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Temperature and time in the geometry of rotating black holes

J. David Brown and James W. York, Jr.

Institute of Field Physics and
Theoretical Astrophysics and Relativity Group
Department of Physics and Astronomy
The University of North Carolina
Chapel Hill, NC 27599-3255

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Introduction

Let us consider a problem with, ostensibly, no time, a problem of thermodynamic equilibrium involving black holes. Whatever the results of the ambitious programs described in this volume, it seems likely that black–hole entropy (Bekenstein 1973) and its place in the larger structure of physics will play an important role in understanding the physical origins of time–asymmetry. Black–hole radiance (Hawking 1975) involves in a relatively concrete context many of the themes that have appeared repeatedly in this Workshop: quantum theory, gravity, statistical and thermal physics, and, perhaps, information theory in some not–yet–clear form. The nature of thermodynamic equilibrium involving black holes, and, more generally, in self–gravitating systems, must be formulated as precisely as possible, we believe, before non–equilibrium processes involving gravity, with corresponding increase of entropy and appearance of time–asymmetry, can be described with confidence. For these reasons it is worthwhile to pursue a well–posed mathematical treatment of the equilibrium case.

The present work began with a clarification (York 1986; Whiting and York 1988) of two very different interpretations of the Euclidean Schwarzschild solution, either as a stationary configuration for the canonical partition function describing a physical black hole (Gibbons and Hawking 1977), or as an instanton characterizing black hole nucleation (Gross *et al.* 1982). The question arose because of the belief that the relevant heat capacity derived from the partition function was always negative. However, it can have either sign, and this fact enabled a reconciliation of the two points of view. That work has progressed in a continuing program of studies, part of which we shall now describe briefly. See Brown *et al.* (1991a, 1991b) and references therein for details.

Physical picture

Gravitational thermodynamics has a number of characteristic features that arise from the long–range, unscreened nature of the gravitational interaction. Gravitating systems are never truly isolated in the traditional sense of thermodynamics.

Although ordinary systems are never actually perfectly isolated either, in practice this leads to small and usually unimportant errors. In contrast, in gravitating systems the failure of perfect isolation has crucial implications, leading to the more prominent role of boundary conditions and the use of finite systems. It should be noted that the use of finite systems, irrespective of gravity, already implies the inequivalence of the various statistical ensembles as well as possible modifications of ordinary thermodynamic relations (Hill 1963, 1964; Horwitz 1966).

Problems of gravitational thermodynamics have to be formulated globally because of the infinite range of the gravitational field, even though at a given spacetime event the principle of equivalence allows us formally to ignore gravity in writing the local thermodynamic laws of gravitating matter (apart from possible quantum corrections depending on the curvature, which we shall not consider explicitly here). Thermodynamic properties of the gravitational field reveal themselves globally, or, alternatively, quasi–locally, subject to definite boundary conditions. (We shall use the term "global" to include "quasi–local with definite boundary conditions".) Black–hole thermodynamics is an extreme case of gravitational thermodynamics and this is where we shall focus attention in this discussion. Here the requirement of a global viewpoint is even more evident. One certainly cannot consider subdividing a black hole! Many of the unusual features of black–hole thermodynamics can be expected to carry over to the case of massive gravitating systems that contain no black holes.

Physical systems are, of course, quantum mechanical by nature, so to speak of a system as containing a black hole is to assume the system can be approximated reasonably well by a single classical configuration. In a "semiclassical" approximation, the system can be viewed as containing a black hole surrounded by radiation described by a non–zero renormalized stress–energy–momentum tensor $< T_{\mu\nu} >$. We shall refer to this radiation as the "environment" of the hole, and assume that it extends to some (possibly large) radius r_0. The black hole plus environment make up the thermodynamic system to be studied. In this paper we focus on the case of thermodynamic equilibrium, which implies, in particular, maintaining appropriate

boundary conditions at r_0. We shall *not* refer to the radiation environment of the hole as a "heat bath", for this terminology brings to mind, misleadingly, a "heat reservoir" as in traditional statistical thermodynamics, whose function is to maintain a definite temperature in the system, as in a canonical ensemble. Once one sees that the radiation–filled environment of a black hole is not to be regarded as a heat reservoir, the argument that only the microcanonical ensemble for black holes can be stable (Hawking 1976; Gibbons and Perry 1978) loses its force (York 1986).

