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$e_L \leftrightarrow e_R$ Transitions due to Thermal Magnetic Field Fluctuations in the Early Universe and the Upper Bound on Majorana Neutrino Masses from B-L Conservation.

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Abstract

It is shown that due to thermal fluctuations of the value of the gauge fields and the magnetic fields generated by their gradients, $e_L \leftrightarrow e_R$ transitions will occur at a rate similar to the rate of electron scatterings from the thermal bath of particles in the early Universe. As a result, the original upper bound on Majorana neutrino masses from the requirement that a B-L asymmetry can survive, $m_\nu \lesssim 1$eV, should be imposed, rather than the weaker upper bound, $m_\nu \lesssim 10$keV, recently suggested by Cline, Kainulainen and Olive on the basis of the low rate of $e_L \leftrightarrow e_R$ transitions mediated by Yukawa interactions.

Over the past few years it has become accepted that in the $SU(3)_C x SU(2)_L x U(1)_Y$ Standard Model (SM), sphaleron fluctuations combined with the B+L anomaly lead to B+L violation being in thermal equilibrium for temperatures $T_{EW} \lesssim T \lesssim 10^{12}$GeV (where $T_{EW} \approx 10^2$GeV is the temperature of the electroweak phase transition), leaving only B-L as a conserved quantity[1,2]. There have been several attempts to generate the observed B asymmetry during the electroweak phase transition in both the SM and its extensions[3]. However, the SM itself seems unable to provide sufficient CP violation during the phase transition to generate the observed B asymmetry[2,4] and in addition appears to be unable to prevent the subsequent washing out of the B asymmetry by B+L violating interactions[5]. If the B asymmetry cannot be generated during the electroweak phase transition, then it will be necessary for a B-L asymmetry to exist prior to the electroweak phase transition, in order to account for the observed baryon asymmetry.

However, if additional sources of B or L violation are introduced into the SM, there is the danger that even a B-L asymmetry could be washed out, making it impossible to account for the baryon asymmetry by way of any B or L asymmetry produced prior to the electroweak phase transition. Such a source of L violation is a Majorana mass for the neutrino, which in the SM originates from an effective interaction of the form LLHH, where L is the lepton doublet and H is the Higgs doublet. Requiring that L violation is out of thermal equilibrium for $10^2\text{GeV} \lesssim T \lesssim 10^{12}\text{GeV}$ leads to an upper bound on the Majorana neutrino mass[6,7], $m_\nu \lesssim 1\text{eV}$, which we refer to as the Fukugita-Yanagida (FY) bound in the following. Recently, Cline, Kainulainen and Olive (CKO) [8] have suggested that this bound is too restrictive. They point out that the B+L violating interactions act only on the left-handed fermions, and that because of the small value of the electron Yukawa coupling, the rate of $e_L \leftrightarrow e_R$ transitions is small. As a result, they come into thermal equilibrium only at a relatively low temperature, $T_* \approx 10\text{TeV}$. Therefore, it is possible for an L asymmetry in e_R to survive so long as the $e_L \leftrightarrow e_R$ transitions are out of thermal equilibrium, which is true for $T > T_*$. Once $T < T_*$, the e_R asymmetry can be converted to an e_L asymmetry via $e_L \leftrightarrow e_R$ transitions and finally to a B asymmetry by B+L violating interactions. Thus in this case the L violation due to the Majorana neutrino masses can remain in thermal equilibrium down to $T_* \ll 10^{12}\text{GeV}$, which in turn allows for much larger Majorana neutrino masses than are allowed by the FY bound, with the upper bound given in general by $m_\nu \lesssim (10^{12}\text{GeV}/T_*)^{1/2}\text{eV}$.

In this letter we will consider an effect not considered in the discussion of CKO, namely the possibility that thermal fluctuations of the gauge fields will produce magnetic fields which will in turn produce rapid $e_L \leftrightarrow e_R$ transitions. We will show that, in general, $e_L \leftrightarrow e_R$ transitions remain in thermal equilibrium throughout the temperature range where B+L violation is in thermal equilibrium, so restoring the original FY upper bound on the Majorana neutrino mass.

At finite temperature, the fields corresponding to the gauge and Higgs particles in the heat bath of the early Universe can be roughly modelled by solutions of the equations of motion which correspond to massless non-interacting gauge and Higgs particles of energy and momentum $\approx T$, such that the total energy density is $\approx T^4$.

