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Modelling solar-like oscillations in η Boo

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ABSTRACT

Kjeldsen et al. (1995, AJ in press) recently obtained strong evidence for solar-like oscillations in the G0 IV star η Boo. Here we present evolution calculations for the star and compare them with the observed oscillation frequencies. Models can be found that are consistent with the star's luminosity and effective temperature and with the large and small frequency separations. The calculations indicate that dipolar ($l = 1$) oscillations behaving like gravity modes in the interior of the star can be excited to observable amplitudes; the observations may show evidence for such modes.

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1. INTRODUCTION

The probable detection of solar-like oscillations in η Boo (Kjeldsen et al. 1995) provides an important opportunity to test stellar evolution theory. Each mode in an oscillating star is characterized by a radial order n and an angular degree l, and the mode frequencies are sensitive to conditions in the stellar interior (see Brown & Gilliland 1994 for a recent review). The frequencies ν_{nl} of low-degree modes satisfy the following asymptotic relation:

$$\nu_{nl} = \Delta\nu \left(n + \frac{l}{2} + \alpha + \frac{1}{4} \right) + \epsilon_{nl} \qquad (1)$$

(Vandakurov 1967; Tassoul 1980). The large frequency separation $\Delta\nu$ is approximately the inverse of the sound travel time across a stellar diameter, while α (which is itself a function of frequency) is determined by conditions near the stellar surface, and the small correction term ϵ_{nl} depends predominantly on conditions in the stellar core (e.g., Gabriel 1989; Christensen-Dalsgaard 1991).

Equation (1) predicts a series of oscillation frequencies with almost uniform separation. Identifying this structure was an essential part of the analysis by Kjeldsen et al. (1995), who determined the large frequency separation for η Boo to be $\Delta\nu = 40.3\,\mu$Hz. They also measured frequencies for thirteen individual modes in the range 750–950 μHz. In this *Letter* we compute models for η Boo and compare the calculated oscillation frequencies with the observations. For the present purpose, we assume that all frequencies identified by Kjeldsen et al. (1995) represent real stellar oscillations.

2. EVOLUTION CALCULATIONS

We have calculated stellar evolution sequences, starting from chemically homogeneous static ZAMS models. The basic computations were as described by Christensen-Dalsgaard (1982), using a mesh with 600 points. We used the Eggleton, Faulkner & Flannery (1973) equation of state and Livermore OPAL opacities (e.g., Iglesias, Rogers & Wilson 1992), matched smoothly, near $T = 10^4$ K, to the low-T opacities of Kurucz (1991). Nuclear reaction parameters were taken from Parker (1986); ^{3}He and the CN part of the CNO cycle were assumed to be in nuclear equilibrium, whereas we followed the gradual conversion of ^{16}O into CN elements with evolution. Convection was treated using the Böhm-Vitense (1958) formulation of mixing-length theory. The ratio of the mixing length to the pressure scale height was taken to be $\alpha_\odot = 1.8337$, obtained from calibrating the corresponding solar model.

We adopt the same parameters for η Boo as Kjeldsen et al. (1995): $L = 9.5 \pm 0.7\,L_\odot$, $T_{\rm eff} = 6050 \pm 60$ K and a logarithmic iron abundance, relative to the solar value, of [Fe/H] $\simeq 0.19$. The composition may be peculiar, with significant overabundances of C, N, O, Na, Mg, Al and Si relative to Fe (Clegg, Lambert & Tomkin 1981; Tomkin, Lambert & Balachandran 1985; Edvardsson et al. 1993). The star is a spectroscopic binary with a low-mass companion (Bertiau 1957); if the companion was originally the more massive star mass transfer in the system might explain the observed abundance anomalies. However, the IUE spectrum of η Boo shows no evidence for a white dwarf companion (Cappelli et al. 1989). For the present, we assume the relative composition of the heavy elements in η Boo to be solar; from the quoted value of [Fe/H] we then obtain a heavy-element abundance by mass of $Z = 0.03$. The hydrogen abundance was chosen to be $X = 0.7$.

