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Can a Brans-Dicke Scalar Account for Dark Matter in Extended Inflation Models?

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Abstract

We discuss the possibility that dark matter in the Universe could be due to the oscillation of the Brans-Dicke scalar in extended inflation models. The constraints following from the requirement that the energy density perturbations due to quantum scalar field fluctuations are within observational limits and from the requirement that the energy density in the oscillating Brans-Dicke scalar induced by the expansion of bubbles at the first-order phase transition is smaller than the critical density at present are discussed for the case of a coupling of the Brans-Dicke scalar to the Ricci scalar of the form ϕ^{n+2}/M^n. General approximate solutions of the equations of motion are given in the slow-rolling approximation, and the case of $n = 0$ (minimal extended inflation) and $n = 2$ are discussed in detail. It is shown that the dominant source of energy density fluctuations produced during inflation are due to isocurvature fluctuations of the Brans-Dicke scalar. The requirement that the fluctuations in the microwave background due to the isocurvature fluctuations are not too large requires that the reheat temperature at the end of inflation is less than 3×10^{13}GeV. It is shown that in the case where gravity is neglected in discussing the expansion of bubbles, the model is consistent with the requirements that the range of non-Newtonian corrections to the gravitational potential is less that 1cm and that there should be no inhomogeneities in the cosmic microwave background radiation due to large bubbles for only a very small range of reheat temperatures at the end of inflation. This suggests that the Brans-Dicke scalar is not favoured as a candidate for dark matter if the isocurvature density fluctuations are to account for the density perturbations. If we do not require that the isocurvature fluctuations account for the density perturbations, then the range of reheat temperatures can be larger. The range of non-Newtonian corrections to the gravitational potential in this case is not expected to be much smaller than the present upper limits from Cavendish-type experiments. The possibility that gravity can suppress the growth of large bubbles is considered, leading to a new scenario for the end of extended inflation in which the phase transition is per-

colated by nucleation alone, without the expansion of bubbles. In this case the average radius of the bubbles at the end of extended inflation may be much smaller than in the conventional case where gravity is neglected. As a result, the bubble fluctuation constraint is removed as well as the constraint from large unthermalized bubbles, allowing for a much larger range of reheat temperatures at the end of extended inflation. The possible advantages of an oscillating Brans-Dicke scalar with respect to structure formation are considered, in particular the possibility that the Brans-Dicke scalar could account for both cold and hot dark matter in a combined CDM+HDM scenario for structure formation, with the hot component arising from excitation of the Brans-Dicke scalar field by the expansion of bubbles at the first-order phase transition.

1. Introduction.

There has been much study in recent times of the possibility that inflation could be successfully terminated by a first-order phase transition if the rate of bubble nucleation per horizon volume increases with time[1,2]. In particular, the idea of inflation in the context of a Brans-Dicke theory of gravity[3,4] ('extended inflation') has attracted much attention[1,2]. In such a model, the Hubble parameter decreases as a power of time, rather than being constant as in the case of Einstein gravity. As a result, the rate of bubble nucleation per horizon volume increases with time, allowing inflation to be terminated by percolation of the first-order transition. Although it was originally hoped that extended inflation would allow the construction of inflation models based on a conventional GUT phase transition with none of the fine-tunings associated with new or chaotic inflation[5,6] scenarios, it became apparent that the simplest extended inflation models, in which there was no potential for the Brans-Dicke scalar, were unsuccessful[2,7]. This was because in the absence of a potential, the distribution of the bubble radii is compatible with the homogeneity of the cosmic microwave background radiation (CMBR) only if the Brans-Dicke parameter ω is less than about 25, in conflict with the non-observation of deviations from Einstein gravity in radar-echo delay experiments[4,8] which required that ω be greater than 500. (Recently this conflict has been questioned[9], as we will discuss later). The simplest solution to this problem is to introduce a potential for the Brans-Dicke scalar giving it a mass, so reducing the range of non-Einstein gravitational effects to a range at least as small as the Earth-Sun distance. Indeed, the omission of a potential for the Brans-Dicke scalar could be interpreted as a fine-tuning of the original extended inflation model, in the sense that one is fine-tuning the potential to be zero.

The inclusion of a potential for the Brans-Dicke scalar naturally suggests the interesting possibility that the Brans-Dicke scalar could account for dark matter in the Universe. If the Brans-Dicke scalar at present is oscillating about the minimum of its potential, then its energy density would behave as a density of non-relativistic matter [10], in other words as cold dark matter (CDM). This would

provide a very simple 'minimalist' model that could in principle both allow for a natural 'GUT driven' inflation (avoiding the need for an ad hoc inflaton sector) and also account for dark matter, which is made necessary by the condition $\Omega = 1$ ($\Omega \equiv \rho/\rho_c$, where ρ_c the critical energy density in the Universe at present) which in turn follows from inflation. (In the following we use the term 'GUT' to refer to a generic matter sector which provides a false vacuum energy density driving extended inflation and in which the standard model is embedded. It need not necessarily be a grand unified theory based on a simple gauge group such as SU(5)). In a sense such a model both enables inflation to be realised in a natural way (in the sense of no tunings of the conventional matter sector of the model, in contrast with new or chaotic inflation models[5,6]) and self-consistently offers a solution to the problem of requiring dark matter in order to account for $\Omega = 1$ which is in turn a side-effect of inflation. The model has the added advantage of being essentially specified by just three parameters, namely the mass of the Brans-Dicke scalar, the reheat temperature at the end of extended inflation, and a parameter specifying the coupling of the Brans-Dicke scalar to the Ricci scalar. This allows for a meaningful investigation of the range of parameters for which the model is consistent with constraints following from the observed isotropy of the CMBR, the energy density of the Universe at present, and limits on the range of non-Newtonian gravitational forces. In this paper we wish to discuss critically whether a Brans-Dicke scalar can account for dark matter and be consistent with these observational constraints.

In a previous paper[11] we discussed the Brans-Dicke scalar dark matter model (which we refer to in the following as the extended inflation dark matter or EIDM model) for the case of the simplest coupling of the Brans-Dicke scalar to the Ricci scalar. There it was shown that requiring that the energy density perturbations arising from scalar field fluctuations during inflation are not too large and requiring also that the energy density at present arising from spatial variations of the Brans-Dicke scalar induced by the expansion of bubbles at end of inflation is not too large placed significant limits on the coupling of the Brans-Dicke scalar to the

Ricci scalar and on the mass of the Brans-Dicke scalar. In the present paper we discuss in more detail the limits on the EIDM model parameters. In particular we consider how the limits are affected by considering more general couplings of the Brans-Dicke scalar to the Ricci scalar, of the form $\frac{\phi^{n+2}}{M^n}R$ (n=0 corresponding to the minimal EIDM model discussed in ref.11). In addition, following the suggestion of ref.9 that the growth of bubbles may be prevented if gravity is strong enough during extended inflation, we consider how the inclusion of gravity can alter the nature of the first-order phase transition (in particular the mean radius of the bubbles at percolation) and so alter the constraints on the model.

The paper is organised as follows. In section 2 we review the EIDM model. In section 3 we discuss the constraints on the model for the case of the minimal (n=0) EIDM model. In ref.11 these constraints were in fact underestimated. This will be corrected here. In section 4 the case of extended inflation where the final 60 e-foldings of inflation are dominated by higher order couplings (n=2) to the Ricci scalar is considered. In section 5 we consider the possibility that gravity dominates the dynamics of bubble growth and percolation during the final 60 e-foldings of inflation. In particular we suggest a new scenario for the end of extended inflation in which the phase transition is percolated by the nucleation of bubbles alone, without their expansion. In section 6 we discuss our conclusions. In appendix A we give a slow-rolling approximation solution of the equations of motion for the case of a coupling to the Ricci scalar of the form $\phi^{n+2}R$. In appendix B we review the constraints coming from the non-observation of deviations from Newtonian gravity, and in appendix C we give a discussion of the anthropic principle determination of the value of ϕ at the end of inflation.

2. The Extended Inflation Dark Matter (EIDM) model

The extended inflation model with a general coupling to the Ricci scalar is described by the action[1]

$$S = \int d^4x\sqrt{-g}\left[-f(\phi)R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + 16\pi L_{\rm matter}\right] \tag{2.1}$$

The effective Planck mass is then $M_{\rm Pl\,eff} = f(\phi)^{1/2}$. Define $\Phi = f(\phi)$, and $\omega(\Phi)$ by

$$\omega(\Phi) = \frac{f}{2(f')^2}\ ; \quad f' = \frac{df}{d\phi} \tag{2.2}$$

(During inflation the potential energy of ϕ can be ignored). The equations of motion are then[1,12]

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{1}{(3+2\omega)}\left[8\pi(\rho - 3p) - \omega'\dot{\Phi}^2\right] \tag{2.3}$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3\Phi} - \frac{\dot{\Phi}}{\Phi}H + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 \tag{2.4}$$

where a is the scale factor, $\omega' = \frac{d\omega}{d\Phi}$, and ρ and p are the energy density and pressure of the matter fields. In the following we consider

$$f(\phi) = \frac{\phi^{n+2}}{M^n} \tag{2.5}$$

where M is a parameter with dimensions of mass (for n=0 we consider $f(\phi) = \frac{\phi^2}{8\omega}$, where ω is the usual constant Brans-Dicke parameter[3,4]). We will consider k=0 throughout in the following.

In the original discussion of the EIDM model[11] it was assumed that the physically significant period of inflation (corresponding to the final 60 e-foldings of inflation) occured when the coupling of ϕ to R was of the form (2.5) with n=0. However, it possible that the cosmologically significant period of inflation ($\Delta N_e \lesssim 60$, where ΔN_e is the number of e-foldings before the end of inflation) will occur when the coupling to R is different from the minimal (n=0) case. This coupling may be regarded as the dominant coupling of ϕ to R when $f(\phi)$ is considered as an expansion in powers of $\frac{\phi}{M}$, with M a mass scale associated with an underlying complete theory for which (2.1) is the effective low energy theory.

From (2.2) and (2.5) we find that in general ω is given by

$$\omega = \frac{M^n}{2(n+2)^2 \phi^n} \tag{2.6}$$

It is straightforward to solve (2.2) and (2.3) in the slow-rolling approximation (Appendix A). It is shown in appendix A that the condition for the validity of the slow-rolling approximation is that $\omega \gtrsim 3$ throughout inflation.

