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Cosmological Limits on Deviations from Square Potentials in Oscillating Scalar Cold Dark Matter Models

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Abstract

We consider the consequences for oscillating scalar cold dark matter models of a scalar field ϕ which is oscillating in a potential which deviates slightly from a ϕ^2 potential. We consider a correction to the ϕ^2 potential of the form $K\phi^2 \ln\phi^2 + \lambda_\phi \phi^4$. Upper bounds on $|K|$ and $|\lambda_\phi|$ are obtained as a function of the ϕ mass by requiring that the effects of the pressure introduced by the correction to the potential do not disrupt the conventional cold dark matter scenario for structure formation. The results are applied to some cosmological models of recent interest. In particular, it is shown that a scalar identified with the pseudo-Goldstone boson of a U(1) symmetry explicitly broken by neutrino masses (suggested in connection with late-time phase transitions) is inconsistent with the MSW solution to the solar neutrino problem if it is to account for dark matter. In addition it is shown that the possibility that the Compton wavelength of a very light scalar particle can account for the large-scale structure of the Universe in a cold dark matter model is also ruled out by the effects of pressure on structure formation.

1.Introduction

In this paper we consider how the cold dark matter (CDM) model based on a coherently oscillating scalar field is modified by oscillations which take place in a potential which deviates slightly from a ϕ^2 potential. It is well-known that a coherently oscillating scalar field, corresponding to a Bose condensate of zero momentum scalars in the particle interpretation of the wave function, can account for dark matter in the Universe, with the most famous example being the cold dark matter model based on axions[1]. A coherently oscillating scalar field will behave from the point of view of structure formation as a pressureless density of non-relativistic matter provided that the scalar field ϕ is oscillating in a potential that is precisely ϕ^2. As shown in ref.2, an oscillating scalar field which oscillates around $\phi = 0$ with a frequency large compared with the expansion rate of the universe will have an energy density and pressure which depends on the exponent of the scalar potential. For example, a field oscillating in a ϕ^4 potential behaves like relativistic matter with $\rho = 3\mathrm{p}$, where p is the pressure. Thus if the potential is different from ϕ^2 then the oscillating scalar field can behave quite differently from conventional CDM. In this paper we consider a correction to the ϕ^2 potential of the form $\mathrm{K}\phi^2 \ln\phi^2 + \lambda_\phi \phi^4$. This represents the most probable form for the corrections to the ϕ^2 potential which can give rise to a pressure for the dark matter (corrections odd in ϕ such as ϕ^3 corrections do not contribute to a pressure). The log term can arise in radiative corrections, and more generally represents the simplest correction to the ϕ^2 potential which cannot be approximated by a polynomial correction, while the ϕ^4 term will be the leading correction in the expansion of a potential around $\phi = 0$. In the following we consider how the evolution of structure in the resulting dark matter model is affected by the pressure from these corrections to the ϕ^2 potential and derive limits on K and λ_ϕ from the requirement that structure formation in the resulting cold dark matter model is not unacceptably disrupted by the effects of pressure. We also consider the consequences of these results for some models of physical interest, usually corresponding to dark matter models based on very light ($\lesssim$ 1eV) scalar particles. We use units with $\hbar = c = k_B = 1$

throughout

2. Evolution of energy density perturbations in dark matter with a pressure.

We first give the equations for the evolution of perturbations in the presence of pressure. We will consider matter with energy density ρ and pressure p, where

$$p = \frac{\theta}{4}\rho \tag{2.1a}$$

$$\rho = a^{-3\left(1+\frac{\theta}{4}\right)}\rho_o \tag{2.1b}$$

where a is the scale factor (equal to 1 at present), ρ_o is the present value of the energy density and $|\theta|$ is a constant small compared to 1. To discuss the evolution of energy density perturbations we consider the equations for perturbations of the Friedmann-Robertson-Walker (FRW) metric and the energy-momentum tensor as given in ref.3,

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} = -8\pi G\left(\delta\rho + 3\delta p\right) \quad ; \quad h = h^{\mu}_{\mu} \tag{2.2}$$

$$\delta\dot{\rho} + 3\frac{\dot{a}}{a}\left(\delta\rho + \delta p\right) = -\left(\rho + p\right)\left(\frac{\dot{h}}{2} + v,^{i}_{i}\right) \tag{2.3}$$

$$\frac{1}{a^3}\frac{d}{dt}\left[a^5 v^i\left(\rho + p\right)\right] = -\delta p,_i \tag{2.4}$$