The role of a heat reservoir in contact with the system at radius r_0, if one is needed, is to maintain certain boundary conditions to the accuracy permitted by physical principles. The reservoir is not thought of as part of the system in this view. It can have a stress–energy tensor entirely different from that of the environment. As one example, meant to be illustrative of principle rather than representative of what is expected in the natural world, consider a heat reservoir for the black hole problem as a large hollow "steel ball" with inner radius r_0, suitably instrumented and equipped to maintain specified boundary conditions at r_0. If necessary, the thick shell could include "rocket ships" to support it against being ruptured and pulled into the black hole. In such a case the radial members of the reservoir would be in a state of tensile, rather than compressive, stress. The traditional view of the foundations of statistical thermodynamics would hold that the meta–system, black hole plus environment plus reservoir, if regarded as an insulated (microcanonical) structure with fixed energy, angular momentum, *etc.*, should be in a state of extremal entropy. If the system is found to be stable, then so should the meta–system. All our results are consistent with this view.

The distinctions we have described above are required precisely because black–hole thermodynamics is indeed global. Thus, even when equilibrium is assumed under the *approximation* $<T_{\mu\nu}>= 0$ between the horizon and the boundary radius r_0, the space around the hole, which is classically a vacuum, is nevertheless part of the system because it has non–trivial properties. This point is especially clear when one considers a rotating black hole and the dragging of local frames in the space around it. The role of this effect in black–hole thermodynamics (Brown *et al.*

1991a, 1991b) will be described below.

It is clear that an equilibrium system with $<T_{\mu\nu}> \neq 0$ cannot have a gravitational field that is asymptotically flat in spacelike directions, with bounded total energy, unless the system itself is finite, that is, spatially bounded, and is enclosed by a perfectly insulating (microcanonical) boundary B. The ordinary thermodynamic limit does not exist because as the boundary radius r_0 increases, the environment will eventually collapse onto the existing hole, thus forming a larger one. In considering a modified thermodynamic limit, one finds from rough estimates that $E^5 \sim r_0^3$, where E is the total energy of the system (Hawking 1976; Gibbons and Perry 1978). The equilibrium is stable only when most of the energy of the system is the black hole's mass, so the hole's environment is not suitable as a heat reservoir and, as we have stated, it is not considered as such. More precise calculations, including back-reaction effects, confirm these results qualitatively whenever r_0 is large compared to the gravitational radius of the black hole, which in turn is large compared to the Planck length (York 1985).

The various *canonical* ensembles that can be defined are of great interest because they link statistical mechanics and quantum gravity through Feynman's prescription for the partition function as a path integral in "periodic imaginary time" (Feynman and Hibbs 1965). In these cases there is no "asymptotically flat spatial infinity" pertaining to the thermodynamic system because of the required intervention of a heat reservoir to maintain canonical boundary conditions. The heat reservoir, and the boundary conditions it maintains, represent the connection between the self–gravitating thermodynamic system and the rest of the universe. Thus, the pioneering effort of Gibbons and Hawking (1977) to implement the functional integral approach, using a canonical ensemble, was flawed by the use of asymptotically flat boundary conditions ($r_0 = \infty$). As a result, they necessarily found a negative heat capacity that spoiled their approach to the canonical ensemble. However, by using $r_0 < \infty$ one finds that the heat capacity can also be positive (York 1986). To go a step further, one knows that in a canonical ensemble one's task is to *find* from the partition function the energy as a function of a given temperature, not to assume

it from the start. This was carried out for any r_0 by Whiting and York (1988), who used a stationary phase approximation.