The scenario of the EIDM model, as discussed in ref.11, is based on having a potential for the Brans-Dicke scalar such that after inflation the scalar can coherently oscillate about the minimum of its potential, corresponding to a density of cold scalar particles in the Universe at present. The scalar field at the end of inflation has to be close to the minimum of its potential ($\delta\phi/\phi \lesssim 10^{-6}/\omega^{1/2}$), in order that it doesn't contribute too much to the energy density at present. This is a fundamental problem for this scenario, since in general we do not expect the value of ϕ at the end of extended inflation to equal the value corresponding to the minimum of its potential. However, if inflation generates domains of effectively constant ϕ of a size much larger than the size of the observable Universe, then it is plausible that the closeness of the scalar field to the minimum can be determined by anthropic selection, similiar to the scenario of the axion plus inflation model of Linde[13]. (This is discussed in more detail in Appendix C). As we will discuss later, this requires that some apparently fixed parameters of the model can vary on scales much larger than the observable Universe. The model is then constrained by the requirements that the energy density perturbations induced by quantum fluctuations during inflation are smaller than the bound from the homogeneity of the cosmic microwave background radiation (CMBR) (a stricter requirement is that these fluctuations can account for the fluctuations required for galaxy formation and possibly observed by COBE[14]); that the space dependent fluctuations of the Brans-Dicke scalar induced by the growth and percolation of bubbles at the end of extended inflation (bubble fluctuations) do not contribute too much to the energy density of the Universe at present; that the range of non-Newtonian gravitational effects, which is determined by the mass of the Brans-Dicke scalar,

is less than about 1cm, which corresponds to the upper limit from Cavendish-type experiments if $\omega \lesssim 10^5$ [8](this is discussed further in Appendix B); and that there should not be too many large unthermalised bubbles (those larger than the horizon) at the time of decoupling of the microwave background, in order to avoid unacceptable inhomogeneities in the CMBR[2,7].

The broad conclusion of ref. 11 was that it is possible to satisfy all constraints on the model, thus making the EIDM model a viable model for dark matter. However, the calculation of the density perturbations due to quantum fluctuations of the scalar field in ref.11 was incorrect and underestimated their magnitude. Therefore in the next section we will consider again the EIDM model for the minimal extended inflation model with n = 0.

3. Constraints on the minimal n=0 extended inflation model

(a) Cosmological constraints on the parameter space.

In general the cosmological constraints on the EIDM model are due to the following requirements

(i) We require that the perturbations of the energy density due to quantum fluctuations of the scalar fields during inflation are within the bounds set by observations of the cosmic microwave background radiation (CMBR).[6,14]

(ii) We require that the energy density due to fluctuations of the Brans-Dicke scalar induced by bubble expansion and percolation at the first-order phase transition (bubble fluctuations) is less than the critical energy density required to close the Universe[11].

(iii) We require that the mass of the Brans-Dicke scalar is large enough to ensure that the range of non-Newtonian gravitational effects is small enough not to have been observed in solar-system or terrestrial tests of Newtonian gravity[8].

(iv) We require that there be no CMBR inhomogeneities due to large unthermalised bubbles[2,7].

These are the main constraints determining the allowed parameter space. In addition it is necessary that the energy density of the false vacuum should be sufficiently small that the classical equations of motion used in our discussion are valid, which requires that $V_o \lesssim M^4_{\mathrm{pl\,eff}}$, where V_o is the false vacuum energy density during inflation.

Following the general slow-rolling approximation solution of Appendix A, the solutions of the equations of motion in the n=0 case are

$$\phi = \phi_o\,(1 + A(t - t_o)) \tag{3.1}$$

$$H = \frac{A\omega}{(1 + A\,(t - t_o))} \tag{3.2}$$

$$a = (1 + A(t - t_o))^{\omega} \tag{3.3}$$

where

$$A = \left(\frac{64\pi V_o}{3\omega\phi_o^2}\right)^{1/2} \tag{3.4}$$

and we take $a_o = 1$ and $t = t_o$ at the beginning of inflation.

We see that for $A(t - t_o) \gg 1$, which will generally be the case if the number of e-folds of inflation is greater that 60, we have $\phi = \left(\frac{64\pi V_o}{3\omega}\right)^{1/2}(t - t_o)$. Thus the value of ϕ at the end of inflation is purely determined by the duration of inflation. Inflation ends when $\Gamma \approx H^4$, where Γ is the bubble nucleation rate per unit volume per unit time. Therefore from (3.2) we have $(t - t_o) = \omega\Gamma^{-1/4}$. Therefore we see that ϕ at the end of inflation will be fixed in all domains if Γ and V_o are the same in all domains. In order that ϕ can be determined by anthropic selection (appendix C) we must allow for the possibility that V_o or Γ can vary on length scales much larger than the observable Universe, thus allowing ϕ to take different values in different domains at the end of inflation. Since such a possibility is difficult to dismiss a priori, it is best to assume that it is the case and then to confront the resulting model with constraints from observations.

(i) Constraints from energy density perturbations due to quantum fluctuations during inflation.

We first determine the dominant energy density perturbations.

I. Adiabatic energy density perturbations.

These were considered in the original discussions of extended inflation[1,2]. The magnitude of an adiabatic energy density perturbation when the perturbation re-enters the horizon after inflation is given by[2,6]

$$\delta_{adi} \approx \left.\frac{H^2}{\dot{\phi}}\right|_{t_{\Delta N_e}} \tag{3.5}$$

where $\delta = \delta\rho/\rho$ and $t_{\Delta N_e}$ is the time at ΔN_e e-foldings before the end of inflation when the perturbation leaves the horizon. From (3.1)-(3.4) we find

$$H^2(t_{\Delta N_e}) = H_e^2 e^{\frac{2\Delta N_e}{\omega}} \tag{3.6}$$

where t_e is the time at which inflation ends and $H_e \equiv H(t_e)$. Therefore

$$\delta_{adi} = \frac{H_e^2}{\phi_o A} e^{\frac{2}{\omega}\Delta N_e} \tag{3.7}$$

II. Isocurvature Perturbations.

The isocurvature perturbations are due to fluctuations in the scalar field which persist to the present and contribute to the energy density in the oscillating scalar field. The magnitude of the quantum fluctuation of the scalar field when it crosses the horizon at $t_{\Delta N_e}$ is

$$\delta\phi_Q(t_{\Delta N_e}) = \frac{H(t_{\Delta N_e})}{2\pi} \tag{3.8}$$

Since $\delta\phi_Q$ grows like ϕ (3.1) when its wavelength is larger than the horizon we have

$$\delta\phi_Q(t_e) = \frac{H(t_e)}{2\pi} e^{\frac{2\Delta N_e}{\omega}} \tag{3.9}$$

We denote the space-independent displacement of ϕ from the minimum of its potential by $\delta\phi(t)$. This is assumed to account for $\Omega = 1$ dark matter once $\delta\phi(t)$ starts oscillating about the minimum. Then the perturbation of the energy density on scales at present which correspond to those crossing the horizon at $t_{\Delta N_e}$ is given by

$$\delta_{iso} \approx \frac{2\delta\phi_Q(t)}{\delta\phi(t)} \tag{3.10}$$

Here t is the time at which the perturbation re-enters the horizon. (In ref.11 this was incorrectly taken to be $\delta_{iso} = (\delta\phi_Q(t)/\delta\phi(t))^2$, which underestimates δ_{iso}). Since when the fluctuation is outside the horizon $\delta\phi_Q(t)$ and $\delta\phi(t)$ both evolve like $a^{-3/2}$ when $H < m$ and are essentially constant when $H > m$, we see that δ_{iso} can be calculated at $t = t_e$

$$\delta_{iso} \approx \frac{2\delta\phi_Q(t_e)}{\delta\phi(t_e)} \tag{3.11}$$

$\delta\phi(t_e)$ is given by

$$\delta\phi(t_e) = \left(\frac{g(T_1)T_1^3}{g(T_\gamma)T_\gamma^3}\right)^{1/2} \delta\phi(t_\gamma) \tag{3.12}$$

where T_γ is the present photon temperature and g(T) is the number of degrees of freedom in thermal equilibrium at T. T_1 is defined as the temperature at which $\delta\phi$ starts oscillating, corresponding to $H(T_1) \approx m$. For T_1 occuring during radiation

domination ($T_1 > T_{eq} \approx 5.5h^2eV$) we find

$$T_1 = \left(\frac{45}{4\pi^3 g(T_1)}\right)^{1/4} (mM_{Pl})^{1/2} \tag{3.13}$$

Thus

$$\delta_{iso} = 2\left(\frac{g(T_\gamma)}{g(T_1)}\right)^{1/2}\left(\frac{T_\gamma}{T_1}\right)^{3/2}\left(\frac{m^2}{2\rho_c}\right)^{1/2}\frac{H(t_e)}{2\pi}e^{\frac{2\Delta N_e}{\omega}} \tag{3.14}$$

where we have used $\frac{1}{2}m^2\delta\phi(t_\gamma)^2 = \rho_c$.

From (3.14) and (3.7) we find

$$\frac{\delta_{iso}}{\delta_{adi}} = \frac{\alpha_1^{1/2}}{\pi}\frac{(mM_{Pl})^{1/4}\,T_\gamma^{3/2}}{\rho_c^{1/2}}\left(\frac{1}{24\omega}\right)^{1/2} \tag{3.15}$$

where

$$\alpha_1 = \frac{6g(T_\gamma)}{\pi g(T_1)}\left(\frac{4\pi^3 g(T_1)}{45}\right)^{3/8} \tag{3.16}$$

The isocurvature perturbation will be dominant if

$$m > \frac{\pi^4}{\alpha_1^2}\frac{(24\omega)^2\,\rho_c^2}{T_\gamma^6\,M_{Pl}} \tag{3.17}$$

Since $\frac{\rho_c^2}{T_\gamma^6\,M_{Pl}} = 3\text{x}10^{-36}h^4\text{GeV}$ (where $\rho_c = 7.5\text{x}10^{-47}h^2\text{GeV}^4$ and h= 0.5 to 1.0), and since for values of $\omega \lesssim 10^5$ we require that $m \gtrsim 10^{-14}$GeV in order not to conflict with bounds on non-Newtonian gravitational forces (Appendix B), we see that the isocurvature perturbation will be dominant over the adiabatic perturbation, with $\delta_{iso}/\delta_{adi} \gtrsim \left(\frac{10^{-14}}{10^{-36}}\right)^{1/4} \approx 10^5$. Therefore the energy density perturbations in the EIDM model will be larger than in conventional extended inflation models, which have only δ_{adi}. In the following we will take the isocurvature perturbations to be dominant.