Here the unperturbed metric $g^{(o)}_{\mu\nu}$ and the energy momentum tensor $T^{\mu}_{(o)\nu}$ are given by (i=1,2,3; μ = 0,1,2,3)

$$ds^2 = -dt^2 + a\left(t\right)^2\delta_{ij}dx^i dx^j \equiv g^{(o)}_{\mu\nu}dx^{\mu}dx^{\nu} \tag{2.5}$$

$$T^{\mu}_{(o)\nu} = \left(\rho + p\right)u^{\mu}_{(o)}u_{(o)\nu} + p g^{\mu}_{\nu} \tag{2.6}$$

and the perturbations in the synchronous gauge ($g_{o\lambda} = 0$) are given by $h_{\mu\nu}$ and δT^{μ}_{ν} where

$$g_{\mu\nu} = g^{(o)}_{\mu\nu} + h_{\mu\nu} \quad ; \quad h_{o\lambda} = 0 \tag{2.7}$$

$$\delta T^{o}_{o} = -\delta\rho \quad ; \; \delta T^{i}_{j} = \delta p g^{i}_{j} \tag{2.8}$$

$$\delta T^{o}_{i} = \left(\rho + p\right)a^2 v_i \quad ; \; v_i = \delta u_i \tag{2.9}$$

Since (2.2) to (2.4) are linear in the small perturbations, we can in general consider a perturbation $\delta\rho_\alpha$, δp_α,... of a single component ρ_α of the total energy density.

The total perturbation will then be a sum over such contributions obtained by solving (2.2) to (2.4) for each seperate component. In the following we will consider perturbations of the dominant cold dark matter component of ρ and drop the subscript α. From (2.2) to (2.4) we can obtain an equation for the evolution of the energy density perturbations $\delta\rho$ in the limit where $|\theta|$ is small compared to 1. We define $\delta\rho_{\underline{k}}$ and $h_{\underline{k}}$ by

$$\frac{\delta\rho}{\rho} = \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{\delta\rho_{\underline{k}}(t)}{\rho} e^{i\underline{k}.\underline{x}} \; ; \; h = \int \frac{d^3\underline{k}}{(2\pi)^3} h_{\underline{k}}(t)\, e^{i\underline{k}.\underline{x}} \tag{2.10}$$

Here $\underline{x}$ is the comoving cooordinate. Defining $\delta_{\underline{k}}(t) = \frac{\delta\rho_{\underline{k}}(t)}{\rho}$, equations (2.2) to (2.4) give

$$\ddot{\delta}_{\underline{k}} + \left(2 - \frac{3\theta}{4}\right) H\dot{\delta}_{\underline{k}} = 4\pi G\rho\,(1+\theta)\,\delta_{\underline{k}} + \frac{\theta}{4}\left(\frac{3H}{2}\dot{h}_{\underline{k}} - \frac{\underline{k}^2}{a^2}\delta_{\underline{k}}\right) \tag{2.11}$$

In obtaining this we have used $\delta p = \frac{\theta}{4}\delta\rho$. Since $|\theta| \ll 1$, we can use the $\theta = 0$ result $h_{\underline{k}} = -2\delta_{\underline{k}}$ in (2.11) to obtain the final form for the equation for the evolution of energy density perturbations to lowest order in θ,

$$\ddot{\delta}_{\underline{k}} + 2H\dot{\delta}_{\underline{k}} = 4\pi G\rho\,(1+\theta)\,\delta_{\underline{k}} - \frac{\theta}{4}\frac{\underline{k}^2}{a^2}\delta_{\underline{k}} \tag{2.12}$$

Since $|\theta|$ is small compared with 1, we can use the $\theta = 0$ result for H and the time-dependence of the scale-factor. (In general, $a \propto t^{\frac{2}{3}\left(1-\frac{\theta}{4}\right)}$ and $H = \frac{2}{3}\left(1 - \frac{\theta}{4}\right) t^{-1}$ for $|\theta| \ll 1$).

The effect of the pressure when $\theta \neq 0$ is to introduce a Jeans length λ_J, corresponding to the value of $|\underline{k}|$ at which the second term on the right of (2.12) becomes dominant[4,5]

$$\lambda_J = \left|\frac{\pi\theta}{4G\rho}\right|^{\frac{1}{2}} \tag{2.13}$$

(The physical wavelength of a perturbation $\delta_{\underline{k}}$ at a given value of a is $\lambda = \frac{2\pi a}{|\underline{k}|}$).

If $\theta > 0$, corresponding to a positive pressure, then the effect of the Jeans term in (2.12) will be to stop the growth of perturbations when they have wavelength (or scale) $\lambda < \lambda_J$. If $\theta < 0$ (corresponding to a negative pressure), then the Jeans term will cause an exponential growth of perturbations which satisfy $\lambda < \lambda_J$.