Evolution tracks computed using these parameters are shown as solid lines in Figure 1 for three masses. These tracks approximately span the error box in ($T_{\rm eff}$, L); they imply that η Boo is in the shell hydrogen-burning phase. To investigate the sensitivity of the models to the assumed parameters, we have computed two additional sets of evolution sequences. In the first, α_c was reduced to 1.5 and all other param-

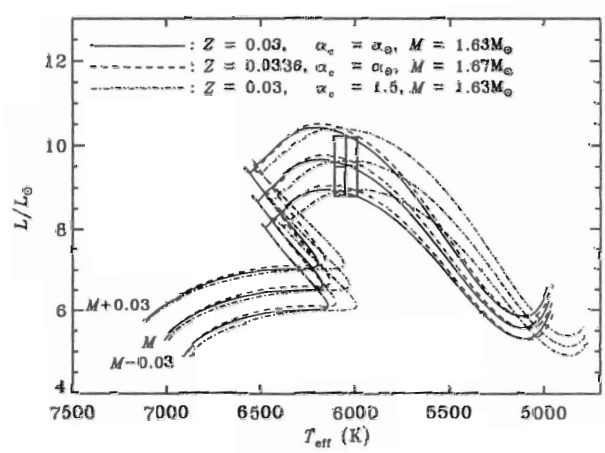

Fig. 1.— Evolutionary tracks in the HR diagram. The error box indicates the observed value of $T_{\rm eff}$ and L.

eters were unchanged. As indicated by the dot-dashed lines in Figure 1, this caused relatively small changes in the region of the observed $(T_{\rm eff}, L)$; in particular, the preferred range of masses is unchanged. However, for more evolved models there is a significant shift towards larger radius, and hence lower effective temperature.

In the second set of sequences, Z was increased to 0.0336, corresponding to an increase of 0.05 in [Fe/H]. This required an increase in mass by $0.04\,M_\odot$ to match the observed location in the HR diagram (dashed lines in Figure 1). These tracks are very close to the original set, indicating that the mass increase almost compensates for the increase in Z.

3. ADIABATIC OSCILLATION FREQUENCIES

Adiabatic oscillations were calculated as described by Christensen-Dalsgaard & Berthomieu (1991). The models were transferred to a 2400-point mesh suitable for resolving the structure of the eigenfunctions. The frequencies were obtained by Richardson extrapolation from solutions obtained with 1200 and 2400 points.

We first consider the large separation. In Figure 2 lines of constant $\Delta\nu$ are superposed on a close-up of the HR diagram. For clarity, only the $Z = 0.03$ models are illustrated. Clearly, models can be found that are consistent both with the location $(T_{\rm eff}, L)$ of η Boo in the HR diagram and with the observed $\Delta\nu$. Comparison of the solid and the dot-dashed lines shows that $\Delta\nu$ is sensitive to changes in α_c at given $(T_{\rm eff}, L)$, reflecting the changes in the internal structure of the star. This indicates the value of observed frequencies as additional constraints on the structure of stellar models. In contrast, we have found that changing Z has little effect on $\Delta\nu$ at fixed location in the HR diagram.

Equation (1) is valid only to the extent that the modes behave like trapped acoustic modes (p modes). This is largely true for the five-minute oscillations in the Sun, but in a more evolved star such as η Boo, additional effects become significant. The steep maximum in the local buoyancy frequency at the edge of the helium core gives rise to modes behaving like trapped internal gravity waves (so-called g modes; e.g., Dziembowski & Pamyatnykh 1991; Ausdard, Provost and Christensen-Dalsgaard 1994). The core contracts as the star evolves, leading to an increase in the buoyancy frequency and hence in the frequencies of the g modes. When the freqnency of a g mode approaches that of a p mode, the two modes undergo an avoided crossing (e.g., Aizenman, Smeyers & Weigert 1977) where they exchange physical nature. This is illustrated in Figure 3, which shows frequency as a function of $T_{\rm eff}$, along the $1.6\,M_\odot$ evolution track. Radial modes ($l = 0$), shown by dashed lines, closely follow equation (1); the decrease in frequency with age reflects the increasing stellar radius. The same trend is seen for modes with $l = 1$ (solid lines), but here we also see the effect of the additional set of g modes, whose frequencies increase with age. At the avoided crossings the modes take on mixed characters between pure p and pure g modes. We found a similar frequency behavior for $l = 2$ and 3; here, however, the density of g modes is higher and the g modes are trapped more efficiently in the stellar interior, leading to weaker coupling with the p modes, a smaller minimum frequency spacing at the avoided crossings and smaller expected surface amplitudes.