III. Bounds on model parameters from requiring that δ_{iso} does not cause unacceptable CMBR inhomogeneities.

In general, requiring that $\delta_{iso} < k$ on re-entering the horizon gives an upper bound on the ϕ mass. Using (3.14) we find

$$m < m_q(T_e, \omega) \tag{3.18}$$

where

$$m_q(T_e,\omega) = \frac{k^4}{f_k^4}\frac{M_{Pl}^7(2\rho_c)^2}{T_\gamma^6\,T_e^8}\left(\frac{M_{Pl}}{R_p T_e T_\gamma}\right)^{8/\omega} \tag{3.19}$$

$$f_k = \frac{1}{\pi}\left(\frac{4\pi^3 g(T_e)}{45}\right)^{1/2}\left(\frac{45}{4\pi^3 g(T_1)}\right)^{3/8}\left(\frac{g(T_\gamma)}{g(T_1)}\right)^{1/2}\left(\frac{4\pi^3 g(T_e)}{45}\right)^{1/\omega}\left(\frac{g(T_\gamma)}{g(T_e)}\right)^{\frac{2}{3\omega}} \tag{3.20}$$

In this we assume that the false vacuum energy density V_o completely thermalises at the end of inflation, with the reheat temperature T_e given by $\frac{\pi^2}{30}g(T_e)T_e^4 = V_o$. R_p is the radius in the Universe at present corresponding to the size of the fluctuation responsible for the perturbation of the CMBR. For R_p we consider the radius in the present Universe which corresponds to a region at the time of decoupling which subtends an angle of about $1'$ of arc at present, the scale of small-angle observations of the microwave background[6]. Thus we consider $R_p = 2\text{x}10^{41}h^{-1}\text{GeV}^{-1}$. (This corresponds to a region about 15 times the size of the horizon at $T_{dec} \approx 0.3\text{eV}$). With $g(T_1) = g(T_e) \approx 100$ (corresponding to the fields of the electroweak model) and $g(T_\gamma) = 2$ we have

$$m_q(\eta,\omega) = 2.9\text{x}10^{-14}\left(5.5\text{x}10^{-27}\frac{h}{\eta}\right)^{\frac{8}{\omega}}\frac{k^4h^4}{\eta^8}\ \text{GeV} \tag{3.21}$$

where we have defined η by $T_e = \eta 10^{16}\text{GeV}$.

IV. Constraints from scalar field fluctuations induced at the first-order phase transition ('bubble fluctuations').

The equation for ϕ in the false vacuum can be written as

$$\phi(t) = \phi(t_B)(1 + A_B(t - t_B)) \tag{3.22}$$

where

$$A_B = \left(\frac{64\pi V_o}{3\omega\phi(t_B)^2}\right)^{1/2} \tag{3.23}$$

and t_B is the time at which a given bubble is nucleated. When a bubble is nucleated in a GUT, the solution for the bubble will be such that the energy density after tunnelling will either be in the wall of the bubble or will be thermalized inside

the bubble in a time-scale short compared with the expansion time H^{-1} (this is true for GUT mass scales small compared with M_{Pl}; see ref.6 p.294). Either way, since $R = 0$ inside a bubble with zero energy density or with a purely radiation energy density, we see that inside the bubble A_B is effectively zero (V_o effectively zero), and so the field ϕ will not evolve once a point in space has crossed inside the bubble. As a result, the value of ϕ at the end of inflation, when all the bubble walls have collided and thermalised, will depend on how large the bubble was at the end of inflation, which is determined by its nucleation time t_B. Since the bubbles will have a distribution of sizes around a mean size which is of order $H^{-1}(T_e)$ [2,11], when inflation ends at T_e there will be space-dependent fluctuations of the value of ϕ with a wavelength $\approx H^{-1}(T_e)$.

If we assume that $\phi(\underline{x}, t)$ is constant once $\underline{x}$ has crossed inside an expanding bubble, then the value of ϕ inside a bubble as a function of r will be

$$\phi(r) = (1 + A_B r)\phi(t_B) \tag{3.24}$$

At the end of inflation, assuming that the initial radius of the bubble is small compared with the horizon at T_e, we have $r_B \approx t_e - t_B$ and so

$$\phi(r_B) \equiv (1 + A_B r_B)\phi(t_B) = \phi(t_e) \tag{3.25}$$

where $\phi(t_e)$ is the value which ϕ has in the false vacuum at t_e in the absence of bubbles. We associate a mean value of ϕ with a bubble of radius r_B

$$< \phi(r) >= \frac{3}{r_B^3}\int_0^{r_B} r^2\phi(r)\,dr \approx (1 - \frac{1}{4}A_B r_B)\phi(t_e) \tag{3.26}$$

Then since most bubbles at percolation have a mean bubble radius $\overline{r_B}$ of the order of $H^{-1}(t_e)$, the fluctuation induced in the otherwise smooth ϕ background by the expansion and percolation of the bubbles will be

$$\delta\phi_B =< \phi(r_B) > - < \phi(\overline{r_B}) >\approx \frac{A_B}{4}(\overline{r_B} - r_B)\phi(t_B) \approx \frac{A_B}{4}f_r H^{-1}(t_e)\phi(t_e) \tag{3.27}$$

where we have defined $r_B - \overline{r_B} = f_r H^{-1}(t_e)$ for the average bubble at the end of percolation. Here $f_r \approx (\bar{r}_B/H^{-1})(\overline{\delta r_B}/\bar{r}_B)$, where $\overline{\delta r_B}$ is the mean deviation of

the bubble radii from the mean radius $\bar{r}_B$. We expect that $\bar{r}_B/H^{-1} > 0.1$ and $\overline{\delta r_B}/\bar{r}_B \sim 1$ are reasonable assumptions, which gives a lower bound for f_r of 0.1. We will therefore consider $0.1 < f_r < 1$. Since for $A_B r_B \ll 1$ we have $\phi(t_B) \approx \phi(t_e)$, we find that $A_B H^{-1}(t_e) = \frac{1}{\omega}$ and so

$$\delta\phi_B \approx \frac{f_r M_{pl}}{\sqrt{2}\omega^{1/2}} \tag{3.28}$$

Thus at T_e we have spatial fluctuations of the otherwise perfectly smooth value of $\phi(t_e)$, imprinted by the expansion and percolation of bubbles at the first order phase transition. The wavelength of these bubble fluctuations is initially of order $f_r H^{-1}(T_e)$, which is much smaller than the size of the observable Universe. (With $T_e = \eta 10^{16}$GeV, a region of radius $H^{-1}(T_e)$ at T_e will correspond to a size in the present Universe $\approx 1/\eta$ meters.)

The evolution of the bubble fluctuations from T_e to the present follows from the equation for ϕ

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - a^{-2}\underline{\nabla}^2\delta\phi = -\left[V'(\delta\phi) + \frac{1}{4\omega}\phi R\right] \tag{3.29}$$

In the limit R=0, V=0, an exact solution of this is

$$\delta\phi = \frac{A_o}{a}\mathrm{Sin}(k\tau)\mathrm{Sin}(\underline{k}.\underline{x}) \quad ; \quad k^2 = |\underline{k}|^2 \tag{3.30}$$

where $d\tau^2 = dt^2/a(t)^2$ The solution is a good approximation if $R \neq 0$ so long as $\frac{|\underline{k}|}{a} \gg H$. This is true in the case of interest, since for radiation or matter domination H decreases faster than $\frac{|\underline{k}|}{a}$ and initially $\frac{|\underline{k}|}{a} \approx H_e$. Thus the amplitude of the space-dependent perturbation $\delta\phi(\underline{x}, t)$ will drop as a^{-1} until the potential term becomes significant. This will occur when $\frac{|\underline{k}|}{a} \approx m$. After this time the spatial gradient term in (3.29) becomes negligible in the evolution of A(t), and the solution is then

$$\delta\phi = A(t)\mathrm{Sin}(mt)\mathrm{Sin}(\underline{k}.\underline{x}) \quad ; \quad A(t) \propto \frac{1}{a^{3/2}} \tag{3.31}$$

Since at the temperature at which $\frac{|\underline{k}|}{a} \approx m$ is satisfied (which we denote by T_1^*) the condition $m \gg H$ is also satisfied, the scalar field will oscillate according to (3.31)

as soon as $T < T_1^*$. Thus the bubble fluctuations have an amplitude which evolves as a^{-1} from T_e to T_1^*, and as $a^{-3/2}$ from T_1^* to T_γ. T_1^* is effectively the temperature at which the Brans-Dicke scalars corresponding to the bubble fluctuations become non-relativistic, the Brans-Dicke field having been effectively heated by bubble expansion at the first-order phase transition. The energy density at present due to spatially averaging over the bubble fluctuations is then $\approx m^2 A(T_\gamma)^2$. Requiring that this be less than the critical energy density of the Universe at present then gives a constraint on the initial amplitude $\delta\phi(\underline{x}, t_e)$ of the bubble fluctuation of ϕ

$$\delta\phi(\underline{x}, t_e) < \frac{g_f^* T_e T_1^{*\,1/2}}{T_\gamma^{3/2}} \left(\frac{\rho_c}{m^2}\right)^{1/2} \tag{3.32}$$

where

$$g_f^* = \left(\frac{g(T_e)}{g(T_1^*)}\right)^{1/3} \left(\frac{g(T_1^*)}{g(T_\gamma)}\right)^{1/2} \tag{3.33}$$

and

$$T_1^* = \left(\frac{45}{4\pi^3 g(T_e)^{1/3}\, g(T_1^*)^{2/3}}\right)^{1/2} \frac{f_r\, m M_{Pl}}{T_e} \tag{3.34}$$

where we take the initial wavelength to be $f_r H^{-1}(t_e)$. From (3.32) we obtain an upper bound on the mass of the Brans-Dicke scalar

$$m < m_b(\eta, \omega) \tag{3.35}$$

where

$$m_b(\eta, \omega) = \frac{g_f^{*\,2} \alpha^*\, T_e\, \rho_c\, \omega}{f_r\, T_\gamma^3\, M_{Pl}} \tag{3.36}$$

and $\alpha^* = \left(\frac{45}{4\pi^3 g(T_e)^{1/3} g(T_1^*)^{2/3}}\right)^{1/2}$. Numerically (3.36) gives

$$m_b(\eta, \omega) = 1.4\text{x}10^{-11}\, \frac{\eta h^2 \omega}{f_r}\ \text{GeV} \tag{3.37}$$

V. Numerical bounds on the n=0 model from quantum and bubble fluctuation constraints.

As discussed in appendix B, for all values of ω less than 10^5, which is the range we will consider here, the non-observation of non-Newtonian forces in Cavendish-type experiments imposes a lower bound on the ϕ mass, $m > m_* = 10^{-14}$GeV. As discussed above, we also have upper bounds on m from the quantum and bubble fluctuation constraints ($m < m_q(\eta, \omega)$ and $m < m_b(\eta, \omega)$). In addition, if we wish that the quantum fluctuations in the ϕ field explain the density perturbations responsible for galaxy formation and possibly observed by COBE[6,14], then we must require also that $m_q < m_b$. In the quantum constraint we will consider $k = 10^{-5}$ corresponding to $\delta\rho/\rho \leq 10^{-5}$ at horizon crossing for CMBR fluctuations on an angular scale of $1'$. In the bubble fluctuation constraint we consider f_r to be in the range 0.1 to 1. We take $h = 0.5$ throughout.