Thermodynamic equilibrium and gravity

The spacetime associated with a self–gravitating system in thermodynamic equilibrium is time–independent in the sense that it possesses a timelike Killing vector field $\partial/\partial t$ that can be either orthogonal to the "preferred" spacelike slices $t = constant$ (static) or not (stationary), the latter case being associated with non–vanishing angular momentum. In either case, such systems are spatially inhomogeneous. The temperature, "chemical potentials", and other "intensive" variables vary in space but not in time (Tolman 1930). This has very interesting implications for a system containing a black hole *and for other self–gravitating systems as well.* Gravity breaks the translational symmetry of space that is typically (implicitly) assumed in standard thermodynamics. As a consequence of this symmetry–breaking, there does not exist an Euler relation (of degree one) relating the intensive and extensive variables. It is then not difficult to show that there is no Gibbs–Duhem formula relating variations of the intensive variables. This alters the usual "Gibbs phase rules". This point was apparently first appreciated, in the particular case when a Schwarzschild black hole and flat–space radiation constitute the system, by Wright (1980). However, the phenomenon is general for self–gravitating systems.

The absence of Gibbs–Duhem relations means that variations of all the intensive variables, as well as variations of the corresponding extensive variables, are linearly independent even when variations are restricted to comparison of equilibrium states. As a result, there is a global non–degenerate symplectic or canonical Hamiltonian structure associated with thermodynamically conjugate pairs in general relativity, in contrast with the usual flat space, flat spacetime, or local formulations of thermodynamics. Furthermore, this thermodynamical conjugacy follows directly from the action integral of general relativity (Brown *et al.* 1990; Brown and York 1992a). The conjugacy one obtains is that associated with the dimensionless Massieu potentials (Callen 1985), except that it is not degenerate. The Massieu

potentials are the usual assortment of "free energies" divided by kT and the entropy divided by Boltzmann's constant k. Massieu potentials correspond to the gravitational action, with suitable boundary terms, divided by Planck's constant $\hbar$.

As a consequence of the features described above, one has canonical conjugacy between the energy E and $\beta = (kT)^{-1}$, between the angular momentum J and $\beta\omega$, where ω is an angular velocity, between the electric charge Q and $\beta\phi$, where ϕ is the electrostatic potential, *etc*. In general relativity, all the traditional constants of motion, or "extensive" variables E, J, Q, ... are given by integrals over the closed spatial two–surface boundary of the system. Likewise, the conjugate "intensive" variables β, $\beta\omega$, $\beta\phi$, ... are also defined by boundary data. The properties (1) non–degenerate symplectic structure, and (2) all thermodynamic variables defined as boundary data, enable a widening and strengthening of Feynman's idea of functional integration as both a technical means of obtaining partition functions and a conceptual unifier of quantum and statistical physics.

In gravity, for a completely isolated system, one can express the density of states $\nu(E, J, Q, \ldots)$ as a Lorentzian functional integral and, for a completely open system, one can express the partition function $Z(\beta, \beta\omega, \beta\phi, \ldots)$ as a "Euclidean" functional integral (Brown and York 1992a). All other partition functions can be obtained systematically from either of these two. (In statistical thermodynamics without gravity, the partition function $Z(\beta, \beta\omega, \beta\phi, \ldots)$ for a completely open system has been stated not to exist (Prigogine 1950).) We believe that the Lorentzian "microcanonical functional integral" may well be a fundamental one through which stronger links joining gravity, statistical physics, "maximum entropy", and the quantum can be forged.

Rotating black hole

Consider a rotating, stationary, axisymmetric black hole with metric $g_{\mu\nu}$ given by

$$ds^2 = -N^2 dt^2 + h_{rr} dr^2 + h_{\theta\theta} d\theta^2 + h_{\varphi\varphi}(d\varphi + \omega\, dt)^2 \ , \tag{1}$$

where the lapse function N, the shift vector $V^i = (0, 0, \omega)$ with $i = r$, θ, φ, and the

spatial metric h_{ij} of the slices of constant time depend only on r and θ. For later convenience, we work in a spatial coordinate system rotating with the horizon's constant angular velocity $\omega_H^\star$ with respect to the distant stars ("$\star$"); thus $\omega = \omega_H^\star - \omega_z^\star(r,\theta)$, where $\omega_z^\star$ is the non–uniform angular velocity of the observers at rest in the "preferred" slices of constant time t. Because the angular Killing vector $\partial/\partial\varphi$ lies in these slices, these "Eulerian" observers (hydrodynamical terminology) have zero angular momentum per unit mass; hence, they are frequently called ZAMOs ("Z"). This property follows from observing that their unit timelike four–velocity field is $u^\mu = N^{-1}(1, -V^i)$ and that $g(u, \partial/\partial\varphi) \equiv u_\varphi = 0$.

