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Filtering in Inversion for Solar Internal Structure

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1 Introduction

The Sun oscillates in many thousands of different modes. The frequencies of these modes are related to the internal structure of the Sun, in particular to sound-speed profile. Hence, the solar oscillation frequencies can be inverted to determine the internal structure of the Sun to a very high precision (e.g. Gough & Thompson 1991). Theoretically, both acoustic waves (p modes) and buoyancy waves (g modes) can be present in the Sun. However, only p modes have been definitely detected so far. Modes of stellar oscillation are characterized by the degree l and azimuthal order m of the spherical harmonic which describes the behaviour of the mode over spherical surfaces, as well as the radial order n. The observed solar modes can be described throughout most of the solar interior by the equations describing linear adiabatic oscillations (cf. Unno et al. 1989), as the superposition of acoustic waves, each travelling in a resonant cavity, with a lower turning point at a depth determined by the degree and frequency of the mode and an upper turning point at a depth just beneath the photosphere.

Unlike the Earth, the structure of the Sun is normally assumed to be sufficiently simple that realistic solar models can be computed from first principles. Matter in the Sun behaves like a gas with only modest interactions between the constituents; thus the ideal gas law, with relatively small modifications, provides an adequate description of the thermodynamic state. In much of the solar interior, energy is transported by radiative diffusion, with a diffusion coefficient which may be computed from atomic physics. By contrast, in the convection zone – which occupies the outer ~ 28 per cent by radius – energy is transported by convection and the temperature gradient is essentially adiabatic (except in a very thin region near the top of the convection zone). Finally, the rates of energy generation and nuclear transmutation are determined by nuclear parameters which are either measured or computed with reasonable accuracy. On the basis of this description of the physics of the solar interior, models of the present Sun can be computed by following solar evolution, resulting from the fusion of hydrogen into helium, from an assumed initially chemically homogeneous model to the present solar age. Given such a model, it is straightforward to compute its

adiabatic oscillation frequencies. By linearising the equations of stellar oscillation around the model, the differences between the structure of the Sun and this model can be determined through inversion of the frequency differences between the Sun and the model. This provides a test of the physics and other assumptions used in the computation. Such procedures have been used extensively to invert for solar structure using the oscillation frequencies (e.g., Gough & Kosovichev 1988, 1990, 1993; Dziembowski, et al. 1990; Däppen et al. 1991; Antia & Basu 1994; Dziembowski et al. 1994).

Unfortunately, although the above procedure apparently represents the Sun's structure and the mode dynamics well throughout most of the solar interior, we run into problems near the surface: here uncertain details of convective energy transport and corresponding dynamical effects should be taken into account; also, the assumption of adiabaticity of the oscillations breaks down, with no definite way of treating the nonadiabatic effects being currently available. Thus we are faced with the challenge of solving an inverse problem, without knowing fully how to solve the forward problem.

Fortunately, although we cannot completely solve the realistic forward problem, we can characterize the effect of the outer layers on the mode frequencies. Our aim is then to invent an inversion procedure the results of which are insensitive to uncertainties of the form of those arising from the superficial layers. Comparison of theoretical p-mode frequencies of different models shows that, when appropriately scaled, the frequency differences for low- to moderate-degree modes due to changes in the surface structure are largely a function of frequency (e.g. Christensen-Dalsgaard & Berthomieu 1991). This comes about because such p modes travel almost vertically in the superficial layers of the Sun, regardless of the degree of the mode; hence the signature of these layers on the frequencies depends little on degree, except for an inverse proportionality to the inertia of the modes. (Shallowly penetrating modes have a small inertia and their frequencies are easily perturbed by near-surface changes; while deeply penetrating modes have a large inertia and are relatively insensitive to changes in the surface layers.) Changes in the equilibrium structure of a solar model introduce an oscillatory signal in the frequencies, which for modes of a given degree can be shown to have a 'frequency' which increases with the acoustic depth τ of the layers in which the signals arise (Gough 1990; Christensen-Dalsgaard & Pérez Hernández 1992). Hence, the effect of unknown physics in the surface layers of the Sun is expected to be the introduction of frequency shifts which are slowly varying functions of frequency.