3. Cold Dark Matter with a pressure: Small corrections to ϕ^2 potentials.

It has been shown that in general the energy density and pressure of a scalar field oscillating coherently in a potential $V(\phi)$ with a frequency large compared to H are given by[2]

$$p = (\gamma - 1)\rho \tag{3.1a}$$

$$\rho \propto a^{-3\gamma} \tag{3.1b}$$

where ρ and p are the energy density and pressure averaged over one oscillation cycle. γ is given by

$$\gamma = \frac{\int 2\left(1 - \frac{V(\phi)}{\rho}\right) dt}{\int dt} \tag{3.2}$$

where the integral is over one oscillation cycle[2]. Note that a contribution to $V(\phi)$ which is odd in ϕ will give zero when integrated over an oscillation cycle. For the case of an exactly ϕ^2 potential, we find $\gamma = 1$ and so the matter described by (3.1) behaves exactly like non-relativistic pressureless matter. This is the usual case (eg. the axion model). If however we consider a potential which deviates slightly from a ϕ^2 potential, for example

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 - K\phi^2 \ln\left(\frac{\phi^2}{\mu^2}\right) + \lambda_\phi\phi^4 \tag{3.3}$$

where the second and third terms are small corrections to the first term and K and μ are constants, then as we will show in the following the CDM model[4,6] can be significantly modified.

(i). Logarithmic correction.

We first consider the case of a log correction corresponding to (3.3) with $\lambda_\phi = 0$. In this case (3.3) can be written as

$$V(\phi) \simeq \frac{1}{2}m_\phi^2\phi^2\left(\frac{\phi^2}{\mu^2}\right)^{\theta/2} \tag{3.4}$$

where $\theta = \frac{4K}{m_\phi^2} \ll 1$. From (3.2) we find that $\gamma = (1 + \theta/4)$ and so $p = \frac{\theta}{4}\rho$ [2]. We can then consider the consequences for structure formation of the pressure arising when $\theta \neq 0$.

a) $\theta > 0$ For the case $\theta > 0$, as long as a perturbation has scale $\lambda > \lambda_J$ it will grow as in a conventional CDM model, with $\delta \propto a$ for a matter dominated Universe. (In the following we drop the subscript $\underline{k}$ on $\delta_{\underline{k}}$, and δ will be assumed to correspond to a particular scale λ. We may take δ to be the value of $\delta\rho/\rho$ averaged over a region of length scale λ, since $\left(\frac{\delta\rho}{\rho}\right)_\lambda$ is proportional to $\delta_{\underline{k}}$ [4]). Once $\lambda < \lambda_J$, the solution of (2.12) will be an oscillating δ and so no growth of the perturbation will occur. Let T_J be the temperature at which a perturbation of size λ_p at present becomes smaller than λ_J. For T less than T_{eq} (where $T_{eq} = 5.5h^2eV$ is the matter-radiation equality temperature when $\Omega = 1$), λ_J can be written as,

$$\lambda_J = \left(\frac{\pi\theta}{4G\rho_o}\right)^{1/2}\left(\frac{T_\gamma}{T}\right)^{3/2} \equiv \lambda_{Jp}\left(\frac{T_\gamma}{T}\right)^{3/2} \tag{3.5}$$

where ρ_o is the present energy density of the Universe, and λ_{Jp} is the present value of the Jeans length. (With $\Omega = 1$ we have $\rho_o = 7.5\text{x}10^{-47}h^2GeV^4$ where $0.4 \lesssim h \lesssim 1.0$ parameterizes the uncertainty in the present value of the Hubble constant. In this case cold dark matter accounts for at least 90 per cent of the matter in the Universe[4,6]). The value of λ_{Jp} is then,

$$\lambda_{Jp} = 6.4\text{x}10^3\frac{\theta^{\frac{1}{2}}}{h}\text{Mpc} \tag{3.6}$$

Thus requiring that the Jeans length of the dark matter at present be less than about 0.1 Mpc in order that galactic halos can form without any suppression of the growth of density perturbations (allowing the model to preserve the successful predictions of the conventional CDM model for halo formation[6]) gives an upper bound on θ, $\theta \lesssim 3\text{x}10^{-10}h^2$. In this case the model will behave exactly like a conventional CDM model on scales greater than 0.1Mpc. This then gives a bound on K as a function of m_ϕ, $K < 8\text{x}10^{-11}h^2m_\phi^2$.