Thus if we consider N gauge or Higgs particles, we can write the physical components of the gauge fields W_μ and the Higgs field ϕ as

$$W_\mu \ , \ \phi \sim \sum_{a=1}^{N} A_a \ \mathrm{Sin}(k_a.x + \delta_a) \ , \qquad k_a^0 \approx |\underline{k}_a| \approx T \qquad (1),$$

where $\underline{k}_a$ has a random direction and δ_a is a random phase. (By an $SU(2)_L x U(1)_Y$ gauge transformation we can choose the value of the non-zero components of the Higgs doublet to correspond to $H = \binom{0}{\phi}$, with ϕ real). The requirement that the average energy density in the gauge or Higgs field is $\approx T^4$ then implies that $A_a \approx \frac{T}{\sqrt{N}}$, which in turn implies, on summing over a, that W_μ and ϕ have fluctuations of amplitude $\approx T$ and size $\delta x \approx T^{-1}$. In the presence of the non-zero Higgs field the $SU(2)_L x U(1)_Y$ symmetry will be broken to $U(1)_{em}$ and the electron will gain a mass $m_e(T) \approx m_e \left(\frac{T}{v}\right)$, where $v = 175\mathrm{GeV}$ is the zero temperature value of the Higgs vacuum expectation value and m_e is the $T = 0$ electron mass. In the following we wish to consider the effects of the thermal background $U(1)_{em}$ gauge field on electrons passing through the thermal fluctuations.

In the presence of a space-dependent $U(1)_{em}$ gauge field there will be electric and magnetic fields. Indeed, the thermal energy density of the photons at finite temperature can be interpreted as the energy density in fluctuations of the electric and magnetic fields, since $\rho_\gamma \equiv \frac{1}{2} < \underline{E}^2 + \underline{B}^2 >$ (the energy density of the photons at T being given by $\rho_\gamma = \frac{\pi^2}{15} T^4$), where $< ... >$ denotes the thermal average. From this and also from (1), by considering the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for an A_μ field changing by $\sim T$ over a distance $\delta x \approx T^{-1}$, we see that the magnitudes of the $\underline{E}$ and $\underline{B}$ field fluctuations are given by $|\underline{E}| \approx |\underline{B}| \approx T^2$. In the following we will use a simplified model of this thermal fluctuation by considering a region of width $\delta x = T^{-1}$ with a constant magnitude for the electric and magnetic fields $|\underline{E}| = |\underline{B}| = T^2$, and with an electron mass due to the Higgs field fluctuation given by $m_e(T) \approx m_e(\frac{T}{v})$.

We next wish to consider the probability that an electron, initially in a pure e_L state, will precess to an e_R state as it crosses the thermal fluctuation. We do this by approximately solving the Dirac equation for an electron in a background A_μ field. The quadratic form of the Dirac equation is given by[9],

$$\left[(i\partial_\mu - eA_\mu)^2 - \frac{e}{2}\sigma^{\mu\nu}F_{\mu\nu} - m_e^2(T)\right]\Psi = 0 \qquad (2),$$

where e is the electromagnetic coupling constant and Ψ is the electron wavefunction. We consider a fluctuation in a region with coordinates $-\delta x/2 \leq x_i \leq \delta x/2$ (i=1,2,3), and we take the magnetic field to be constant and the electric field to be zero. (The $\underline{E}$ field will produce effects only of the same order of magnitude as those of the $\underline{B}$ field, and so will not alter the order of magnitude of our results). The A_μ field can then be taken to be $A_\mu = (0, -B_2x_3, B_1x_3 - B_3x_1, 0)$. In order to solve (2) we use the standard approach as used in discussing neutrino oscillations, and consider a plane wave propagating in the z direction[10]. We substitute $\Psi = e^{iE_e(t-z)}U(z)$ and

A_μ into (2), where $E_e \approx 3.2T$ is the mean electron energy at temperature T, and obtain (with the spinor in the Dirac representation),

$$i\left[\frac{d}{dz} + \frac{i}{2E_e}\frac{d^2}{dz^2} - \frac{i}{2E_e}e^2(A_1^2 + A_2^2)\right]U(z) = -\frac{m_e^2(T)}{2E_e}U(z) - \frac{e}{2E_e}\begin{bmatrix} -\underline{\sigma}.\underline{B} & 0 \\ 0 & -\underline{\sigma}.\underline{B} \end{bmatrix}U(z) \quad (3).$$

The $\underline{\sigma}.\underline{B}$ terms relate the spin $\pm$ 1/2 components of the electron. From (3) we obtain the equation determining the evolution of the e_L and e_R states,

$$i\frac{d}{dz}\begin{pmatrix} e_R \\ e_L \end{pmatrix} = \begin{bmatrix} -\frac{m_e(T)^2}{2E_e} + \mu B_3 & \mu B_1 + i\mu B_2 \\ \mu B_1 - i\mu B_2 & -\frac{m_e(T)^2}{2E_e} - \mu B_3 \end{bmatrix}\begin{pmatrix} e_R \\ e_L \end{pmatrix} \quad (4),$$

where $\mu = \frac{e}{2E_e}$, and we have assumed that $|2E_e\partial_z U(z)|$ is large compared with $|\partial_z^2 U(z)|$ and $e^2(A_1^2 + A_2^2)$. On average we expect $|B_1| \approx |B_2| \approx |B_3| \approx T^2/\sqrt{3}$, corresponding to $\underline{B}^2 = T^2$. Since $\frac{m_e^2(T)}{2E_e} \approx \frac{1}{6}(\frac{m_e}{v})^2 T \ll \mu B_i \approx \frac{e}{6\sqrt{3}}T$, we can neglect the mass term. Also, since $|\partial_z U| \approx |\mu B_3 U| \approx |\frac{eT}{6\sqrt{3}}U|$, we find that $|2E_e\partial_z U(z)| \approx eT^2/\sqrt{3}$, $|\partial_z^2 U(z)| \approx e^2T^2/108$ and $e^2(A_1^2 + A_2^2) \lesssim e^2T^2/2$ (where we have used $|x_i| \lesssim (2T)^{-1}$ within the region of the fluctuation), and so the approximations made in arriving are (4) are valid for e small compared to 1. Solving (4) for the probability of an electron initially in a pure e_L state at $z = 0$ being in an e_R state at z then gives,