The constraint $m_q > m_*$, required for a range of allowed masses to exist, gives an upper bound on η, η_u, for a given value of ω. The constraint $m_b > m_q$ gives a lower bound $\eta > \eta_l$. This lower bound applies if we require that it is possible for the quantum fluctuations to account for the observed density perturbations, which requires that $m = m_q$. If we do not require this, then we obtain a lower bound on η from $m_b > m_*$, which gives a lower bound $\eta > \eta_l'$ where

$$\eta_l' = 7.35\text{x}10^{-4}\frac{f_r}{\omega h^2} \tag{3.38}$$

In table 1 we give the bounds on η for various vales of ω for the cases $f_r = 0.1$ and 1. In table 1 we also give the upper bound on the ϕ mass m_{max} following from $m_q = m_b$. Since in general we require that $m < \text{Min}(m_q, m_b)$, the largest value of m as a function of η corresponds to $m_q = m_b$.

From table 1 we see that in general the value of η is constrained to be quite small ($\eta < 3\text{x}10^{-3}$) for the range of ω we have considered. This corresponds to a reheat temperature after the first-order phase transition T_e which is less than $3\text{x}10^{13}$GeV, which could pose problems for models which have baryon number violating interactions which can induce proton decay if the GUT mass scale is of the order of the reheat temperature[6]; this depends on how the masses of particles mediating proton decay are related to the false vacuum energy of the GUT. In

addition, if we impose the bound from requiring that there be no distortion of the CMBR due to horizon sized bubbles at the time of decoupling, which requires that $\omega < 25$ for the n=0 model[2,7], then we see from table 1 that it is possible to have $\omega < 25$, but the range of allowed η between η_u and η_l is very small, making the model seem rather unnatural. Also, for $\omega < 25$ the upper bound on η gives a very low reheat temperature $T_e < 3x10^{12}$GeV. In general as ω becomes large η_u tends towards an ω independent limit, $\eta_u = 1.1k^{1/2}h^{1/2}$, which is seen from (3.21) by letting $m_q = m_*$ and taking ω to infinity. With the above values of h and k this gives an upper limit on T_e of $2.6x10^{13}$GeV.

If we do not demand that the quantum fluctuations can account for the observed density perturbations ($m \neq m_q$), then we consider η_l' rather than η_l. In the case with $f_r = 0.1$ we see that for $\omega = 25$ the lower bound becomes $1.2x10^{-5}$, allowing a much larger range of η and so making the model more natural, although some other mechanism for generating the density fluctuations required for galaxy formation and possibly observed by COBE is then necessary.

Thus we see that the constraints imposed by the quantum and bubble fluctuations, together with the constraints from the non-observation of non-Newtonian forces and the absence of CMBR inhomogeneities due to horizon size bubbles at decoupling, disfavour in the context of the EIDM model the possibility that the Brans-Dicke scalar can account for dark matter if quantum fluctuations are responsible for the density perturbations leading to galaxy formation and the COBE observations, and in addition require that the reheat temperature following inflation be not much larger than $3x10^{12}$GeV.

We see also from table 1a) that for $\omega < 25$ the largest value of the mass of the Brans-Dicke scalar corresponds to $2x10^{-13}$GeV. This corresponds to a range for the non-Newtonian force of 0.1cm. Since this is a lower bound on the range when $\omega < 25$, it seems quite possible that non-Newtonian forces will be observed experimentally in this model in the near future.

Clearly the EIDM model would be more attractive if the energy density perturbations could be explained by the isocurvature quantum fluctuations of ϕ without

any unnatural looking tuning of η (essentially corresponding to a tuning of the mass scale of the GUT). In the following we consider two possible ways to alter the constraints on η. Firstly we consider the constraints in the case with n=2 rather than n=0. Then we consider the possibility that gravity is dominant in the nucleation and growth of the bubbles[9], unlike our discussion up to now which has ignored the effects of gravity on bubble growth.

4. Constraints on the n=2 extended inflation model.

In this section we consider how the above constraints are altered when we assume that during the physically significant final 60 e-foldings of inflation the coupling of ϕ to R is dominated by a higher order term corresponding to n=2. The natural interpretation of this is that the model described by (2.1) is an effective theory of a more fundamental theory which has a mass scale M associated with it. Then the coupling of ϕ to R may be expanded in powers of $\left(\frac{\phi}{M}\right)^2$.

From Appendix A, the slow-rolling solutions for n = 2 are

$$\phi = \phi_o \exp\left(\frac{K}{4}(t - t_o)\right) \tag{4.1}$$

$$H = H_o \exp\left(-\frac{K}{2}(t - t_o)\right) \tag{4.2}$$

$$\ln\left(\frac{a}{a_o}\right) = \frac{2H_o}{K}\left(1 - \exp\left(-\frac{K}{2}(t - t_o)\right)\right) \tag{4.3}$$

where

$$K = 64\left(\frac{8\pi V_o}{3}\right)^{1/2}\frac{1}{M} \tag{4.4}$$

The analysis of this model follows the same steps as in the n=0 case. We will simply state the results.

I. Quantum fluctuations.

As in the case of the n=0 model the isocurvature perturbations are found to be the dominant perturbations. For the adiabatic energy density perturbation we find

$$\delta_{adi} = 2^{17/2}\left(\frac{8\pi V_o}{3}\right)^{1/2}\frac{(\Delta N_e + \omega_f)^{5/2}}{M^2} \tag{4.5}$$

where in general ω in this case is a function of ϕ

$$\omega = \frac{M^2}{32\phi^2} \tag{4.6}$$

and, using $M_{Pl\,eff} = \frac{\phi^2}{32M}$, the value of ω at the end of inflation is given by $\omega_f = \frac{M}{32M_{Pl}}$.

For the case of the isocurvature perturbations we have

$$\delta_{\rm iso} = 2\left(\frac{g(T_\gamma)}{g(T_1)}\right)^{1/2}\left(\frac{T_\gamma}{T_1}\right)^{3/2}\left(\frac{m^2}{2\rho_c}\right)^{1/2}\frac{K(\Delta N_e+\omega_f)^{3/2}}{4\pi\sqrt{\omega_f}} \tag{4.7}$$

From these we find that $\delta_{\rm iso} > \delta_{\rm adi}$ if

$$m \gtrsim 1.1\text{x}10^{-22}\,\frac{h^4\omega_f^2}{\epsilon^4}\left(\frac{\Delta N_e+\omega_f}{60}\right)^4 \text{ GeV} \tag{4.8}$$

where we have defined ϵ by $M = \epsilon M_{\rm Pl}$. The validity of the slow-rolling approximation, which requires $\omega_f \gtrsim 3$, implies that $\epsilon \gtrsim 96$.

Since, as in the case n=0, $m > m_* \equiv 10^{-14}$GeV is necessary in order that non-Newtonian forces have a short enough range to remain unobserved, we find that (4.8) is satisfied and that $\frac{\delta_{\rm iso}}{\delta_{\rm adi}} \gtrsim 600\,\frac{\epsilon^{1/2}}{h}\left(\frac{60}{\Delta N_e+\omega_f}\right)$.

From (4.7) we obtain an upper bound, $m < m_q(\eta,\epsilon)$, where

$$m_q(\eta,\epsilon) = \frac{k^4 M_{\rm Pl}^3 M^4 \omega_f^2 (2\rho_c)^2}{\beta^4\, T_e^8\, T_\gamma^6\, (\Delta N_e+\omega_f)^6} \tag{4.9}$$

and

$$\beta = \frac{32}{\pi}\left(\frac{g(T_\gamma)}{g(T_1)}\right)^{1/2}\left(\frac{45}{4\pi^3 g(T_1)}\right)^{-3/8}\left(\frac{4\pi^3 g(T_e)}{45}\right)^{1/2} \tag{4.10}$$

Numerically this gives

$$m_q(\eta,\epsilon) = 5.9\text{x}10^{-34}\,\frac{\epsilon^6 h^4 k^4}{\eta^8}\left(\frac{60}{\Delta N_e+\omega_f}\right)^6 \text{ GeV} \tag{4.11}$$

II. Bubble fluctuations.

The arguement is essentially the same as for the n=0 case. For a bubble nucleated at t_B the value of ϕ inside the bubble assuming that ϕ doesn't evolve once a point crosses inside the bubble is given by

$$\varphi = \phi(t_B)\exp\left(\frac{Kr}{4}\right) \tag{4.12}$$

As before we take the mean bubble radius $\overline{r_B}$ after percolation of the first-order phase transition to be $\approx H(t_e)^{-1}$. Then

$$Kr_B \approx KH(t_e)^{-1} = \frac{2}{\omega_f} \tag{4.13}$$

Since $\omega_f > 3$ is needed for the slow-rolling approximation to hold we see that $\frac{Kr_B}{4}$ will be smaller than 1. Then

$$\phi \approx \phi(t_B)\left(1 + \frac{Kr}{4}\right) \tag{4.14}$$

and the mean fluctuation in ϕ due to expansion of the bubbles during the first-order phase transition will be

$$\delta\phi \approx \frac{K}{16}\left(r_B - \overline{r_B}\right)\phi(t_e) \approx \frac{4f_r M_{Pl}^{3/2}}{M^{1/2}} \tag{4.15}$$

where as before we take $r_B - \overline{r_B} = f_r H(t_e)^{-1}$, and $\delta\phi$ is defined as in (3.27). Requiring that the energy density from this fluctuation is less than the critical density gives an upper bound, $m < m_b(\eta, \epsilon)$, where

$$m_b(\eta, \epsilon) = \frac{g_f^{*\,2}\alpha^* T_e \rho_c M}{32 T_\gamma^3 M_{Pl}^2 f_r} \tag{4.16}$$

Numerically this gives

$$m_b(\eta, \epsilon) = 4.2\text{x}10^{-13}\,\frac{\eta h^2 \epsilon}{f_r}\,\text{GeV} \tag{4.17}$$

III. Constraints on the model from quantum and bubble fluctuations.

The constraint from non-Newtonian forces gives as before $m > m_* = 10^{-14}$GeV. Since the expression for $m_q(\eta, \epsilon)$ in this case is simpler than the corresponding expression in the n=0 case (3.21) we can give analytic expressions for the upper and lower bounds on η. From $m_q > m_*$ we find $\eta < \eta_u$ where

$$\eta_u = 3.9\text{x}10^{-3}\,\epsilon^{3/4} h^{1/2} k^{1/2}\left(\frac{60}{\Delta N_e + \omega_f}\right)^{3/4} \tag{4.18}$$

We consider first the case where the quantum fluctuations are able to account for the density perturbations leading to galaxy formation. If quantum fluctuations

are to account for the density perturbations then we need $m_q < m_b$ which gives $\eta > \eta_l$ where

$$\eta_l = 4.8\text{x}10^{-3}\epsilon^{5/9}h^{2/9}k^{4/9}f_r^{1/9} \tag{4.19}$$

Note that as ϵ increases, η_u increases faster than η_l, so increasing the range of allowed η.