These arguments indicate that the inverse problem of finding the structure of the solar interior can be written as

$$\frac{\delta\omega_i}{\omega_i} = \int K_i^{(c,\rho)}(r)\frac{\delta c^2(r)}{c^2(r)}\mathrm{d}r + \int K_i^{(\rho,c)}(r)\frac{\delta\rho(r)}{\rho(r)}\mathrm{d}r + \frac{F_{\mathrm{surf}}(\omega_i)}{S_i} \ , \qquad (1)$$

where $\delta\omega_i$ is the difference in the frequency ω_i of the ith mode between the solar data and a reference model, and c and ρ are, respectively, sound speed and density in the interior. The kernels $K_i^{(c,\rho)}$ and $K_i^{(\rho,c)}$ are known functions of the reference model which relate the changes in frequencies of adiabatic oscillations

to the changes in c and ρ respectively. The term in $F_{\rm surf}$ results from the near-surface errors in the physics; thus we assume that $F_{\rm surf}$ is a slowly varying function of frequency, with S_i being proportional to the inertia of the mode (cf. Christensen-Dalsgaard 1991).

Although techniques have been developed to remove the surface term during inversion, these depend on the technique of inversion used. In the regularised least-squares technique, for instance, the function $F_{\rm surf}(\omega)$ is obtained simultaneously with the functions $\delta c^2(r)/c^2(r)$ and $\delta\rho(r)/\rho(r)$ in the least-squares fit to the data (cf. Dziembowski et al. 1990; Antia & Basu 1994). When using optimally localised averaging or its variants, where the inferred solution is obtained by forming explicitly linear combinations of the data, the surface term has been taken care of by constraining those linear combinations to be insensitive to the presence of slowly varying functions of frequency of the form given in equation (1) (e.g., Däppen et al. 1991; Kosovichev et al. 1992; Christensen-Dalsgaard & Thompson 1995). Since the surface term is not treated in the same way, it often becomes very difficult to compare the inversion results. We aim to present a way of filtering out the effects of near-surface uncertainties from the frequency differences, prior to the inversion. Once the surface term is removed, any standard inversion procedure can be applied. Since the method unavoidably also suppresses those components of $\delta\omega_i/\omega_i$ which arise from the actual differences near the surface in structure (i.e., $\delta c^2/c^2$ and $\delta\rho/\rho$ in equation 1), the corresponding contributions from the kernels $K^{(c,\rho)}$ and $K^{(\rho,c)}$ are also suppressed for consistency. We do this by constructing a filter which removes low-frequency components from the frequency differences while simultaneously suppressing the kernels at the surface.

2 Formulation

If we have a *low-pass* filter $F_{\alpha\beta}$ on a suitable frequency mesh $\{\omega_\alpha\}$, then we can construct a suitable filter $\mathcal{F}$ which can be applied directly to the frequency differences $\delta\omega/\omega$. The details of this procedure can be found in Basu et al. (1995).

By supposition, the low-pass filter has no effect on $F_{\rm surf}$. Hence, applying the filtering to (1) and subtracting the result from (1) we obtain

$$\sum_j \mathcal{G}_{ij}\frac{\delta\omega_j}{\omega_j} = \int\left[\tilde{K}_i^{(c,\rho)}(r)\frac{\delta c^2(r)}{c^2} + \tilde{K}_i^{(\rho,c)}(r)\frac{\delta\rho(r)}{\rho(r)}\right]\mathrm{d}r \ , \qquad (2)$$

where

$$\mathcal{G}_{ij} = (\delta_{ij} - \mathcal{F}_{ij})\frac{S_j}{S_i} \ , \quad \text{and} \quad \tilde{K}_i = \sum_j \mathcal{G}_{ij}K_j \ . \qquad (3)$$

Hence we achieve the desired consistent filtering.