We can estimate the oscillation amplitudes as follows. The kinetic energy of a mode can be expressed as $A^2\mathcal{E}$, where A is the surface amplitude, and the normalized energy $\mathcal{E}$ is an integral over the model that involves the square of the mode eigenfunction, normalized to unit surface amplitude. If we assume that the modes are excited stochastically by convec-

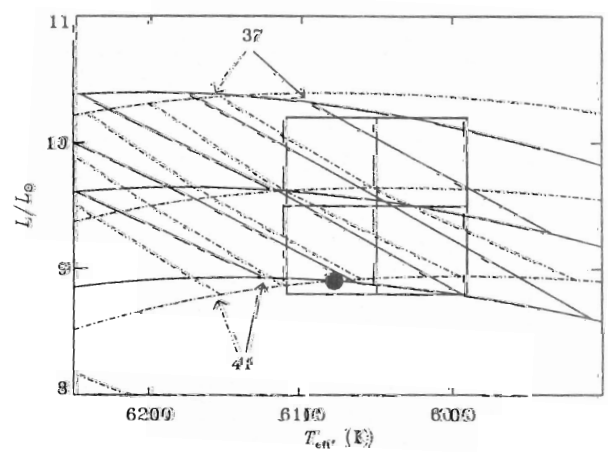

Fig. 2.— Close-up of the HR diagram in the vicinity of η Boo. Evolution tracks for $Z = 0.03$ are shown for $\alpha_c = \alpha_\odot$ (solid lines) and $\alpha_c = 1.5$ (dot-dashed lines), for masses 1.60, 1.63 and $1.66\,M_\odot$. The diagonal lines, using the same line styles, show contours of constant $\Delta\nu$ (37, 38, ..., 41 μHz), labeled in a few cases by $\Delta\nu$ (in μHz). The filled circle indicates the model illustrated in Figure 4.

tion (e.g., Houdek et al. 1994), the mode energy is likely to be independent of degree, at fixed frequency. It follows that the amplitude A_{nl} of a mode of degree l, order n and normalized energy $\mathcal{E}_{nl}$ satisfies

$$\frac{A_{nl}}{A_0(\nu_{nl})} \simeq \left[\frac{\mathcal{E}_{nl}}{\mathcal{E}_0(\nu_{nl})}\right]^{-1/2} , \qquad (2)$$

where $A_0(\nu)$ and $\mathcal{E}_0(\nu)$ are obtained by interpolating to frequency ν in the results for radial modes.

The fine structure in the observed and computed frequency spectra may be analyzed by means of an *echelle diagram* (e.g., Grec et al. 1983). Here, the frequencies are reduced modulo $\Delta\nu$ by expressing them as

$$\nu_{nl} = \nu_0 + k\Delta\nu + \tilde{\nu}_{nl} , \qquad (3)$$

(where ν_0 is a suitable reference frequency and k is an integer), such that $\tilde{\nu}_{nl}$ is between 0 and $\Delta\nu$. In the echelle diagram $\nu_0 + k\Delta\nu$ is plotted against $\tilde{\nu}_{nl}$. The result is shown in Figure 4. The open symbols are for a $1.6\,M_\odot$ model that was chosen to have $\Delta\nu \simeq 40.3\,\mu$Hz; the reference frequency was $\nu_0^{(\mathrm{mod})} = 856\,\mu$Hz. The sizes of the symbols have been scaled by the amplitude ratio A_{nl}/A_0 determined by equation (2).

The model results for $l = 0$, 2 and 3 clearly reflect the behavior predicted by equation (1). In particular, the points for $l = 0$ and 2 run parallel, with a small separation $\delta\nu_{02} \simeq 3.3\,\mu$Hz resulting from the difference in ϵ_{nl}. For $l = 1$, equation (1) predicts an almost vertical series of points separated by roughly $\Delta\nu/2$ from those for $l = 0$. The model frequencies clearly deviate from this. Comparison with Figure 3 (where the location of this model is indicated by a vertical solid line) indicates that this behavior is associated with the avoided crossings, which change the frequency separation and therefore shift the frequencies relative to the location expected from p-mode asymptotics. Remarkably, the $l = 1$ modes that behave partly like g modes, with some trapping in the deep interior, still have sufficiently small normalized energy $\mathcal{E}$ that their estimated amplitudes are close to those of the pure p modes. (The figure shows a single exception: a mode at 730 μHz shifted almost to the $l = 0$ line, with somewhat reduced amplitude.) In contrast, the g modes of degree 2 and 3 are trapped quite efficiently in the deep interior; thus the estimated amplitudes are so small as to make the points virtually invisible in Figure 4.