If we are to have an allowed range of η then we require $\eta_u > \eta_l$. Using as before h=0.5 and $k = 10^{-5}$, this requires that

$$\epsilon > 215f_r^{4/7} \tag{4.20}$$

This also implies that $\omega_f > 6.7f_r^{4/7}$. From (4.18), with the above values of h and k, we find $\eta_u = 8.7\text{x}10^{-6}\epsilon^{3/4}\left(\frac{60}{\Delta N_e+\omega_f}\right)^{3/4}$. Thus the upper bound on η will imply a low value for the reheat temperature $T_e = \eta 10^{16}$GeV unless ϵ is quite large. The largest possible value for η_u occurs for $\omega_f \gtrsim \Delta N_e \approx 60$. This gives an upper bound on η_u of $2.6\text{x}10^{-3}$ (which is the same result as for the n=0 model). However this limit would probably be a problem for the number of horizon-sized bubbles at the time of decoupling, since it requires $\omega_f \gtrsim 60$. Since during extended inflation ω decreases with time in the n=2 model, this implies that ω is larger than the CMBR large-bubble constraint $\omega < 25$.

As shown in the preceeding section, a problem with the n=0 model is that for $\omega < 25$ the range of η between η_u and η_l is unnaturally small. In the n=2 case, if we consider $\omega_f = 25$ then $\epsilon = 800$, which gives a range of η

$$1.3\text{x}10^{-3} > \eta > 1.0\text{x}10^{-3}f_r^{1/9} > 7.7\text{x}10^{-4} \tag{4.21}$$

where in the last term we have used $f_r > 0.1$. Thus the range of values of η remains small even if we consider n=2. Comparing with the $\omega = 25$ result from table 1a), we see that the range in the n=2 model is only slightly larger than in the n=0 model. The upper bound on the reheating temperature in the n=2 case for $\omega < 25$ is $1.3\text{x}10^{13}$GeV, which is larger than in the n=0 case ($3\text{x}10^{12}$GeV).

We conclude that considering an n=2 model does not improve significantly on the η tuning problem of the n=0 model in the case where quantum fluctuations

explain the energy density perturbations responsible for galaxy formation. The n=2 model becomes better with larger values of ϵ, with a larger range of allowed η for a given ϵ, but is likely to have problems with the constraint from too many horizon-sized bubbles at decoupling. We note that larger ϵ corresponds to larger values of $M = \epsilon M_{Pl}$. The slow-rolling approximation requires that $\epsilon \gtrsim 96$, which corresponds to a mass scale $M \gtrsim 10^{21}$GeV.

If we do not require that quantum fluctuations explain the galaxy-forming density fluctuations, then the lower bound on η comes from $m_b > m_*$. This gives $\eta > \eta_l'$ where

$$\eta_l' = 2.4\text{x}10^{-2}\frac{f_r}{h^2\epsilon} \tag{4.22}$$

For example, $\omega_f = 25$ and h=0.5 gives $\eta_l' = 1.2\text{x}10^{-4}f_r$. With $f_r = 0.1$ we have $\eta_l' = 1.2\text{x}10^{-5}$, which is much smaller than the corresponding value of η_l, $\eta_l = 7.7\text{x}10^{-4}$, thus allowing a much wider and so more natural range of values of η between η_u and η_l'.

We conclude that higher-order couplings of ϕ to R do not significantly improve on the models with n=0, and still require η to occur within a rather unnaturally small range of values if the galaxy-forming density perturbations are to be explained by quantum fluctuations produced during inflation. The slow-rolling approximation requires a mass scale M (associated with some complete theory) which is at least $96M_{Pl}$.

5. Consequences for the phase transition and the EIDM model of including the effects of gravity on bubble growth.

In this section we consider the possibility that gravity could significantly relax the constraints on the EIDM model as compared with the situation when the effects of gravity on the expansion of bubbles is neglected.

In the n=0 model it was originally believed that in order for large unthermalised bubbles not to perturb the isotropy of the CMBR it is necessary for ω to satisfy $\omega < 25$ [2,7]. However, the bubble distribution leading to this conclusion was calculated neglecting the effects of gravity on bubbles nucleated at early times, in particular those which grow to be larger than the horizon at decoupling. It was subsequently shown in ref.9 that since the effective Planck mass (and so the gravitational energy of the bubbles) is increasing with time during extended inflation, it is possible for bubbles to collapse before the end of inflation due to the effect of gravity on their evolution. We briefly review the results of ref.9 and then consider how they may alter the nature of the phase transition and the constraints on the EIDM model.

From ref.9 the equations determining the growth of bubbles when the effects of gravity are included are

$$\ddot{R} = \frac{-2\left(1+\dot{R}^2\right)}{R} - U\left[\frac{\dot{t}^2}{t} - \frac{3U}{R}\right] - \left[\frac{3}{4}\frac{2\omega}{3+2\omega}\frac{8\pi\sigma}{\Phi} - \frac{\rho}{\sigma}\right]\left(1+\dot{R}^2-U^2\right)^{1/2} \quad (5.1)$$

where $U = R(da/dt)/a$ and

$$\dot{t}^2 - a^2\dot{r}^2 = 1 \quad (5.2)$$

$$a(\tau)r(\tau) = R(\tau) \quad (5.3)$$

$$\dot{t} = \frac{-U\dot{R} + \left(1+\dot{R}^2-U^2\right)^{1/2}}{1-U^2} \quad (5.4)$$

Here 'dot' denotes differentiation with respect to τ, the comoving time on the bubble wall, and the metric in the false vacuum outside the bubble is

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega^2) \quad (5.5)$$

R is the physicial radius of the bubble, ρ is the energy density in the false vacuum, and σ is the surface energy density of the bubble wall. Φ is as defined in section

2, with $M^2_{Pl\,eff} \equiv \Phi$. Using the assumption that bubble nucleation occurs as in conventional Einstein gravity[15], with M_{Pl} replaced by the effective Planck mass $M_{Pl\,eff}$, the initial radius of the bubble is

$$R_o = 3\left[\frac{3+2\omega}{2\omega}\frac{\rho}{\sigma} + \frac{3}{4}\frac{2\omega}{3+2\omega}\frac{8\pi\sigma}{\Phi(t_n)}\right]^{-1} \tag{5.6}$$

where the bubble is nucleated at t_n and initially $\dot{R} = 0$. In order to see the dynamics of the bubble we solve (5.1) approximately. We assume that initially $U \ll 1$, which is equivalent to $RH \ll 1$, which is true if the initial bubble radius is small compared to the horizon. Then (5.1) can be written as

$$\ddot{R} \approx \frac{-2\left(1+\dot{R}^2\right)}{R} - \left[\frac{3}{4}\frac{2\omega}{3+2\omega}\frac{8\pi\sigma}{\Phi} - \frac{\rho}{\sigma}\right]\left(1+\dot{R}^2\right)^{1/2} \tag{5.7}$$

and

$$\dot{t} \approx \left(1+\dot{R}^2\right)^{1/2} \tag{5.8}$$

From (5.7) we see that R will start to collapAse if initially

$$\frac{3}{4}\frac{2\omega}{(3+2\omega)}\frac{8\pi\sigma}{\Phi} > \frac{\rho}{\sigma} \tag{5.9}$$

Also, from (5.6) the condition that the initial radius of the bubble is primarily determined by gravitational effects is

$$\frac{3}{4}\left(\frac{2\omega}{(3+2\omega)}\right)^2\frac{8\pi\sigma}{\Phi} > \frac{\rho}{\sigma} \tag{5.10}$$

which is essentially the same as (5.9) for $\omega \gg 1$. In terms of $f = \rho H^{-1}(t_n)/\sigma$,[9] (5.9) can be written as

$$f < \frac{3}{2}\left(\frac{2\omega}{3+2\omega}\right)^{1/2} \tag{5.11}$$

(In ref.9 using an exact numerical solution of the equations of motion the condition for bubbles to recollapse is $f < 1$). To see the time-scale of the collapse we assume $U < 1$ throughout, and we assume that the time scale of collapse is small enough that Φ can be taken as constant throughout i.e. $\delta t < \left(\frac{\dot{\Phi}}{\Phi}\right)^{-1}$. We will make an overestimate of the collapse time by neglecting the first term in (5.7), i.e. we consider

$$\ddot{R} = -k_b\left(1+\dot{R}^2\right)^{1/2} \tag{5.12}$$

where

$$k_b = \left(\frac{3}{4}\frac{2\omega}{(3+2\omega)}\frac{8\pi\sigma}{\Phi} - \frac{\rho}{\sigma}\right) \tag{5.13}$$

For $\dot{R}^2 < 1$, the solution is

$$R = R_o - \frac{k_b}{2}\delta\tau^2 \tag{5.14}$$

where $\tau = \tau_n + \delta\tau$ and the bubble is nucleated at τ_n. Thus if $\dot{R} < 1$ throughout, the time for the bubble to collapse would be $\delta\tau_c = (2R_o/k_b)^{1/2}$. Since, from (5.8), for $\dot{R} < 1$ we have $\dot{t} \approx 1$, we see that $\delta\tau \approx \delta t$. Thus the time δt_c for collapse to occur is $\delta t_c = \left(\frac{2R_o}{k_b}\right)^{1/2}$. For $f < \frac{3}{2}$, $k_b = \frac{3}{R_o}$. Thus we find

$$\delta t_c \approx \sqrt{\frac{2}{3}}R_o < H^{-1} \tag{5.15}$$

Since $\left(\frac{\dot{\Phi}}{\Phi}\right) = \frac{2H}{\omega}$, the assumption $\delta t_c < \left(\frac{\dot{\Phi}}{\Phi}\right)^{-1}$ is justified. Therefore the bubble will recollapse in a time short compared with the expansion rate of the Universe. (This result assumes that $\dot{R} < 1$ throughout the collapse. In fact, $\dot{R} > 1$ will occur when the bubble has collapsed only to $\frac{5}{6}R_o$. However, a better estimate of the overestimate for the collapse timescale which allows for $\dot{R}^2 > 1$ gives essentially the same result, $\delta t_c \approx \frac{7}{6}R_o$). This holds for $f < \frac{3}{2}$ ($f < 1$ in the exact numerical solution).