If $\theta \gtrsim 3\text{x}10^{-10}h^2$ then the growth of perturbations on galactic scales will be suppressed relative to the conventional CDM model. The length of a scale at temperature T which has a size λ_p at present is

$$\lambda(T) = \left(\frac{T_\gamma}{T}\right)\lambda_p \tag{3.7}$$

so the temperature below which λ is smaller than λ_J is

$$T_J(\lambda_p) = \left(\frac{\lambda_{Jp}}{\lambda_p}\right)^2 T_\gamma \tag{3.8}$$

T_J will be smaller than the temperature $T_H(\lambda_p)$ at which the scale λ_p enters the horizon if $T_H(\lambda_p) > T_J(\lambda_p)$, where

$$T_H(\lambda_p) = \left(\frac{3}{8\pi G\rho_o}\right)\frac{T_\gamma}{\lambda_p^2} \tag{3.9}$$

$T_J < T_H$ is true if $\theta < 3/2\pi^2$, which will be satisfied for the cases of interest to us here. The total growth of a perturbation up to the present is determined by T_J. Scales with $\lambda_p > \lambda_{Jp}$ experience full growth as in the conventional CDM model. For scales at present $\lambda_p < \lambda_{Jp}$, a suppression of their growth by a factor $\frac{T_\gamma}{T_J} \equiv \left(\frac{\lambda_p}{\lambda_{Jp}}\right)^2$ compared with the conventional CDM model will occur. In light of recent bounds on the magnitude of perturbations of the cosmic microwave background radiation (CMBR) temperature from COBE[7], which are only marginally consistent with the growth of the primordial energy density perturbations which account for galaxy and structure formation, we expect that any suppression of the growth of the energy density perturbations will require larger initial density perturbations at matter-radiation equality, resulting in unacceptably large perturbations of the CMBR temperature. We therefore conclude that we must have $\theta \lesssim 3\text{x}10^{-10}h^2$ when $\theta > 0$.

b) $\theta < 0$ In this case there will be a negative pressure for the dark matter. As discussed in the appendix, the effect of the negative pressure is to cause an exponential growth of the perturbations

$$\delta = \delta_{eq}\left(\frac{T_{eq}}{T_*}\right)\exp\left(\frac{3}{2}\left[\left(\frac{T_*}{T}\right)^{1/2} - 1\right]\right) \tag{3.10}$$

where δ_{eq} is the size of the perturbation at matter-radiation equality, $T_* = \frac{2}{3}T_J$ and it is assumed that the perturbation enters the horizon at $T > T_{eq}$, as is the case for scales less than 1Mpc. Since T_* is proportional to λ_p^{-2}, we see that the exponential growth of the perturbations will result in small scales going non-linear at an early stage in the evolution of structure. We can obtain bounds on

$|\theta|$ by requiring that the exponential growth of the perturbations does not lead to cosmologically unacceptable consequences. If we require that λ_{Jp} is less than about 0.1Mpc, in order that the exponential growth of the perturbations does not affect the growth of galaxy sized perturbations and the predictions of the conventional CDM model, then we obtain the same bound as in the $\theta > 0$ case, $|\theta| \lesssim 3\text{x}10^{-10}\text{h}^2$. A stronger bound on $|\theta|$ may be obtained by requiring that the largest mass scale going non-linear due to exponential growth of the perturbations is small enough in order not to cause the early formation of stars or black holes and so produce significant explosions during the evolution of structure (explosive structure formation being disfavoured by COBE data on the blackbody spectrum of the microwave background radiation[7,8]). We define a scale λ_{ex} to correspond to the largest scale at present for which significant exponential growth occurs. We take this to correspond to a value of T_* which satisfies

$$\exp\left(\frac{3}{2}\left[\left(\frac{T_*}{T_\gamma}\right)^{1/2} - 1\right]\right) \simeq 10 \tag{3.11}$$

This is satisfied by $T_*/T_\gamma \simeq 4.8$. This value of T_* corresponds to a scale at present

$$\lambda_{ex} = 0.37\lambda_{Jp} \tag{3.12}$$

Thus scales at present greater than about $\frac{1}{3}\lambda_{Jp}$ will have experienced little exponential growth. If we require that the mass scale corresponding to λ_{ex} is less than about 10^2 $M_\odot$ in order to avoid explosions (where the mass corresponding to a perturbation λ_p is $1.2\text{x}10^{12}\text{h}^2\left(\frac{\lambda_p}{1\text{Mpc}}\right)^3 M_\odot$), then we obtain the bound on θ

$$|\theta| \lesssim 3\text{x}10^{-14}\text{h}^{2/3} \tag{3.13}$$

This then gives a bound on K as a function of m_ϕ, $|K| < 8\text{x}10^{-15}\text{h}^{2/3}m_\phi^2$. Thus for $|\theta| \gtrsim 10^{-10}$ there will be unacceptable cosmological consequences of the negative pressure occuring when $\theta < 0$, while if $|\theta| \lesssim 10^{-14}$, the effect of the negative pressure will be cosmologically negligible.