The physical significance of the rotating spatial coordinates is that they are co–moving with the elements of the "radiation fluid" environment that is required to equilibrate the black hole. That this environment must rotate rigidly *with respect to the distant stars* at the rate $\omega_H^\star$ can be shown using local thermodynamic arguments (Brown and York 1992b) based on extremal entropy in the manner of Israel and Stewart (1980), or similarly to equations (4)–(6) (only!) in Zannias and Israel (1981). A quantum field–theoretic argument establishing this same rigid rotation was given by Frolov and Thorne (1989). Because the environment cannot rotate with finite angular velocity for a too–large radius, the system's boundary B must lie inside the speed–of–light surface. Again, we see the necessity of using a finite system.

The world lines of the elements of the environment are parallel to the timelike "co–rotating" Killing vector $\partial/\partial t$ (Carter 1979). The unit four–velocity $\bar{u}^\mu$ of the radiation elements is given by $(\partial/\partial t)^\mu = \bar{N}\bar{u}^\mu$ where $\bar{N}^2 = -g_{tt}$. Local observers with four velocity field $\bar{u}^\mu$ are called "Lagrangian" observers because they are comoving with the fluid environment. They see a locally isotropic thermal distribution of quanta (Frolov and Thorne 1989) and *seem* to be the natural observers to describe the equilibrium of a system containing a black hole, in imitation of the usual local formulation of thermodynamic laws in general relativity. But because $\bar{u}_{[\mu}\nabla_\nu\bar{u}_{\sigma]} \neq 0$, there is no way for them globally to synchronize their clocks ("barber pole effect") and therefore there is no global time–slicing with respect to which they are at rest. Thus, while Lagrangian observers may seem "ideal" for local physics

and the "recognition" of equilibrium thermal properties, they are not actually suitable for defining global properties of the system, such as black–hole entropy and its changes. Consequently, the local temperature $\bar{\beta}^{-1} = \bar{T}$ they measure (Frolov and Thorne 1989, Eq. (4.3)) is not a suitable thermodynamic temperature for the system. (This implies that Zaslavskii's (1991) Lagrangian treatment of the rotating black hole is, in our view, unsatisfactory.)

Using the thermodynamic arguments that establish "rigid rotation", one also finds that $\bar{\beta}\bar{N}^{-1}$ must be a constant; this is the Tolman (1930) result for the red–shifting of Lagrangian temperature. For a system with a black hole, the constant is $2\pi\kappa^{-1}$, where κ is the surface gravity of the black hole in absolute units where $G = \hbar = c = k = 1$. What is the globally correct Eulerian or ZAMO temperature? Following Israel and Stewart (1980), define the inverse temperature four–vector $\bar{\beta}^{\mu} = \bar{\beta}\bar{u}^{\mu}$, which is easily shown to satisfy Killing's equation. Define the spacelike velocity of the environment's elements with respect to the ZAMOs in ZAMO proper time: $\hat{V}^{\mu} = (0, \hat{V}^{i})$ with $\hat{V}^{i} = N^{-1}V^{i}$ given by the quotient of shift vector by lapse function. Note that $\hat{V}^{\mu}u_{\mu} = 0$ and $u^{\mu} + \hat{V}^{\mu} = N^{-1}(1,0)$. Then (see, for example, Smarr and York (1978)) we have

$$\bar{u}^{\mu} = \gamma(u^{\mu} + \hat{V}^{\mu}) \ , \tag{2}$$

$$\bar{\beta}^{\mu} = \bar{\beta}\bar{u}^{\mu} = (\bar{\beta}\gamma)u^{\mu} + (\bar{\beta}\gamma)\hat{V}^{\mu} \equiv \beta u^{\mu} + \beta\hat{V}^{\mu} \ , \tag{3}$$