Once $f \gtrsim 1$ the bubble can expand after nucleation. This gives a new scenario for the end of extended inflation. The usual condition for the end extended inflation is that the rate of bubble nucleation per horizon volume per expansion time $\frac{\Gamma}{H^4}$ becomes of order 1. Then in a given horizon volume it becomes possible for two or more bubbles to expand and collide. However, once the effect of gravity is taken into account, we see that it is also necessary that $f \gtrsim 1$ also occurs, otherwise the bubbles cannot expand after nucleation. One can then consider the possibility that initially $f < 1$ during extended inflation but $\frac{\Gamma}{H^4} \gg 1$. Since H decreases during inflation, f will eventually increase until $f \approx 1$. Then the bubbles can expand and, since $\frac{\Gamma}{H^4} \gg 1$, the phase transition will percolate and so extended inflation will be brought to an end. The important feature of this scenario for the EIDM model is that since $\frac{\Gamma}{H^4} \gg 1$, the mean seperation of the bubbles once

the transition has percolated will generally be much smaller than $H(t_c)^{-1}$, i.e. the factor f_r introduced in the previous sections will be much smaller than 1. This will effectively remove the bubble fluctuation constraint from the model, leaving only the upper bound η_u from the quantum fluctuations. Note that this scenario for ending extended inflation also eliminates the problem pointed out in ref.9, where it was suggested that the condition $f \approx 1$ at the end of extended inflation requires a fine-tuning of the parameters of the model. We see from our discussion that this condition is in fact a natural consequence of this mechanism for ending extended inflation.

In fact the scenario described above, in which it is assumed that $f \approx 1$ at the end of inflation, is not quite correct. The initial radius of the bubbles at nucleation is R_o, where from (5.6)

$$\frac{R_o}{H^{-1}} = \frac{4}{3}\left(\frac{3+2\omega}{2\omega}\right) f \tag{5.16}$$

for $f < 1$. Thus R_o/H^{-1} reaches a value ≈ 1 as $f \to 1$. This would suggest that the average radius of the bubbles at nucleation is $\approx H^{-1}$, if $f \approx 1$ at the end of extended inflation. What actually happens in this case is that the transition ends purely by rapid nucleation (without any bubble expansion) before $f \approx 1$. For $\omega \gg 1$, the initial radius is $R_o \approx \frac{4f}{3}H^{-1}$. The number of bubbles nucleated per unit volume in a time interval δt is $\Gamma\delta t$. The condition (ignoring the recollapse of the bubbles) for the bubbles nucleated during δt to percolate the transition is $\Gamma R_o^3 \delta t \gtrsim 1$. The bubbles recollapse in a time $\delta t_c \approx R_o$. So δt_c is the longest time interval we can consider for the percolation condition without the bubbles recollapsing. Thus the percolation condition will first be satisfied when $\Gamma R_o^4 \approx 1$. Therefore when percolation occurs purely by nucleation we find $f \equiv \frac{3}{4}\frac{R_o}{H^{-1}} \approx \frac{3}{4}\left(\frac{\Gamma}{H^4}\right)^{-1/4}$. Since by assumption $\frac{\Gamma}{H^4} \gg 1$, we see that in this case f can be much smaller than 1 when inflation ends.

Thus we see that including the effects of gravity not only eliminates the problem of large unthermalised bubbles and so the bound $\omega < 25$, but can also result in a phase transition in which the bubbles at the end of inflation are much smaller than the horizon, so eliminating the bubble fluctuation constraint on the EIDM model.

As a result, the lower bounds on the reheat temperature are effectively removed. The upper bound can also be relaxed, since we can now take ω as large as we wish, which allows η_u to take its largest possible value $\eta_u = 1.1h^{1/2}k^{1/2}$. With the previously used values for h and k we find that T_e can be as large as $2.6\text{x}10^{13}$GeV.

The condition for gravity to dominate the formation of bubbles and prevent percolation initially, $f \lesssim \frac{3}{4}\left(\frac{\Gamma}{H^4}\right)^{-1/4}$, requires

$$M^2_{\text{Pleff o}} < \frac{3\pi\sigma^2}{2V_o}\left(\frac{\Gamma}{H^4}\right)^{-1/2} \equiv \frac{\pi V_o^{1/2}}{6\lambda^2}\left(\frac{\Gamma}{H^4}\right)^{-1/2} \tag{5.17}$$

where $M_{\text{Pleff o}}$ is the effective Planck mass at the beginning of extended inflation. To obtain the last term in (5.17) we use the thin-wall approximation model for the bubble used in ref.9 in which $\sigma = \frac{V_o^{3/4}}{3\lambda}$, with λ a self-coupling for the scalar field which undergoes the first-order phase transition. The validity of the classical equations of motion requires that $V_o < M^4_{\text{Pleff o}}$. Thus a range of V_o, $\left(\frac{6\lambda^2}{\pi}\right)^2\left(\frac{\Gamma}{H^4}\right)M^4_{\text{Pleff o}} < V_o < M^4_{\text{Pleff o}}$, will exist if λ is small compared with 1.

6. Conclusions and Discussion

In this paper we have discussed in some detail the question of whether a Brans-Dicke scalar in the context of extended inflation models could account for the dark matter in the Universe. Such a question is important given that the extended inflation scenario is the only inflation scenario which can be driven by the energy density of the matter sector which contains the fermions, Higgs bosons and gauge bosons of the standard model (the only sector we know about apart from gravity), and given that the Brans-Dicke scalar is an obvious possibility for dark matter in such a scenario. In general for the Brans-Dicke scalar to be a natural dark matter candidate we require that the false vacuum energy or bubble nucleation rate vary over length scales much larger than the observable Universe, in order that the value of ϕ at the end of inflation can be determined naturally by anthropic selection. In the case of the minimal n=0 extended inflation model we found that the range of reheating temperatures at the end of extended inflation is strongly constrained by the requirements that the energy density perturbations generated during inflation (which were shown to be dominated by the isocurvature fluctuations of the Brans-Dicke scalar) are acceptably small, that the energy density due to fluctuations of the Brans-Dicke scalar induced by bubble expansion at the first-order phase transition is not too large, and that non-Newtonian corrections to the gravitational potential are of a short enough range to be unobservable. In general we find that there is an upper limit on the reheat temperature at the end of inflation of about $3\text{x}10^{13}$GeV. In addition, if we require that the isocurvature fluctuations account for the density perturbations leading to galaxy formation, then there is a lower bound on the reheat temperature which is very close to the upper bound for values of ω which are small enough to avoid the creation of CMBR inhomogeneities due to unthermalized horizon-sized bubbles at decoupling ($\omega < 25$). In this case the small range of allowed reheat temperatures makes the n=0 extended inflation dark matter model seem rather unnatural. Also, in this case the upper bound on the reheat temperature is low, about $3\text{x}10^{12}$GeV.

The small range of reheat temperatures allowed when quantum fluctuations

account for the density perturbations clearly disfavours the Brans-Dicke scalar as a dark matter candidate in this case. If we do not require that the isocurvature fluctuations account for the density perturbations then the range of reheat temperatures may be increased, making the model seem more natural. However, in this case we need a new mechanism to account for the density perturbations.

A potentially significant observation is that the range of non-Newtonian corrections in models where the effect of gravity on the growth of bubbles is small is not likely to be much larger than the present upper bounds of about 1cm on their range. From table 1 we see that for $\omega < 25$ the range is expected to be not much smaller than about 0.1cm ($m < 1.9\text{x}10^{-13}$GeV). So in this case we expect non-Newtonian corrections to the gravitational potential to be observable in not too distant future.

In order to overcome the unnaturalness of the n=0 model in the case where the density perturbations are due to quantum scalar field fluctuations we considered two possible departures from the minimal extended inflation model. We first considered the possibility that during the final 60 e-foldings of inflation the coupling of the Brans-Dicke scalar to the Ricci scalar is dominated by a higher-order n=2 term in an expansion of the coupling in powers of $\frac{\phi}{M}$, with M a mass scale associated with some underlying complete theory. We found that the unnaturalness of the n=0 model in the case where the isocurvature fluctuations account for galaxy formation is not significantly improved on in the n=2 model, although the upper bound on reheat temperature in the case where the value of ω at the end of inflation ω_f is less than 25 is slightly larger than for the n=0 model at $1\text{x}10^{13}$GeV compared with $3\text{x}10^{12}$GeV. In order that the slow-rolling approximation necessary for inflation is valid we require that $M \gtrsim 96M_{Pl}$. In general we conclude that considering an n=2 extended inflation model produces no major improvement over the case of the minimal n=0 model.

The second possibility we considered was that gravity could play a significant role in the dynamics of bubble expansion and the percolation of the first-order phase transition. This leads naturally to a new scenario for the end of extended

inflation. Rather than a picture in which the nucleation rate per horizon volume increases with time until more than one bubble is nucleated in a horizon volume in a time H^{-1}, leading to percolation of the phase transition with bubbles of a mean radius of the order of the horizon, we can now have a picture in which gravity prevents the growth of bubbles and the phase transition completes when the radius of the bubbles at nucleation is large enough that nucleation alone can percolate the transition, without requiring growth of the bubbles. In this case the mean radius of the bubbles at the end of extended inflation can be much smaller than the horizon, which eliminates the lower bounds on the reheating temperature arising from the bubble fluctuation constraint. Therefore if gravity dominates the dynamics of the phase transition then not only is the constraint on ω from large unthermalized bubbles ($\omega < 25$) eliminated, but also the model no longer requires unnatural tuning of the GUT scale. However, it should be noted that the upper bound on the reheating temperature $3\text{x}10^{13}$GeV will remain. This could be a problem for GUTs which have interactions leading to proton decay if the mass scale of particles mediating proton decay is approximately equal to the reheating temperature.

The EIDM model for dark matter has some potential advantages over most conventional particle models of dark matter. The model allows (in the spirit of the original extended inflation model) for a GUT without requiring the fine-tuning of couplings needed by the new and chaotic inflation scenarios in order to avoid large density perturbations. In addition, the fact that the amount of dark matter in this model is determined by the initial value of a scalar field following inflation allows for an anthropic selection explaination of the similarity of the dark matter and baryon densities in the observed Universe[13] (appendix C). This is in contrast with particle physics explainations of dark matter in particle physics models which have fixed parameters throughout the Universe, in which the dark matter and baryon densities are determined by completely unrelated physical processes (although there are exceptions[18]).