These bounds on θ when θ is a constant also provide a simple test to see whether a general correction to the ϕ^2 potential can significantly affect structure

formation as a result of the pressure introduced. We simply define an effective θ by $\theta_{\rm eff} = \frac{\phi}{V}\frac{dV}{d\phi} - 2$. Define $\theta_{\rm max}$ to be the value of $\theta_{\rm eff}$ with the largest magnitude between $T_{\rm eq}$ and T_γ. If $|\theta_{\rm max}|$ is less than the upper bounds given for the case of constant $|\theta|$ then we can be sure that there will be no cosmologically significant effects due to the pressure introduced by the correction.

(ii). ϕ^4 correction.

We next consider the case of a ϕ^4 correction, corresponding to (3.3) with K=0.

a) $\lambda_\phi > 0$ In this case the pressure is given by (3.1a) with [2]

$$\gamma = 1 + \frac{3}{2}\frac{\lambda_\phi \rho}{m_\phi^4} \tag{3.14}$$

where the energy density is $\rho = \frac{1}{2} m_\phi^2 \, \phi_{\rm max}^2$ and $\phi_{\rm max}$ is the maximum amplitude during the oscillations at a particular time. We can then define an effective θ, $\theta_{\rm eff} = 4(\gamma - 1)$, which gives

$$\theta_{\rm eff} = \frac{6\lambda_\phi \rho}{m_\phi^4} \tag{3.15}$$

For $\Omega = 1$ dark matter ρ is given by

$$\rho = \left(\frac{T}{T_\gamma}\right)^3 \rho_o \tag{3.16}$$

and so

$$\theta_{\rm eff} = k_1 \left(\frac{T}{T_\gamma}\right)^3 \quad ; \quad k_1 = \frac{6\lambda_\phi \rho_o}{m_\phi^4} \tag{3.17}$$

In this case θ is a function of t, with $\frac{\dot\theta}{\theta} \approx H$. Although (2.12) was derived assuming that θ is a constant, we expect it to also hold for a time-dependent θ. This is because physically the Jeans length at a given time (which is given by the right hand side of (2.12)) is determined purely by the pressure and density of the the Universe at this time. The effect of the time dependence of θ and so of the pressure will then be given by replacing θ by $\theta(t)$ in (2.12). The Jeans length is then given by

$$\lambda_J = k_1^{1/2} \left(\frac{\pi}{4G\rho_o}\right)^{1/2} \tag{3.18}$$

So in this case the Jeans length is a constant with respect to temperature, and at present $\lambda_{Jp} \equiv \lambda_J$ is given by

$$\lambda_{Jp} = 6.4\text{x}10^3 \frac{k_1^{1/2}}{h} \text{Mpc} \tag{3.19}$$

In the case of the log correction the ratio of the size of a scale as a function of T to the Jeans length increases with temperature $\lambda(T)/\lambda_J \propto T^{1/2}$. Therefore if a scale is larger than the Jeans length at present it will be larger at all previous times. In the case of the ϕ^4 correction we find $\lambda(T)/\lambda_J \propto T^{-1}$. In this case the temperature T_J at which $\lambda(T) = \lambda_J$

$$T_J(\lambda_p) = \left(\frac{\lambda_p}{\lambda_{Jp}}\right) T_\gamma \tag{3.20}$$

corresponds to the temperature above which a perturbation is smaller than the Jeans length, unlike the case of the log correction where it corresponded to the temperature below which the perturbation is smaller than the Jeans length. Therefore even if a length scale at present is larger than the Jeans length, at an earlier time it may have been smaller than the Jeans length. Thus in order to ensure that there is no suppression of the growth of perturbations on the scale of galactic halos 0.1 Mpc, we require that the corresponding length scale is larger than the Jeans length at the time of matter-radiation equality T_{eq}, when the energy density perturbations begin to grow. This requires that

$$0.1\text{Mpc} > \frac{T_{eq}}{T_\gamma} \lambda_{Jp} \tag{3.21}$$

With $T_{eq} = 5.5h^2\text{eV}$, we find that this gives an upper bound on the coupling λ_ϕ as a function of m_ϕ

$$\lambda_\phi < 1\text{x}10^{-9}\, h^{-4} \left(\frac{m_\phi}{1\text{eV}}\right)^4 \tag{3.22}$$

.