The filled circles in Figure 4 show the frequencies observed by Kjeldsen et al. (1995), again plotted with $\Delta\nu = 40.3\,\mu$Hz, but with the reference frequency $\nu_0^{(\mathrm{obs})} = 846\,\mu$Hz. We can immediately identify modes

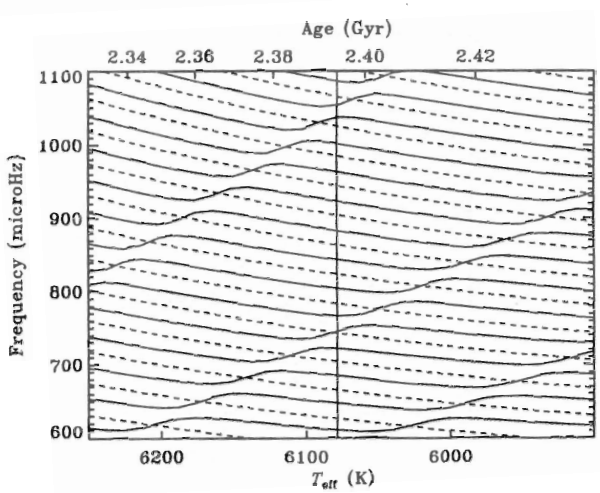

Fig. 3.— Evolution of adiabatic frequencies for a model of mass $1.60\,M_\odot$, with $Z = 0.03$ and $\alpha_c = \alpha_\odot$. The lower abscissa shows the effective temperature $T_{\rm eff}$, the upper abscissa the age of the model in Gyr. The dashed lines correspond to modes of degree $l = 0$, and the solid lines to $l = 1$. The vertical solid line indicates the location of the model whose frequencies are illustrated in Figure 4.

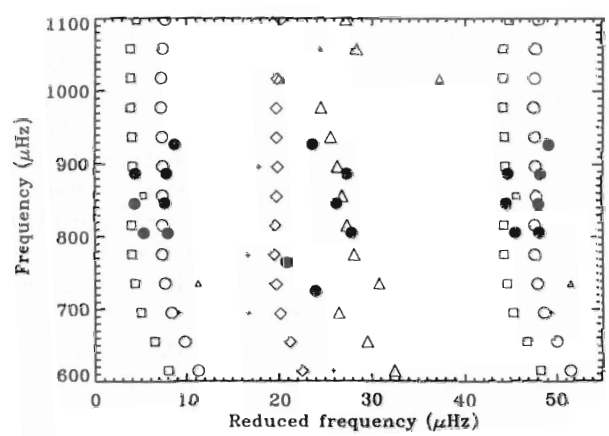

Fig. 4.— Echelle diagram with a frequency separation of $\Delta\nu = 40.3\,\mu$Hz, showing computed frequencies for a model with $M = 1.60\,M_\odot$ and $Z = 0.03$ (open symbols). Circles are used for modes with $l = 0$, triangles for $l = 1$, squares for $l = 2$ and diamonds for $l = 3$. The size of the symbols indicates the relative amplitude of the modes (see text); symbols that would otherwise be too small have been replaced by crosses. The filled circles show observed frequencies from Kjeldsen et al. (1995).

with degrees $l = 0$ and 2, and the small frequency separation found by Kjeldsen et al. ($\delta\nu_{02} = 3.1 \pm 0.3\,\mu$Hz) is in excellent agreement with the model value. We note that $\delta\nu_{02}$ is largely insensitive to the modifications of α_c and Z. The remaining six observed frequencies coincide quite well with $l = 1$ modes in the model and display an irregularity similar to the model frequencies (although differing in detail). Thus we appear to be seeing evidence in η Boo for avoided crossings involving g modes. Note, however, that some of the observed frequencies may arise from modes with $l = 3$.

4. CONCLUSIONS

The results reported here indicate that "classical" evolution models can account for most of the observed properties of η Boo, including important aspects of the oscillation spectrum observed by Kjeldsen et al. (1995). The location of the star in the Hertzsprung-Russell diagram identifies it as being in the post-main-sequence phase of evolution. Models can be found which are consistent with both the observed values of $T_{\rm eff}$ and L and the large frequency separation $\Delta\nu$. From this fit, the mass of the star is determined to be in the range 1.6–1.7 $M_\odot$. The calculated small frequency separation $\delta\nu_{02}$ between modes of degree 0 and 2 also agrees with the observations. A remaining difficulty is a shift of $\sim 10\,\mu$Hz between the observed and computed frequencies.

Interestingly, the computed frequencies of modes with $l = 1$ show irregularities induced by a resonance with a region just outside the stellar core, where a large peak in the buoyancy frequency causes the modes to behave locally as trapped gravity waves. A similar behavior is found in models of main-sequence stars (e.g., Dziembowski & Pamyatnykh 1991), although at substantially lower frequency. In the present case, the effect occurs in the frequency range where modes are likely to be excited stochastically by turbulent convection; furthermore, the resonance does not lead to a substantial decrease in the surface amplitude predicted from this process. This indicates that the core structure may have strong observable effects on frequencies, which makes these modes important as diagnostics. Indeed, it appears that this phenomenon has been observed in η Boo.