The EIDM model also provides some interesting possibilities for structure for-

mation scenarios which deviate from the conventional CDM scenario based on scale-invariant density perturbations[6]. There have been indications from observations of large-scale structure that the power spectrum of density perturbations from the conventional CDM model provides insufficient power on large scales (larger than about $20h^{-1}$Mpc)[19,20]. The EIDM model allows for two possible ways to have a power spectrum which deviates from the conventional CDM model. The first possibility, which is not specific to the EIDM model but is a general feature of extended inflation models, is that the spectrum of fluctuations is not scale-invariant[1,2]. For the minimal n=0 extended inflation model the density perturbations $\delta_{\rm iso}$ (3.14) on a scale R_p at present satisfy $\delta_{\rm iso} \propto R_p^{2/(\omega-1)}$, where we have used $e^{\Delta N_e/\omega} \propto R_p^{1/(\omega-1)}$. In order to have an increase in power by a factor of 2 between $10h^{-1}$Mpc and $25h^{-1}$Mpc we therefore require $\omega \lesssim 3.7$. However, we see from table 1 that this is possible only if gravity effects remove the lower bounds on η coming from bubble fluctuations and if the value of η is very small $\eta < 6\text{x}10^{-5}$, corresponding to a very low reheat temperature after inflation $< 10^{12}$GeV.

The second possibility for deviations from the conventional CDM model of structure formation is one which is specific to the EIDM model. This is the possibility of the Brans-Dicke scalar providing both a cold dark matter and hot dark matter (HDM) component of dark matter simultaneously. The idea is that the coherent oscillations of the Brans-Dicke scalar can provide a cold component of the dark matter, while the energy density of the space-dependent bubble fluctuations can provide a hot component of dark matter. From (3.29) and (3.30) we see that when the gradient energy of the space-dependent scalar field fluctuations dominates their evolution, which occurs when $T > T_1^*$ (3.34), the scalar field behaves like an effectively massless scalar field (the Brans-Dicke scalars having been effectively heated by bubble expansion at the first-order transition). Once $T < T_1^*$, the evolution of the fluctuations will be dominated by their mass, with their energy density evolving like a density of massive scalars. Thus as far as considering the free-streaming length associated with the energy density of particles corresponding to the space-dependent scalar fluctuations is concerned, the scalars behave similar

to a density of neutrinos of mass around T_1^*. For the free-streaming length to occur at scales of interest to structure formation we require that T_1^* be of order 10eV[6]. The remarkable feature of the EIDM model is that this is a natural value for T_1^* in the model. From (3.34) we find that $T_1^* = 2\text{x}10^{-3}(m/10^{-14}\text{GeV})(f_r/\eta)\text{eV}$. Thus with $\eta \approx 10^{-4}$ (a typical value from table 1 for $\omega < 25$) and with $f_r = 0.1$ we find $T_1^* = 2(m/10^{-14}\text{GeV})\text{eV}$. Thus with m around 10^{-13}GeV (a typical value of m_{max} from table 1) we obtain $T_1^* \approx 20\text{eV}$. This shows that it is quite natural for a hot dark matter component due to bubble fluctuations to have a free-streaming scale which is of the right size to allow density perturbations in the hot dark matter to contribute to large-scale structure formation while being smooth on smaller scales, where only perturbations in the coherent oscillation cold component of the dark matter will contribute. Thus we see that the EIDM model allows a combined CDM+HDM structure formation model with only a single dark matter particle, the Brans-Dicke scalar. The only problem with this scenario is how to understand why the energy density of the CDM and HDM components of the dark matter should be similar, but this is a likely to be problem for all CDM+HDM scenarios. We note from table 1 that the typical value of m_{max}, which corresponds to the value of the Brans-Dicke scalar mass for which the HDM energy density is of order the critical density when quantum fluctuations account for density perturbations, is between 10^{-11}GeV and 10^{-14}GeV for $\omega < 100$, which is just the range at which T_1^* leads to an interesting free-streaming length. So a combined CDM+HDM scenario seems plausible in the context of the EIDM model. We note that this requires that gravity is not significant in determining the growth of bubbles during inflation, so that the bubble fluctuations exist. We also note that it may be possible to generalize this mechanism to other oscillating scalar CDM candidates if they are non-minimally coupled to gravity in an extended inflation scenario.

We conclude that in reply to the question 'Can the Brans-Dicke scalar account for dark matter in extended inflation models?' the answer is probably not if we wish that quantum scalar field fluctuations generated during inflation can account for the energy density perturbations in the Universe, unless the effects of

gravity dominate the dynamics of bubble nucleation and growth during inflation. If we allow for the possibility that some other mechanism generates the density perturbations then the model in which the effect of gravity on bubble dynamics is negligible becomes more plausible. In this case we expect non-Newtonian corrections to the gravitational potential to be observable in the near future.

The Brans-Dicke scalar as a dark matter candidate is an important possibility. We have shown that the resulting extended inflation dark matter model can be severely constrained by observations, and has new and interesting features with respect to the influence of inflationary first-order phase transitions on the physics of light scalar dark matter and with respect to the possibilities for modification of the conventional cold dark matter model of structure formation.

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Appendix A. Extended inflation models and general slow-rolling solutions of their equations of motion.

In this appendix we give a general slow-rolling approximation solution to the equations of motion for the case of a coupling to the Ricci scalar of the form $\phi^{n+2}R$.

The extended inflation model with a general coupling between the Brans-Dicke scalar ϕ and the Ricci scalar R is described by the action (2.1) As discussed in section 2, the equations of motion of the extended inflation model are

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{1}{(3+2\omega)}\left[8\pi\left(\rho - 3p\right) - \omega'\dot{\Phi}^2\right] \qquad (A.1)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho}{3\Phi} - \frac{\dot{\Phi}}{\Phi}H + \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 \qquad (A.2)$$

where $\Phi = f(\phi) = \frac{\phi^{n+2}}{M^n}$ (for n=0, $f(\phi) = \frac{\phi^2}{8\omega}$), $\omega' = \frac{d\omega}{d\Phi}$ and ρ and p are the energy density and pressure due to the matter fields. (The potential energy of ϕ may be neglected during inflation).

We consider the equations in the limit where a) $\ddot{\Phi} \ll 3H\dot{\Phi}$ and b) $\frac{\dot{\Phi}}{\Phi} \ll H$ (slow-rolling approximation). In addition we make the assumptions that c) $32\pi V_o \gg \omega'\dot{\Phi}^2$ and d) $H^2 \gg \frac{\omega}{6}\left(\frac{\dot{\Phi}}{\Phi}\right)^2$. (In all these the modulus of the quantites appearing in the inequalities should be understood). It will be seen that c) and d) are satisfied whenever the slow-rolling approximation is satisfied.

The equations of motion are then

$$3H\dot{\Phi} = 32\pi\left(n+2\right)^2\left(\frac{\Phi}{M^2}\right)^{\frac{n}{n+2}} V_o \qquad (A.3)$$

and

$$H^2 = \frac{8\pi V_o}{3\Phi} \qquad (A.4)$$

Together these give

$$\dot{\Phi} = K\Phi^{\frac{3n+2}{2(n+2)}} \qquad (A.5)$$

where K is given by

$$K = \frac{4\left(n+2\right)^2}{M^{\frac{2n}{n+2}}}\left(\frac{8\pi V_o}{3}\right)^{1/2} \qquad (A.6)$$

(For n=0 K is given by $K = \frac{2}{\omega}\left(\frac{8\pi V_o}{3}\right)^{1/2}$). The solution for Φ is then

$$\Phi^{\frac{2-n}{2(n+2)}} = \Phi_o^{\frac{2-n}{2(n+2)}} + \frac{(2-n)}{2(n+2)} K(t - t_o) \qquad (A.7)$$

except for the case n=2, which has the solution

$$\Phi = \Phi_o \exp(K(t - t_o)) \qquad (A.8)$$

Thus for n=0 the solution is

$$\phi = \phi_o (1 + A(t - t_o)) \qquad (A.9)$$

where

$$A = (2\omega)^{1/2} \frac{K}{\phi_o} \qquad (A.10)$$

and

$$a = a_o (1 + A(t - t_o))^{\omega} \qquad (A.11)$$

For n=2 the solution is

$$\phi = \phi_o \exp\left(\frac{K}{4}(t - t_o)\right) \qquad (A.12)$$

$$\ln\left(\frac{a}{a_o}\right) = \frac{2H_o}{K}\left(1 - \exp\left[-\frac{K}{2}(t - t_o)\right]\right) \qquad (A.13)$$

The conditions for the assumptions a) to d) to be valid are as follows.

a):

$$\omega \gg \frac{(3n+2)}{3(n+2)} \qquad (A.14a)$$

b):

$$\omega^2 \gg 4 \qquad (A.14b)$$

c):

$$\omega \gg \frac{n}{3(n+2)} \qquad (A.14c)$$

d):

$$\omega \gg \frac{2}{3} \qquad (A.14d)$$

Thus (A.14b) gives the tightest constraint. This condition will be satisfied for $\omega \gtrsim 3$. We see from the expression for ω

$$\omega = \frac{1}{2\,(\mathrm{n}+2)^2}\left(\frac{\mathrm{M}^2}{\Phi}\right)^{\frac{\mathrm{n}}{\mathrm{n}+2}} \tag{A.15}$$

that since Φ increases with time for $\mathrm{n} \geq 0$, ω decreases with time. Thus the slow-rolling approximation is justified if ω is large compared to 1 initially and if during extended inflation it doesn't become smaller than about 3.

Appendix B. Bounds on m from non-Newtonian gravity experiments.

The deviation of the gravitational force from the Newtonian form in a Brans-Dicke model can be obtained by a Weyl scaling to the Einstein frame[16]

$$g_{\mu\nu} \rightarrow \Omega^2(t)\,\overline{g}_{\mu\nu} \tag{B.1}$$

In order to obtain the conventional form for the gravitational action we define a scalar field ψ and choose Ω to satisfy

$$\Omega^2 = \frac{M_{Pl}^2}{16\pi\Phi} \equiv \exp\left(-\frac{\psi}{\psi_o}\right) \tag{B.2}$$

Then the action (2.1) becomes (with the metric now defined by $\overline{g}_{\mu\nu}$)

$$\int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{16\pi}R - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - W(\psi) - e^{-\frac{\psi}{\psi_o}}\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - e^{-\frac{2\psi}{\psi_o}}V(\sigma)\right] \tag{B.2}$$

where $\psi_o^2 = \frac{2\omega+3}{16\pi}\,M_{Pl}^2$ and $W(\psi)$ is the Weyl-scaled potential from $V(\phi)$. At present $\Omega = 1$ so $\psi \rightarrow 0$. At the low momentum transfers involved in non-Newtonian gravity effects we can expand the exponential as $e^{-\frac{2\psi}{\psi_o}} \approx (1 - \frac{2\psi}{\psi_o})$. Then the coupling of ψ to nucleons, coming from the Weyl-scaled quark mass term $e^{-\frac{2\psi}{\psi_o}}m_q\overline{q}q$, will be

$$-\frac{2\psi}{\psi_o}m_N\overline{N}N \tag{B.4}$$

A coupling of a scalar to nucleons of the form

$$\frac{C_N\phi}{f}m_N\overline{N}N \tag{B.5}$$

will give rise to a correction to the gravitational potential between two masses of the form[17]

$$\Delta V = -\frac{G_N M_1 M_2}{r}\,\alpha\,\exp(-\mu r) \tag{B.6}$$

where

$$\alpha = \frac{C_N^2}{4\pi}\frac{M_{Pl}^2}{f^2} \tag{B.7}$$

G_N is Newton's constant and μ is the mass of the scalar. From (B.4) we find C_N = -1 and $f = \frac{\psi_o}{2}$, which finally gives

$$\alpha = \frac{16}{2\omega+3} \tag{B.8}$$

This result is true for all n, since the Weyl-scaling does not depend on ω being a constant with respect to t. One simply replaces the constant ω of the n=0 case by the value of $\omega(t)$ at the end of inflation ω_f. This gives the correct normalization for the ψ kinetic term.