The filter $F_{\alpha\beta}$ is specially formulated to have a sharp cut-off frequency. The resulting filter $\mathcal{G}$ suppresses all signals arising from the near surface region with $r \geq 0.995\ \mathrm{R}_\odot$. The details of the construction of the filter F can be found in Pérez Hernández & Christensen-Dalsgaard (1994).

The frequency response function of the high-pass filter $\mathcal{G}$ is shown in Fig. 1. We see that it indeed has a sharp cut-off frequency.

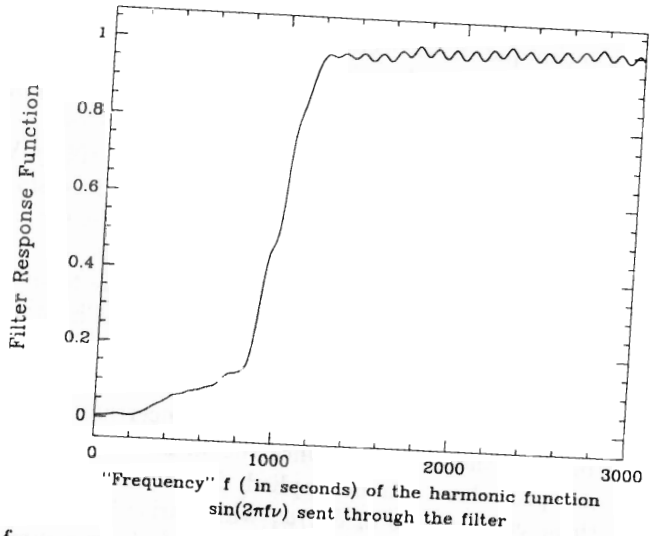

Fig. 1. The frequency response function of the filter $\mathcal{G}$ used in this work, defined as the normalized response of the filter to a harmonic function. Note that the filter has a fairly sharp cut-off frequency

We have used two solar models to test the method. In the test model, taking the role of the unknown Sun, the surface contributions to the frequencies have been artificially increased by modifying the atmosphere, and adding an extra frequency dependent component to the scaled frequencies. We use about 1900 modes for which observed solar frequencies are available. This enables us to use observational errors, although we invert artificial data. Most of the solar frequencies in this range have been measured with an accuracy of a few parts in 10^5. By using the observers' error estimates, we can get a realistic estimate of the accuracy with which we can invert helioseismic data.

3 The Inversion Technique

We use the method of subtractive optimally localised averages (henceforth SOLA; cf. Pijpers & Thompson 1994; Christensen-Dalsgaard & Thompson 1995), to carry out the inversions. Thus we choose coefficients $c_i(r_0)$ in such a way that the linear combination

$$\mathcal{K}_{c,\rho}(r_0, r) = \sum_i c_i(r_0) K_i^{(c,\rho)}(r) \tag{4}$$

is a well-localised averaging kernel at target radius r_0. In the SOLA technique this is achieved by minimizing the difference between $\mathcal{K}_{c,\rho}(r_0, r)$ and a target

kernel $\mathcal{T}(r_0, r)$. We have used target kernels of Gaussian shape. The minimization is carried out subject to the condition that the sum $\mathcal{C}_{c,\rho}(r_0, r) \equiv \sum_i c_i(r_0) K_i^{(\rho,c)}(r)$ and the error in the corresponding combination of the data are small (for details, see Christensen-Dalsgaard & Thompson 1995). In this case the sum $\left(\delta c^2/c^2\right)_{\rm inv} = \sum_i c_i \delta\omega_i/\omega_i$ provides a well-localised average of the first function $\delta c^2/c^2$. Normally in SOLA, the surface term is removed by putting an additional constraint that $\sum c_i \Phi_\lambda(\omega_i) = 0$ for $\lambda = 1, \ldots, \Lambda$, where Φ_λ is a polynomial of degree λ and Λ is the largest degree of polynomial used. However, if the surface term is removed prior to the inversion, then of course, this condition is not require.