where γ is the local kinematical special–relativistic factor $\left(1 - h_{ij}\hat{V}^{i}\hat{V}^{j}\right)^{-1/2} = -g(u, \bar{u})$. From (3) one can identify the ZAMO–measured local proper inverse temperature as $\beta = \bar{\beta}\gamma$. This result is obtained by considering the Doppler effect for zero angular momentum (transverse) photons. The second term in (3) corresponds to the presence of a purely kinematically induced dipole "blue/red" longitudinal Doppler shift that a ZAMO would "see" as the radiation environment flows past (kinematically analogous to a determination from Earth of the $2.7K$ "isotropic" cosmic microwave background). From the definitions of $\bar{N}$, N, and V^{i}, it is easily shown that $N = \gamma\bar{N}$, from which we establish that the red–shift law for ZAMO–measured temperature is

$$\beta N^{-1} = (\gamma\bar{\beta})(\gamma\bar{N})^{-1} = \bar{\beta}\bar{N}^{-1} = 2\pi\kappa^{-1} \ . \tag{4}$$

The result (4) has a geometrical interpretation, as well as the direct physical one that at fixed r and θ, a ZAMO detects a lower temperature than a Lagrangian ("moving") observer ($\gamma > 1$), except at the poles ($\theta = 0, \pi$).

The geometrical interpretation is described in real Lorentzian–signature spacetime as an analog of the aging–of–identical–twins effect that is well known in special relativity. Fix r and θ (not at the north or south poles) and consider two identical twins: call them "Euler" and "Lagrange". Euler stays home while Lagrange follows a circular orbit at constant angular velocity $\omega_H^\star$ and returns home after the passage of P (physical period of trip) units of "universal" time t. The proper time duration of Euler's world line is, restoring ordinary units,

$$\beta' = \int_0^P N\,dt = NP = \int_0^P \gamma\bar{N}\,dt = \gamma\bar{N}P = \gamma\bar{\beta}' \ , \tag{5}$$

where $\bar{\beta}'$ is the total proper time for Lagrange's trip. Note that $\beta' > \bar{\beta}'$: the stay–at–home ZAMO ("inertial") twin Euler has aged more than Lagrange. The time β' (in seconds) is converted formally to an inverse temperature β (in K^{-1}) according to $\beta \leftrightarrow \beta' k\hbar^{-1}$. Thus from (5) we recognize the anology between the ZAMO and Lagrangian temperatures and the Lorentzian "twin effect".

The geometrical interpretation also can be associated directly with the complexification $N \to -iN$, $V^i \to -iV^i$, of the metric (1) whose corresponding action is used to obtain the stationary phase approximation to the partition function (Brown *et al.* 1991). (In another context, Mellor and Moss (1989) have also considered particular complex stationary black hole metrics.) In the complex geometry, in order to obtain an everywhere regular geometry, the slices defined by the scalar t are periodically identified with a period P given by $2\pi c\kappa^{-1}$ (κ contains the gravity constant G) yielding a topology $S^2 \times R^2$ for the complexified spacetime (Gibbons and Perry 1978; Gibbons and Hawking 1977). Multiplying (5) by $k\hbar^{-1}$, we obtain the temperature relation in terms of the proper lengths (in K^{-1}) of the Lagrangian and Eulerian curves in the complex geometry. (The topology of the three–boundary ${}^3B$ of the complex spacetime is $S^2 \times S^1$, where S^2 is the topology of the axisymmetric two–boundary B. A Lagrangian curve in ${}^3B$ can be closed with the identification of

points (t, r, θ, φ) and $(t+2\pi\kappa^{-1}, r, \theta, \varphi)$, and an Eulerian curve can be closed with the identification of (t, r, θ, φ) and $(t+2\pi\kappa^{-1}, r, \theta, \varphi - 2\pi\kappa^{-1}\omega)$. The difference is geometrically irrelevant because it amounts to a rotation of B along $\partial/\partial\varphi$, which is an isometry of B.)