In ref.8 the values of α allowed by terrestrial Cavendish-type experiments and geological data and by various satellite and solar system tests of the gravitational potential are discussed. From fig.1 and 2 of ref.8 it is seen that for $\alpha \gtrsim 10^{-4}$ (corresponding to $\omega \lesssim 10^5$) non-Newtonian forces are ruled out on length scales larger than about 1cm. Larger values of ω allow non-Newtonian forces to exist on larger length scales. (If $\omega \gtrsim 10^{10}$ there is no limit on the range of non-Newtonian forces from terrestrial or solar-system tests). In our discussion we concentrate on the case of $\omega < 10^4$ and require the most generally safe condition that the range of non-Newtonian corrections (corresponding to μ^{-1}) be less than 1cm. This requires that the mass of the Brans-Dicke scalar be greater than about $m_* = 10^{-14}$GeV.

Appendix C. Anthropic principle determination of ϕ at the end of inflation

In this appendix we discuss the analogue in the EIDM model of the anthropic principle arguement originally proposed by Linde in the context of an axion model in which PQ symmetry is broken before the end of inflation[13]. This arguement explained the smallness of the deviation of the axion from the minimum of its potential at the end of inflation. The arguement for the EIDM model is slightly different because in the axion model the energy density perturbations leading to the formation of galaxies are independent of $\delta\phi_e$ (where $\delta\phi_e$ is the deviation of the scalar of interest from the minimum of its potential at the end of inflation), whereas in the EIDM model the energy density perturbation δ_{iso} is proportional to $\frac{2\delta\phi_Q}{\delta\phi_e}$, and so is not constant with respect to $\delta\phi_e$.

From (3.12) and $\phi_e = \sqrt{8\omega}\ M_{Pl}$ we find that if ϕ oscillations account for dark matter with $\Omega \leq 1$ at present then we need $\delta\phi_e/\phi_e \lesssim 10^{-6}/\omega^{1/2}$, where $m > 10^{-14}$GeV has been assumed. We therefore need to explain why $\delta\phi/\phi$ is so small at the end of inflation.

We take the baryon number density to entropy ratio n_B/s to be fixed throughout the Universe. We assume that inflation naturally generates domains where ϕ_e takes an effectively constant value (different in different domains of the Universe) over regions much larger than the size of the observable Universe. This requires that either V_o or Γ vary on scales much larger than the observable Universe. In the EIDM model dark matter is due to coherent ϕ oscillations. Thus in a given domain we have for the density of dark matter ρ_{dm}

$$\rho_{dm} = \left(\frac{T}{T_1}\right)^3 \rho_{dm\,o} \qquad (C.1)$$

when $T < T_1$, where T_1 is the temperature at which $\delta\phi$ starts oscillating ($H(T_1) \approx m$), and $\rho_{dm\,o} \approx \frac{1}{2}m^2\delta\phi_e^2$ is the initial dark matter density at T_1.

The matter-radiation equality temperature T_{eq} in this domain (at which temperature the density perturbations leading to galaxy formation begin to grow[4,6])

is given by

$$T_{eq} = \frac{\rho_{dm\,o}}{aT_1^3}\ ; \quad a = \left(\frac{4\pi^3 g\,(T_{eq})}{45M_{Pl}^2}\right)^{1/2} \tag{C.2}$$

Since the growth of the perturbations δ is proportional to a [6], the temperature T_{nl} at which the perturbations δ_{iso} go non-linear ($\delta_{iso} \approx 1$) is then

$$T_{nl} = T_{eq}\delta_{iso} \tag{C.3}$$

The density of a galaxy remains roughly equal to the value of the density of the Universe when it goes non-linear[6,13]. Thus

$$\rho_{dm\,gal} = \left(\frac{T_{nl}}{T_1}\right)^3 \rho_{dm\,o} \quad \propto \quad \rho_{dm\,o}^4 \delta_{iso}^3 \tag{C.4}$$

The density of baryons in a galaxy is (with n_B/s constant)

$$\rho_{B\,gal} \ \propto \ T_{nl}^3 \ \propto \ \rho_{dm\,o}^3 \delta_{iso}^3 \tag{C.5}$$

In the case of axions δ_{iso} is independent of $\delta\phi_e$. In the EIDM case $\delta_{iso} \propto \delta\phi_e^{-1}$. Thus we find (assuming $\rho_{dm} > \rho_B$)

$$\rho_{dm\,gal} \ \propto \ \delta\phi_e^5 \quad (\delta\phi_e^8) \tag{C.6}$$

$$\rho_{B\,gal} \ \propto \ \delta\phi_e^3 \quad (\delta\phi_e^6) \tag{C.7}$$

where for comparison we give in brackets the expressions for the axion case[13]. From this we see that if we look at a region of the Universe where for example $\delta\phi_e$ is ten times the value in our region, we find that in this region

$$\frac{\rho_{dm\,gal}}{\rho_{dm\,gal\,U}} = 10^5$$

$$\frac{\rho_{B\,gal}}{\rho_{B\,gal\,U}} = 10^3$$

where the subscript U denotes our Universe. So a small increase in $\delta\phi_e$ results in galaxies with much higher baryon and dark matter densities than in our Universe. It is then argued (although it is difficult to prove) that such dense galaxies will result in a physical enviroment which will be unfavourable to the evolution of life[13]. It follows from the above arguement that there should be some upper limit

to the value of $\delta\phi_e$ for which life can evolve. The anthropic principle arguement amounts to saying that we are most likely to exist in a Universe with $\delta\phi_e$ near that upper limit. (Anthropically acceptable regions of the Universe with smaller $\delta\phi_e$ are less likely if ϕ at the end of inflation can take a random value in a given domain, since there will then be more domains with $\delta\phi_e$ around its largest possible value, with the probability being proportional to $\delta\phi_e$ [13]).

It is worth emphasizing that this anthropic selection, whilst relying on assumptions regarding the probability of life forming within galaxies, is natural if a domain structure with smooth $\delta\phi_e$ in regions the size of the observable Universe occurs in the EIDM model. This scenario also has the advantage of allowing the possibility of the similarity of ρ_B and ρ_{dm} in the observed Universe being understood as a result of anthropic selection. This is because the most likely domain will have $\rho_{dm} \gtrsim \rho_B$ if the domain with the largest $\delta\phi_e$ permitted by anthropic selection is the most probable, since anthropic selection plays a role in selecting $\delta\phi_e$ only when $\rho_{dm} \gtrsim \rho_B$. Therefore the most probable ratio of baryons to dark matter in a galaxy will occur when ρ_{dm} is of the order of ρ_B if it should be the case that smaller ratios of baryons to dark matter, corresponding to larger $\delta\phi_e$, are anthropically disfavoured. As discussed above, because the galaxies that result for larger $\delta\phi_e$ are much more dense, it is plausible that this anthropic selection does occur.

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Table 1a. Constraints on η for $f_r = 0.1$

ω	η_u	η_l	$\eta_l^{'}$	$m_{max}\ (GeV)$
10^4	$2.6x10^{-3}$	$7.2x10^{-4}$	$3.8x10^{-8}$	$2.5x10^{-10}$
1000	$2.4x10^{-3}$	$8.9x10^{-4}$	$3.0x10^{-7}$	$3.1x10^{-11}$
500	$2.3x10^{-3}$	$9.1x10^{-4}$	$6.0x10^{-7}$	$1.6x10^{-11}$
250	$2.1x10^{-3}$	$9.0x10^{-4}$	$1.2x10^{-6}$	$7.9x10^{-12}$
100	$1.5x10^{-3}$	$7.5x10^{-4}$	$3.0x10^{-6}$	$2.6x10^{-12}$
50	$8.7x10^{-4}$	$5.0x10^{-4}$	$6.0x10^{-6}$	$8.8x10^{-13}$
25	$3.1x10^{-4}$	$2.2x10^{-4}$	$1.2x10^{-5}$	$1.9x10^{-13}$
11.0	$2.6x10^{-5}$	$2.6x10^{-5}$	$2.6x10^{-5}$	$1.0x10^{-14}$
5.0	$6.1x10^{-6}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		
3.0	$6.1x10^{-6}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		

Table 1b. Constraints on η for $f_r = 1$

ω	η_u	η_l	$\eta_l^{'}$	$m_{max}\ (GeV)$
10^4	$2.6x10^{-3}$	$9.3x10^{-4}$	$3.0x10^{-7}$	$3.2x10^{-11}$
1000	$2.4x10^{-3}$	$1.1x10^{-3}$	$3.0x10^{-6}$	$4.0x10^{-12}$
500	$2.3x10^{-3}$	$1.2x10^{-3}$	$6.0x10^{-6}$	$2.1x10^{-12}$
250	$2.1x10^{-3}$	$1.2x10^{-3}$	$1.2x10^{-5}$	$1.0x10^{-12}$
100	$1.5x10^{-3}$	$9.6x10^{-4}$	$3.0x10^{-5}$	$3.4x10^{-13}$
50	$8.7x10^{-4}$	$6.5x10^{-4}$	$6.0x10^{-5}$	$1.1x10^{-13}$
25	$3.06x10^{-4}$	$2.77x10^{-4}$	$1.2x10^{-4}$	$2.4x10^{-14}$
18.7	$1.55x10^{-4}$	$1.55x10^{-4}$	$1.55x10^{-4}$	$1.0x10^{-14}$
15	$8.2x10^{-5}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		
10	$1.7x10^{-5}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		
5	$6.1x10^{-5}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		
3	$6.1x10^{-5}$	*RULED OUT BY BUBBLE FLUCTUATIONS*		