In addition to the inversion results between two solar models, we also show the results of inverting real solar data. The data we used consist of low-degree ($l = 0$ to $l = 3$) frequencies from Elsworth et al. (1994) and data from Libbrecht et al. (1990) on intermediate- and high-degree modes in the range $l = 4$ to $l = 100$.

4 The Results

The effect of filtering the frequency differences is shown in Fig. 2. We can see that filtering dramatically reduces the spread in the scaled frequency differences by removing the smooth frequency-dependent components. The residual frequency differences are almost purely a function of $w = \omega/(\ell+1/2)$, which is a measure of the lower-turning point of the mode. Indeed it is largely this variation with w that the inversion uses to infer the variation with radius of the structural differences. The remaining frequency dependence are the higher frequency components than are not cut off by the filter: these correspond to differences situated close to the surface but beneath the very superficial layers whose effect we have sought to suppress.

The results of the inversion for the sound-speed difference between the two solar models are shown in Fig. 3. We show the results obtained both with and without filtering the data beforehand. As can be seen from the figure, the inversion results after filtering are better than those obtained without filtering.

The vertical error bars are an indication of the error in the inversion due to errors in the data. As can be seen, the errors are very small indeed. The horizontal error bars indicate the radial resolution of the inversion.

In Fig. 4, we show the relative sound-speed difference between the Sun and a reference solar model, in the sense (Sun minus model). The precision with which we are able to deduce the solar sound speed is evident. The reference model used for this inversion had been constructed with up-to-date physics including gravitational settling of helium. Note that the difference in sound speed between the Sun and the model is less than 0.5 % through most of the solar interior. The dip in the inferred difference around 0.7 $R_\odot$ indicates that the model has a slightly deeper convection zone than the Sun.

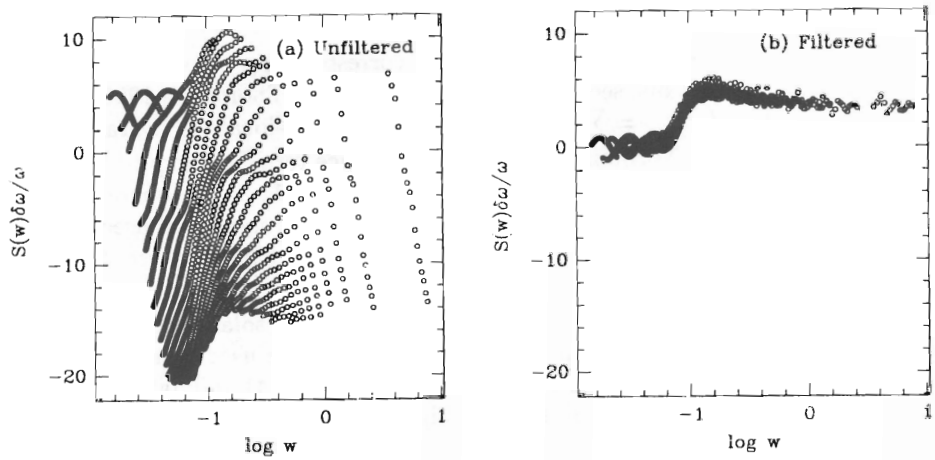

Fig. 2. The scaled relative frequency differences between the reference and test model in the sense test model minus reference model plotted as a function of $w(=\omega/(l+1/2))$. Panel (a) shows the original frequency differences, while panel (b) shows the filtered frequency difference. Note that the filtered scaled frequency differences are almost purely a function of w alone.