In this *Letter* we model the star using "standard" stellar evolution theory. In particular, we neglect diffusion and settling, as well as convective overshoot and other sources of mixing outside convectively unstable regions. Also, the near-surface region of the star is treated rather crudely; thus convection is described by means of a local mixing-length theory neglecting turbulent pressure, and the oscillations are assumed to be adiabatic. Similar approximations are likely to be responsible for much of the discrepancy between the observed and computed frequencies of solar oscillation (e.g., Balmforth 1992). In a subgiant such as η Boo, the neglected effects may be stronger than for the Sun; it is plausible that the shift of $10\,\mu$Hz between the observed and computed frequencies might result from these inadequacies of the computations. We note that they could also have some effect on the computed value of $\Delta\nu$. We are currently investigating these aspects using more elaborate mixing-length treatments, and also using detailed hydrodynamical calculations.

The present results provide an indication of the power of asteroseismic investigations. Observations of other stars are being planned, and further data on η Boo, ideally from multiple sites, would be extremely valuable. We note, however, that the equivalent-width technique used by Kjeldsen et al. (1995) is restricted to bright stars. The results for η Boo indicate the wealth of information to be obtained from observation of a substantial number of fainter stars, such as would be possible from space using broad-band photometry with a telescope of moderate aperture.

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REFERENCES

Aizenman, M., Smeyers, P. & Weigert, A. 1977, A&A, 58, 41

Audard, N., Provost, J. & Christensen-Dalsgaard, J. 1994, A&A, in the press.

Balmforth, N. J. 1992, MNRAS, 255, 632

Bertiau, F. C. 1957, ApJ, 125, 696

Brown, T. M. and Gilliland, R. L. 1994, ARA&A, 32

Böhm-Vitense, E. 1958, ZAp, 46, 108
Cappelli, A., Cerruti-Sola, M., Cheng, C. C. & Pallavicini, R. 1989, A&A, 213, 226
Christensen-Dalsgaard, J. 1982, MNRAS, 199, 735
Christensen-Dalsgaard, J. 1991, Geophys. Astrophys. Fluid Dynamics, 62, 123
Christensen-Dalsgaard, J. & Berthomieu, G. 1991, in Solar interior and atmosphere, eds A. N. Cox, W. C. Livingston, & M. Matthews, Space Science Series, (University of Arizona Press), 401
Clegg, R. E., Lambert, D. L. & Tomkin, J. 1981 ApJ, 250, 262
Dziembowski, W. A. & Pamyatnykh, A. A. 1991, A&A, 248, L11
Edvardsson, B., Andersen, J., Gustafsson, B., Lambert, D. L., Nissen, P. E. & Tomkin, J. 1993, A&A, 275, 101
Eggleton, P. P., Faulkner, J. & Flannery, B. P. 1973, A&A, 23, 325
Gabriel, M. 1989, A&A, 226, 278
Grec, G., Fossat, E. & Pomerantz, M. 1983, Solar Phys., 82, 55
Houdek, G., Rogl, J., Balmforth, N. J. & Christensen-Dalsgaard, J. 1994, in Proc. GONG'94: Helio- and Astero-seismology from Earth and Space, eds R. K. Ulrich, E. J. Rhodes Jr & W. Däppen, PASP Conf. Ser., in press.
Iglesias, C. A., Rogers, F. J. & Wilson, B. G. 1992, ApJ, 397, 717
Kjeldsen, H., Bedding, T. R., Viskum, M. & Frandsen, S. 1995, AJ, in the press.
Kurucz, R. L. 1991, in Stellar atmospheres: beyond classical models, eds L. Crivellari, I. Hubeny & D. G. Hummer, NATO ASI Series (Dordrecht: Kluwer), 441
Parker, P. D. 1986, in Physics of the Sun, Vol. 1, eds P. A. Sturrock, T. E. Holzer, D. Mihalas & R. K. Ulrich (Dordrecht: Reidel), 15
Tassoul, M. 1980, ApJS, 43, 469
Tomkin, J., Lambert, D. L., & Balachandran, S. 1985, ApJ, 290, 289
Vandakurov, Yu. V. 1967, AZh, 44, 786 (English translation: Soviet Ast., 11, 630)

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