<u>b) $\lambda_\phi < 0$</u> In this case there will be a negative pressure for the dark matter. As discussed in the appendix, the effect of the negative pressure is to cause an exponential growth of the perturbations

$$\delta = \delta_{eq} \exp\left(\sqrt{\frac{3}{2}}\left[\frac{T_{eq}}{T_J} - \frac{T}{T_J}\right]\right) \tag{3.23}$$

where it is assumed that the perturbation is smaller than the Jeans length at T_{eq}. Therefore if we demand that a perturbation doesn't grow by more than a factor 10 before becoming smaller than the Jeans length at T_J then we require that $T_{eq}/T_J \leq 2.7$, which gives a lower bound on the value of λ_p which scales at present must exceed in order to avoid large exponential growth

$$\lambda_p > \frac{T_{eq}}{2.7\ T_\gamma}\lambda_{Jp} \tag{3.24}$$

From this we obtain an upper bound on λ_{Jp} by requiring that (3.24) is satisfied for λ_p corresponding to mass scales $\leq 10^2 M_\odot$

$$\lambda_{Jp} \leq \frac{4.6\text{x}10^{-8}}{h^{8/3}}\text{Mpc} \tag{3.25}$$

Thus since the Jeans length when λ_ϕ is negative given by (3.18) with λ_ϕ replaced by $|\lambda_\phi|$, we obtain from (3.25) an upper bound on $|\lambda_\phi|$ as a function of m_ϕ,

$$|\lambda_\phi| < \frac{1.2\text{x}10^{-13}}{h^{16/3}}\left(\frac{m_\phi}{1\text{eV}}\right)^4 \tag{3.26}$$

which is tighter than the bound coming from simply requiring that the evolution of perturbations on scales greater than 0.1Mpc is unaffected by the exponential growth of perturbations, which gives the upper bound (3.22) but with λ_ϕ replaced by $|\lambda_\phi|$.

4. Applications

In this section we consider some implications of these results for models of interest to cosmology, which usually means models which have extremely light scalar bosons of mass $\lesssim$ 1eV as dark matter candidates. For the case of the ϕ^4 potential results, which will be of most interest in realistic models, we see that if in general we wish an oscillating scalar to account for dark matter without having to tune λ_ϕ to be small (for example if we take λ_ϕ to be of order 0.1) then from (3.22) we obtain a lower bound on m_ϕ

$$m_\phi \gtrsim 1.0\text{x}10^2\text{h eV} \tag{4.1}$$

(For negative λ_ϕ we obtain from (3.26) $m_\phi \gtrsim 9.6\text{x}10^2\text{h}^{4/3}$ eV). Smaller m_ϕ will require fine-tuning of λ_ϕ in order that the resulting CDM model is not disrupted by the effects of pressure.

There has recently been some discussion of the possibility that neutrino masses are associated with the explicit breaking of a spontaneously broken global U(1) symmetry, enabling a late-time phase transition[9] to occur. In this case ϕ may be identified with the resulting pseudo-Goldstone boson. The potential for ϕ around a local minimum will have a mass $m_\phi = \frac{\tilde{m}^2}{f}$ and $\lambda_\phi = \frac{\tilde{m}^4}{12f^4}$, where the U(1) breaking scale f is typically associated with a large symmetry breaking scale (for example the grand unification scale) and $\tilde{m}$ is approximately equal to the neutrino mass scale, where all neutrinos have approximately the same masses[9]. (More precisely, $\tilde{m} \approx (m\epsilon)^{1/2}$, where m is a U(1) symmetric contribution to the neutrino mass and ϵ is a contribution which explicitly breaks the U(1) symmetry. Then the larger of m and ϵ will set the scale of the neutrino masses, and $4m\epsilon$ sets the scale of the mass squared splitting of the neutrino masses (see eqs. (5.1)-(5.5) of ref.9). The ϕ potential is then $V(\phi) = \tilde{m}^4\cos(\phi/f))$. From (3.22) we then obtain a lower bound on $\tilde{m}$

$$\tilde{m} \gtrsim 95\text{h eV} \tag{4.2}$$

Thus we see that the possibility that $\tilde{m}$ is associated with neutrino masses or mass squared splittings in the range suggested by the MSW solution to the solar

neutrino problem, $\tilde{m} \approx 10^{-2}$eV [10], is inconsistent with the possibility that the associated pseudo-Goldstone boson could account for cold dark matter, since the effects of pressure will prevent the growth of perturbations. In addition, since $\tilde{m}$ is less than or about equal to the neutrino masses, we see that the neutrino masses when (4.2) is satisfied may violate the cosmological bound on stable neutrino masses $m_\nu \lesssim 100h^2$ eV. We also note that in the case of the axion model, in which $\tilde{m} \approx \Lambda_{QCD}$, the bound (4.2) is easily satisfied. Therefore there are no significant pressure effects in the axion cold dark matter model.