Finally, let us point out a further aspect of characterizing the system by the ZAMO–measured temperature $T = \beta^{-1}$ at the boundary B. In the statement of the thermodynamic law of equilibrium change

$$dS = \beta\, dE - \beta\omega\, dJ + \cdots \tag{6}$$

for the system, the temperature will not be correctly identified without a corresponding correct identification of ω. Therefore, let us consider a process whereby energy is injected into the system from outside and through B such that the angular momentum J is unaffected. For this to occur, the φ–component of the four–momentum per unit mass injected, w_μ, must vanish. Clearly, this means that

$$(d\varphi/dt)\big|_B = (w^\varphi/w^t)\big|_B = -(g_{t\varphi}/g_{\varphi\varphi})\big|_B = -\omega\big|_B\ . \tag{7}$$

Therefore, the injection must be described by a four–velocity "aimed at the rotation axis" by the ZAMOs. Thus, the ZAMO–measured variables are singled out.

That the angular velocity $\omega_H^\star$ of the system is spatially uniform while the "chemical potential" ω depends on the location of B deserves comment. The former is necessary for maximum entropy of the system subject to conservation of total energy and angular momentum, while the latter refers to injection of matter from *outside* the system that conserves its total angular momentum but not its total energy E.

In Brown *et al.* (1991a, 1991b), it was shown that the complexified action of general relativity singles out the proper ZAMO–measured boundary values β and $\hat{\omega} = N^{-1}\omega$ at B, as well as their respective conjugates which are also boundary data at B. One also obtains directly the Bekenstein–Hawking value $(A_H/4)(kG^{-1}\hbar^{-1})$ for the black–hole entropy, where A_H is the area of the event horizon. The harmonious relationships among the thermodynamics implied by extrema of the "classical"

action, the quantum principle, and applications of the maximum entropy principle are striking. Coupled with the fact that general relativity enables one to define all the microcanonical and canonical ensembles in terms of "black box" problems specified completely by boundary data and appropriate "actions", our results suggest "general relativity as thermodynamics" and may imply that general relativity can be derived from some presumably deeper underlying framework such as string theory, information theory, or both.

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Discussion

J.A. Wheeler: Your suggestion that general relativity may be derived someday from entropy or information–theoretic considerations is truly inspiring. What do you foresee as the next step along the road?

B&Y: Unlike what our questioner appears to be able to do, *we* do not feel able to "leap tall buildings in a single bound". We hope next to turn our attention to non–equilibrium processes (dynamics) to see if the idea of "spacetime geometry as statistical thermodynamics" seems still to be valid.

C. Teitelboim: There are "lower dimensional black holes" that come out of string theory. Have you looked at those from your point of view?

B&Y: We have considered the possibility, but those two–dimensional black holes do not arise as extrema of some classical action in the usual way. It is therefore difficult to see how our methods, in which the classical action functional plays a central role, might be applied.

J.B. Hartle: Do your results extend to the Weyl, static, axisymmetric, distorted black holes?

B&Y: Yes. [See Geroch, R. and J.B. Hartle (1982) Distorted black holes. *Journal of Mathematical Physics*, **23**, 680–692.] We also expect to be able to treat stationary, axisymmetric, distorted black holes. We have not used any explicit properties of the Kerr metric in this discussion. Similarly, we have not used the Kerr–Newman fields explicitly in our referenced recent work, which includes the Maxwell field.

R.W. Griffiths: Suppose there is a big star which blows up to form a black hole. Before the explosion I know how to find the total entropy in the part of the universe near this star—adding it up cubic meter by cubic meter. After the explosion I can calculate the entropy outside the black hole (or far away from it) as before. If I then add in the entropy you calculate, has the entropy of this part of the universe gone up?

B&Y: Yes. Bekenstein's "generalized second law", that the entropy of all the matter outside black holes, plus the entropy of black holes, is non–decreasing, appears to be valid when Hawking's results on black–hole radiance are taken into account.