm	0.32 fm	0.47 fm	0.55 fm	0.63 fm
Gupta	0.22 GeV	0.24 GeV	0.21 GeV	0.20 GeV	0.19 GeV	0.14 GeV
SVZ	0.47 GeV	0.38 GeV	0.30 GeV	0.23 GeV	0.19 GeV	0.16 GeV
MS	0.71 GeV	0.51 GeV	0.34 GeV			

At time intervals 0.47 fm, 0.55 fm and 0.63 fm we left blank spaces for the MS case since for such large value $\nu = 0.5$ GeV the value of the time-variable τ_0 at which the Fourier transform of the zero three-momentum gluon propagator is equal to zero is $\tau_0^{MS} = 0.44$ fm. The same time-variable where the Fourier transform of the zero three- momentum gluon propagator vanishes is for the SVZ case equal to $\tau_0^{SVZ} = 0.66$ fm.

Note, that at the value of the time-variable $\tau = 0.55$ fm $= 7a$ the agreement between the lattice data of Gupta et al.[1] and the SVZ[2] value is perfect. Usually, at the first few time intervals of the lattice data there are some distortions due to the finite size effects and higher masses, so the agreement at the time interval around $7a = 0.55$ fm should be considered as the safe one. It is also worthwhile to notice from Table 1 that except for the first value of the derivative at $\tau = 0$ all the lattice data show systematically the decreasing values of the time derivative in accord with the predictions of the analytical formula. If we would include the renormalization group equation improvement logarithmic factor then the value of τ_0 where the

Fourier transform of the gluon propagator at zero three-momentum vanishes moves slightly towards larger values. (Such calculations have to be done numerically.)

The curve shown in Fig.12 of Gupta et al.[1] exhibits the CONVEX character what corresponds to the fact that Gupta et al.[1] in order to fit their data had to use a superposition of two exponentially decreasing terms, one with a positive and the other one with the negative (screened ghost-like) coefficient. The masses in these decreasing exponentials are 600 ± 90 MeV for the positive definite contribution and 1250 ± 80 MeV for the screened ghost-type term.

As Gupta et al.[1] emphasize the transverse gluon " correlator can-not be fit by a sum of contributions from positive-definite states only". In the above analytical formula for the Fourier transform of the zero three-momentum transverse gluon propagator there are also two exponentially decreasing terms first with the positive coefficient $\cos\left(\frac{\nu\tau}{\sqrt{2}}\right)$ and the second with the negative coefficient $-\sin\left(\frac{\nu|\tau|}{\sqrt{2}}\right)$ for $\tau < \tau_0$. This also indicates the CONVEX character of the curve shown in Fig. 12 of Gupta et al. Note however that the trigonometric coefficients in the analytical formula multiplying the exponentially decreasing terms themselves vary with the time-variable and either increase or decrease the same exponential fall off of both exponents in the analytical expression.

The CONVEX character of the curve drawn in Fig. 12 of Gupta et al.[1] corresponds directly to the increasing value of the time derivative of the logarithm of the Fourier transform of the transverse gluon propagator at the zero value of the three-momentum. At the same time intervals as in Table 1 there are given in Table 2 the increasing values of the time derivative of the logarithm of the Fourier transform of the gluon transverse propagator at the zero three- momentum. Finally, in Table 3 there are given the increasing values of the "effective gluon mass" at the same time intervals as in Tables 1 and 2.

Table 2 $\left| \frac{d}{d\tau} \text{Logarithm Fourier tr. } [p^2(p^4+\nu^4)^{-1} |_{\vec{p}=0}] \right| = \frac{\nu\sqrt{2}}{1-\tan(\nu\tau/\sqrt{2})}$

τ	0	0.16 fm	0.32 fm	0.47 fm	0.55 fm	0.63 fm
SVZ	0.47 GeV	0.58 GeV	0.71 GeV	1.26 GeV	2.02 GeV	6.65 GeV
MS	0.71 GeV	1.00 GeV	1.94 GeV			

Table 3 $M(\tau) = \frac{\nu}{\sqrt{2}} - \frac{1}{\tau}\ln\left[\cos\left(\frac{\nu\tau}{\sqrt{2}}\right) - \sin\left(\frac{\nu\tau}{\sqrt{2}}\right)\right]$

τ	0	0.16 fm	0.32 fm	0.47 fm	0.55 fm	0.63 fm
SVZ	0.47 GeV	0.52 GeV	0.59 GeV	0.72 GeV	0.84 GeV	1.16 GeV
MS	0.71 GeV	0.83 GeV	1.09 GeV			

Note, that the "effective gluon mass" for the SVZ[2] case ranges between two values 0.5 GeV and 1.2 GeV which roughly correspond to the two mass scales 0.6 GeV and 1.3 GeV used by Gupta et al.[1] for the fixed exponential masses in the positive-definite and the negative-definite terms, respectively.

It is also important to recognize from Table 3 the steady increase of the "effective gluon mass" in accord both with the CONVEX character of the curve in Fig. 12 of Gupta et al.[1], and with the increasing absolute values of the time derivative of the logarithm of the Fourier transform of the transverse gluon propagator, as well as with the decreasing absolute values of the time derivative of just the Fourier transform of the transverse gluon propagator at the zero value of three-momentum.

The spontaneous generation of the non-zero mass scale for gluon makes also the physical sense of going to the frame of reference where the three-momentum is equal to zero.