Finally we comment on the possibility suggested in ref.11 that the Compton wavelength m_ϕ^{-1} of extremely light scalar boson dark matter particles could be large enough to affect structure formation on scales of tens of Megaparsecs. For a Compton wavelength of greater than 30 kpc we require $m_\phi < 2\text{x}10^{-28}$eV. (The scale 30 kpc is the scale at the time of matter domination which corresponds to a scale in the present Universe 30 Mpc, which is the present scale of large scale structure. This will smooth out the initial perturbations on scales corresponding to scales smaller than 30 Mpc today). If the effects of pressure are to be negligible in this case (so allowing the scalars to behave as cold dark matter particles on scales larger than 30 kpc at present) we must require from (3.22) that $\lambda_\phi < 10^{-119}$. The form of potential considered in ref.11 corresponds to $m_\phi^2 = 8V_o/\phi_o^2$ and $\lambda_\phi = V_o/\phi_o^4$, with $V_o \approx 10^{-4}\text{eV}^4$ (in order that the energy density at present accounts for dark matter) and $\phi_o \approx 10^{17}$GeV, a value typical of grand unification and which also gives a ϕ mass $3\text{x}10^{-28}$eV, which is of the correct magnitude for the Compton wavelength to be significant for large-scale structure. This then gives $\lambda_\phi \approx 10^{-108}$. Thus we see that with couplings which give a Compton wavelength significant for cosmology, the effects of the pressure due to the ϕ^4 term will prevent the growth of structure, so ruling out this particular explaination of large scale structure in a cold dark matter model.

5.Conclusions

We have considered the effect on oscillating scalar cold dark matter of a potential which deviates slightly from a ϕ^2 potential. Bounds on the coefficients of a $\phi^2 \ln\phi^2$ correction and a ϕ^4 correction as a function of the ϕ mass were obtained by requiring that the effect of the pressure introduced by the correction to the ϕ^2 potential does not affect the conventional CDM scenario for galaxy formation, and, for the case where a negative pressure is introduced, by requiring that the exponential growth of perturbations does not lead to explosions. For the case of a logarithmic correction the potential can be written as $\phi^{2+\theta}$ with θ constant. It was shown that in order not to disrupt the conventional cold dark matter model the value of θ must satisfy $\theta < 3\text{x}10^{-10}\text{h}^2$ ($\theta > 0$) and $|\theta| < 3\text{x}10^{-14}\text{h}^{2/3}$ ($\theta < 0$). These bounds on θ also allow one to easily check whether a given general correction to a ϕ^2 potential can result in a significant deviation from the conventional cold dark matter model due to pressure. For the case of a $\lambda_\phi \phi^4$ correction we obtained upper bounds on $|\lambda_\phi|$ as a function of m_ϕ. In addition we showed that the requirement that the pressure introduced by the correction does not prevent the formation of structure can rule out as cold dark matter candidates pseudo-Goldstone bosons where the scale of explicit U(1) symmetry breaking is set by neutrino masses or mass splittings of order 0.01eV (consistent with the MSW solution to the solar neutrino problem), and can rule out also the possibility that the Compton wavelength of an extremely light scalar particle can account for the large scale structure of the Universe.

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Appendix. Growth of perturbations in the presence of a negative pressure.

(i) Logarithmic correction

In this case we can consider the potential in the form (3.3) with $\theta = \frac{4K}{m_\phi^2}$. The equation for the growth of perturbations when $\theta < 0$ is

$$\ddot{\delta}_{\underline{k}} + 2H\dot{\delta}_{\underline{k}} = 4\pi G\rho\,(1+\theta)\,\delta_{\underline{k}} + \frac{|\theta|}{4}\frac{\underline{k}^2}{a^2}\delta_{\underline{k}} \qquad (A.1)$$

where a=1 at present. For $\lambda < \lambda_J$, with λ_J given by (2.13), (A.1) becomes

$$\ddot{\delta} + 2H\dot{\delta} = \frac{A}{a^2}\delta \qquad (A.2)$$

where we have dropped the subscript on $\delta_{\underline{k}}$ and where $A = \frac{|\theta|\underline{k}^2}{4}$. To solve this approximately we substitute $\delta = e^\eta$ into (A.2) which gives

$$\ddot{\eta} + \frac{4}{3t}\dot{\eta} + \dot{\eta}^2 = \frac{A}{\alpha^2 t^{4/3}} \qquad (A.3)$$

where we have used $H = \frac{2}{3t}$ and $a = \alpha t^{2/3}$ for a matter dominated Universe ($\alpha = (6\pi G\rho_o)^{1/3}$). Suppose initially $\ddot{\eta} + \frac{4}{3t}\dot{\eta} \gg \dot{\eta}^2$. Then the solution of (A.3) with $\dot{\eta}^2 = 0$ on the left hand side is