$$P_{L\to R} = \left(\frac{B_1^2 + B_2^2}{\underline{B}^2}\right)\ \mathrm{Sin}^2(\mu|\underline{B}|\ z) \quad (5).$$

With $\delta z \approx T^{-1}$, we find that $\mu|\underline{B}|\ \delta z \approx e/6$. Thus on crossing the fluctuation region we find, with $e \approx 0.3$,

$$P_{L\to R} \approx \frac{e^2}{54} \approx 2\mathrm{x}10^{-3} \quad (6).$$

Since this is due to a single fluctuation region, we expect that this will give a lower bound on $P_{L\to R}$. The rate of measurement of the state of the electron is given by the rate of thermal scatterings with particles in the heat bath[11]. We then obtain from (6) a lower bound on the rate of $e_L \leftrightarrow e_R$ transitions,

$$\Gamma_{L\leftrightarrow R} = P_{L\to R}\Gamma_{wk} \gtrsim 2\mathrm{x}10^{-3}\Gamma_{wk} \quad (7).$$

In this we have taken the rate of scattering of the electrons from particles in the heat bath to be approximately equal to the rate of weak interaction mediated scatterings $\Gamma_{wk} \approx \alpha_W^2 T$, where $\alpha_W = \frac{g^2}{4\pi} \approx 3\mathrm{x}10^{-2}$. The $e_L \leftrightarrow e_R$ transitions will come into thermal equilibrium at a temperature T_{eq} corresponding to $\Gamma_{L\leftrightarrow R} \approx H$, where H is the expansion rate of the Universe. Using (7) we then obtain a lower bound on T_{eq}, $T_{eq} \gtrsim (2\mathrm{x}10^{-3}\ \alpha_W^2/17)M_{Pl} \approx 1\mathrm{x}10^{12}$GeV. We can also roughly estimate the actual value of T_{eq}. We see from (4) that the electron wavefunction will gain a component of e_R of amplitude proportional to $\mu(B_1 + iB_2)\delta z$ after crossing a fluctuation of width δz. The phase of this contribution depends on B_1 and B_2 and is random. Thus each fluctuation will contribute to a random walk of the amplitude in

the complex plane. Thus after crossing N fluctuations, the net amplitude of e_R will have a magnitude roughly $\sqrt{N}$ times the magnitude of the average amplitude gained from one fluctuation, and so $P_{L\to R}$ will increase roughly by a factor of N. Since the electron will cross O(1000) thermal fluctuation regions before it undergoes a weak scattering, we expect that $P_{L\to R}$ will reach its maximum possible average value of 0.5, in which case $T_{eq} \approx 3\text{x}10^{14}$GeV. We conclude that L violating interactions should be out of thermal equilibrium during the whole time that B+L violation is in thermal equilibrium if a B-L asymmetry is to survive, in which case the original FY upper bound on the Majorana mass of the neutrino is restored.

A possible objection to the above is that we have considered a solution of the Dirac equation for the case of a plane wave crossing a constant magnetic field over a distance $\delta z \approx T^{-1}$, but with no change in the magnetic field in the x or y direction, whereas in the real situation the magnetic field will change over a distance δx , $\delta y \approx T^{-1}$. The approximation of ignoring the variation in the x and y directions will be good if the electron wavelength is small compared with the distance over which the variation occurs. In our case the electron wavelength is $E_e^{-1} \approx (3T)^{-1}$, so we don't expect the approximation to be a very good one. However, we expect that the order of magnitude of $P_{L\to R}$ will not be altered significantly. A second point is that we have not considered a thermo-field dynamics (TFD) calculation of the electron propagating at finite temperature[11,12], but rather considered directly the interaction of the electron with the gauge and Higgs fields of the heat bath. The TFD calculation gives the properties of a quasiparticle corresponding to the electron propagating in the plasma of the heat bath[12]. However, since we are considering distance and time scales $\lesssim T^{-1}$, corresponding to the size of the thermal fluctuations, such a thermodynamical calculation is not appropriate for the case of interest to us here, and a direct dynamical analysis of the electron propagating in the heat bath of gauge and Higgs particles is probably the best approach.

In conclusion, we have shown, on the basis of a heuristic but we believe physically correct arguement, that the interaction of electrons with thermal fluctuations of the gauge fields in the standard model will cause rapid $e_L \leftrightarrow e_R$ transitions, implying that the original Fukugita-Yanagida upper bound on the mass of a Majorana neutrino in the Standard Model, $m_\nu \lesssim 1$eV, is the correct upper bound.

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