There are two papers by Mandula and Ogilvie[2,5] in which the lattice gluon propagator is calculated in the Landau gauge. The lattices used in these papers are: 4^3x8, 4^3x10, 4^3x8, *and* 4^4 which are much smaller than the lattice 18^3x42 of the Gupta et al calculations. The lattice results of Mandula and Ogilvie[2,5] do show the increasing values with time of the "lattice effective gluon mass" which in the continuum is equivalent to the

negative of the time derivative of the logarithm of the Fourier transform of the zero three-momentum gluon propagator. For the 4^3x8 lattice in Mandula and Ogilvie[2] the value of the "lattice effective gluon mass" is 0.38 ± 0.04 GeV at the time equal to 0.7 fm what corresponds to $6.0a^{-1} = 1.69$ GeV.

Very recently Bernard, Parrinello and Soni[6] presented "A Lattice Study of the Gluon propagator in Momentum Space". There, the pure glue QCD was studied on the 16^3x40 lattice at $\beta = 6.0$, on the 24^4 lattice at $\beta = 6.3$ and on the 16^3x24 lattice at $\beta = 5.7$. Their gluon propagator at the zero three-momentum presented in their Fig.2 for the lattice 16^3x40 at $\beta = 6.0$ and $a^{-1} = 2.1$ GeV also shows the increasing "lattice effective gluon mass" however with the values smaller than the one corresponding either to the lattice data of Gupta et al.[1] or to the analytical expression with the SVZ[2] value for $\nu = 0.331$ GeV. In Fig. 2 of Bernard et al.[6] the "lattice effective gluon mass" starts from the value 0.22 GeV at $\tau = 0$ and at $\tau = 2a \approx 0.2$ fm it is ≈ 0.42 GeV, at $\tau = 4a \approx 0.4$ fm it is 0.57 GeV, and at $\tau \approx 0.6fm$ is approximately equal to 0.65 GeV. All of these values are determined with rather large errors.

In Fig. 6 of Bernard et al.[6] for the 24^4 lattice at $\beta = 6.3$ and $a^{-1} = 3.2$ GeV the "lattice effective gluon mass" has the value 0.21 GeV at zero time, the value 0.45 GeV at $3a = 0.18$ fm, the value 0.55 GeV at the time equal to $5a = 0.31$ fm, and a slightly smaller value equal to 0.54 GeV at the time $8a = 0.49$ fm also with very large errors. The values of the "lattice effective gluon mass" of Bernard et al.[6] either in their Fig. 2 or in Fig. 6 are much smaller than the values of the time derivative of the logarithm of the Fourier transform shown in Table 2 particularly at the times between 0.4 fm and 0.6 fm which are sufficiently far away from the zero time. Also in Fig. 6 of Bernard et al.[6] the "lattice effective gluon mass" starts to decrease for the increasing time at around 0.4 fm in conflict both with the lattice data of Gupta et al.[1] and with the predictions of the analytical continuum formula of Gribov-Stingl-Zwanziger.

Some source of errors in the results of Bernard et al.[6] is shown in their Fig. 1 where the results for one gauge-fixed configurations of the time-component of the gluon field which should remain constant in time in the Landau gauge show a large variation from the zero time up to the time-variable around $5a \approx 0.5$ fm and only then remain constant until around 1 fm where they start growing up very rapidly. In contrast to this largely varying behaviour of the one gauge-fixed configuration of the time-component of

the gluon field of Bernard et al.[6] the time-time gluon correlation function shown in Fig. 12 of Gupta et al.[1] remains very much time-independent, slightly decreasing with time at around $\tau \approx 10a \approx 0.8$ fm. Over 20 lattice spacings the variation of the time-time gluon correlation is around 4 % in Fig. 12 of Gupta et al.[1] while in Fig. 1 of Bernard et al.[6] the variation of one gauge-fixed configuration of the time-component of the gluon field is 33 % with a very similar lattice spacing $a^{-1} = 2.1$ GeV for Bernard et al.[6] and $a^{-1} = 2.5$ GeV for Gupta et al[1].

Bernard et al.[6] show also the lattice results for the gluon propagator in the momentum space. For small momenta they are able to fit their results in Fig. 4 with the analytical expression corresponding to the Gribov-Stingl-Zwanziger gluon propagator with the value $\nu = 0.333 \pm 0.012$ GeV which is practically the Shifman- Veinshtein- Zakharov[2] value. The large momentum behaviour of their lattice gluon propagator for momenta above 1 GeV can not be fit by the analytical formula of the Gribov-Stingl-Zwanziger gluon propagator but instead requires a power law fall off of the form $1/p^{2+\gamma}$ with $\gamma \approx 0.7$.

Such fast fall off is not in accord with the much slower fall off of the Gribov-Stingl-Zwanziger gluon propagator together with the renormalization group equation logarithmic factor raised to the anomalous exponent. In the domain of momenta between 1 and 2 GeV this logarithmic term increases the $1/p^2$ fall off of the Gribov-Stingl-Zwanziger gluon propagator but only approximately by the factor $1/p^{0.23}$ in this range of momenta. There is however a problem of taking the lattice data results for such large momenta in the presence of the lattice ultra-violet cut off. In general the lattice data are much more reliable in the position space than in the momentum space where one needs all informations for the gluon propagator also at the very small distances which are correlated with its high momentum behaviour and therefore effected by the ultra-violet cut off.

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