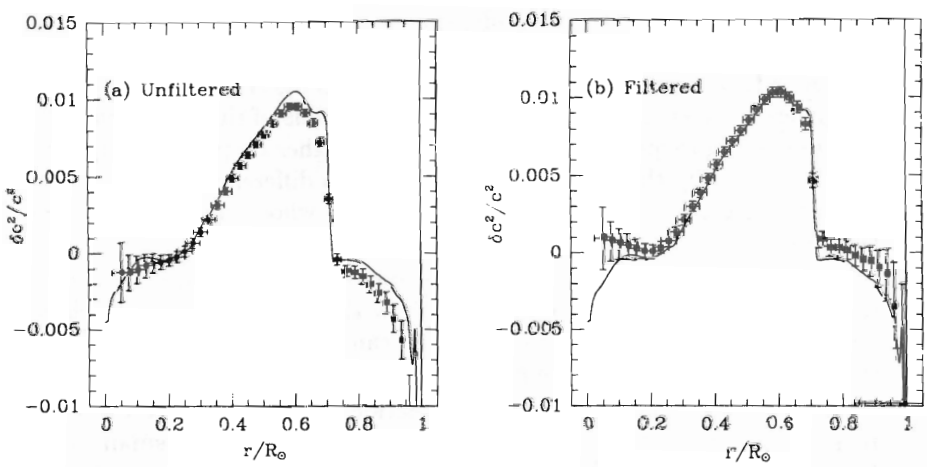

Fig. 3. The sound-speed inversion results for the test model. Panel (a) shows the results of inversion without removing the surface terms from the data, panel (b) shows the inversion results of the filtered data. In each panel the continuous line is the exact relative sound-speed difference between the two models. The points mark inversion results at each target radius. The vertical error-bars indicate the error in inversion due to the errors in the data. The horizontal error-bars are half the distance between the quartile points of the averaging kernel at that radius, and is an indication of the resolution of the inversion.

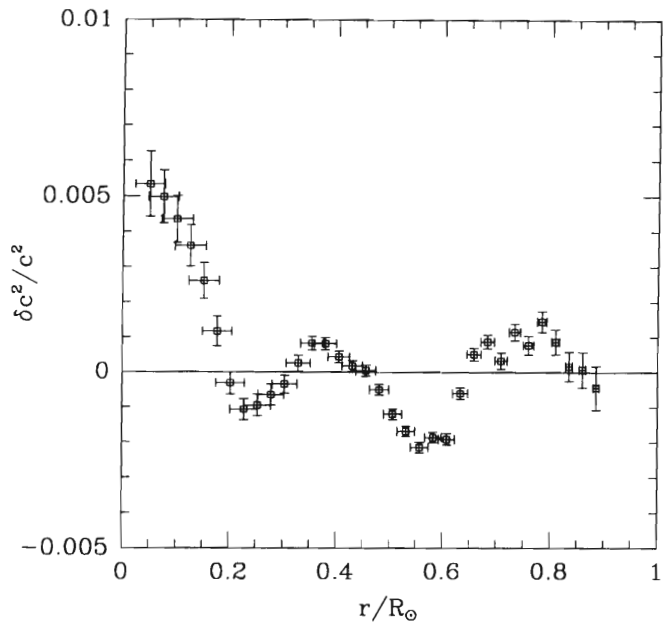

Fig. 4. The sound-speed inversion results for the Sun, relative to a solar model with up-to-date physics. The inversion used a combination of 1899 frequencies from Elsworth et al 1994 ($l = 0$ to $l = 3$) and from Libbrecht et al. 1990 ($l = 4$ to $l = 100$)

5 Conclusions

We have shown here that by applying suitable filters to the frequency differences and the mode kernels it is possible to preprocess the frequency differences prior to applying any of the standard inversion techniques in order to eliminate the frequency-dependent component which arises from the near surface uncertainties. In the process we also suppress the corresponding contributions from the kernels relating frequency differences to differences in structure. The results of inverting such pre-processed data compare very favourably with those obtained without removing the surface contributions.

Although other ad hoc methods have been developed to handle the surface terms while inverting helioseismic data (e.g., Dziembowski et al. 1990; Däppen et al. 1991), we believe that filtering out the surface contribution before doing the inversion provides us with the advantage of being able to compare directly the results obtained by different inversion techniques. The ease with which the filter response can be tuned to the problem at hand makes this approach very flexible.

We have also shown that we are able to invert real solar data to determine the sound speed within the Sun to high precision.

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