$$\eta = \frac{3A}{2\alpha^2}t^{2/3} \qquad (A.4)$$

For $\ddot{\eta} + \frac{4}{3t}\dot{\eta} \ll \dot{\eta}^2$ the solution of (A.3) is

$$\eta = \frac{3A^{1/2}}{\alpha}t^{1/3} \qquad (A.5)$$

The condition for the solution (A.4) to hold is

$$t < t_* \equiv \left(\frac{\alpha^2}{A}\right)^{3/2} \qquad (A.6)$$

Thus solution (A.4) will hold at early times. Let the temperature at which $t = t_*$ occurs be T_*. Then from (A.6) T_* is given by

$$\frac{T_*}{T_\gamma} = \frac{\pi\theta}{6G\rho_o\lambda_p^2} \equiv \frac{2}{3}\frac{T_J}{T_\gamma} \qquad (A.7)$$

Thus we find that shortly after a perturbation enters the Jeans length and so begins to grow exponentially due to the negative pressure, $T < T_*$ will occur

and so most of the growth will be described by the solution (A.5). In terms of the temperature we can write (A.4) and (A.5) as

$$\eta = \frac{3}{8}\frac{T_*}{T} \quad (T_J > T > T_*) \tag{A.8}$$

and

$$\eta = \frac{3}{2}\left(\frac{T_*}{T}\right)^{1/2} \quad (T_* > T) \tag{A.9}$$

From (A.8) the total growth of the perturbation between T_J and T_* is $\exp(\frac{3}{8} - \frac{3T_*}{8T_J}) \simeq 1.1$. Thus in the following we will ignore the exponential growth between T_J and T_*. For the cases of interest to us (scales at present smaller than 1Mpc) the perturbation will enter the horizon during radiation domination. In this case the growth of the perturbation is given by

$$\delta = \delta_{eq}\left(\frac{T_{eq}}{T_*}\right)\exp\left(\frac{3}{2}\left[\left(\frac{T_*}{T}\right)^{1/2} - 1\right]\right) \; ; \; T_{eq} > T_* > T \tag{A.10}$$

$$\delta = \delta_{eq}\exp\left(\frac{3}{2}\left[\left(\frac{T_{eq}}{T}\right)^{1/2} - 1\right]\right) \; ; \; T_* > T_{eq} > T \tag{A.11}$$

where δ_{eq} is the value of the perturbation at T_{eq}.

(ii) $\lambda_\phi\phi^4$ correction

In this case a negative pressure will arise when $\lambda_\phi < 0$. The equation for the growth of perturbations is then

$$\ddot{\delta} + 2H\dot{\delta} = \frac{A}{a^5}\delta \tag{A.12}$$

where $A = \frac{k_1 k^2}{4}$. As before we substitute $\delta = e^\eta$ into (A.12) which gives

$$\ddot{\eta} + \frac{4}{3t}\dot{\eta} + \dot{\eta}^2 = \frac{A}{\alpha^5 t^{10/3}} \tag{A.13}$$

We follow the same steps as before. If $\ddot{\eta} + \frac{4}{3t}\dot{\eta} < \dot{\eta}^2$, then the solution is

$$\eta = \frac{-3A^{1/2}}{2\alpha^{5/2}t^{2/3}} \equiv \frac{-3A^{1/2}}{2\alpha^{3/2}}\left(\frac{T}{T_\gamma}\right) \tag{A.14}$$

If $\ddot{\eta} + \frac{4}{3t}\dot{\eta} > \dot{\eta}^2$, then the solution is

$$\eta = \frac{3A}{4\alpha^5 t^{4/3}} \equiv \frac{3A}{4\alpha^3}\left(\frac{T}{T_\gamma}\right)^2 \tag{A.15}$$

(A.15) is valid for $t > t_* \equiv \left(\frac{A}{\alpha^5}\right)^{3/4}$. The corresponding temperature is $T_* = \sqrt{\frac{3}{2}}T_J$. In this case T_J is the temperature below which the perturbation becomes larger than the Jeans length. Thus (A.14) is valid essentially during the whole time that the perturbation is smaller than the Jeans length. For the perturbations of interest (smaller than 1Mpc at present) the perturbations enter the horizon during radiation domination and begin growing at T_{eq}. The growth of a perturbation for $T_J < T_{eq}$ is then given by

$$\delta = \delta_{eq}\exp\left(\sqrt{\frac{3}{2}}\left[\frac{T_{eq}}{T_J} - \frac{T}{T_J}\right]\right) \qquad (A.16)$$

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