EFFECTS OF OPACITY ON STELLAR OSCILLATIONS

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1 Introduction

In his book "The pulsation theory of variable stars" Rosseland (1949) made a comprehensive overview of the theory of stellar pulsation as known at the time, with special emphasis on the dynamics and non-linear behaviour of the oscillations. In the introduction he distinguished between three idealized pure types of internal movements of a star: pure rotation, meridional circulations and "purely radial motions, either a unidirectional outward stream or a periodic pulsation or both". The implication is clearly that periodic pulsations involve purely radial, i.e., spherically symmetric, motion. Indeed, in the following he considered only radial motion; however, he stated (p. 18) that "[i]t is quite possible, though, that non-radial oscillations will prove to play a part also for stellar variability, but thus far such cases have not emerged into the foreground of astronomical interest".* As will become clear in the following, nonradial oscillations have now very much "emerged into the foreground of astronomical interest".

Stellar oscillations are characterized by the restoring forces that bring the star back towards equilibrium, if that equilibrium is perturbed. Crudely speaking, there are two such forces: pressure which acts to expand the stellar material if it is compressed; and buoyancy, which acts on density differences along horizontal surfaces. Pressure induces oscillations which locally have the nature of sound waves; these oscillations are called \( p \) modes. The oscillations dominated by buoyancy have locally the character of internal gravity waves and are called \( g \) modes. Pressure acts even for spherically symmetric perturbations, corresponding to radial modes, whereas buoyancy requires departure from spherical symmetry. The behaviour over spherical surfaces of an oscillation is given by a spherical harmonic \( Y_{lm}(\theta, \phi) \), as a function of co-latitude \( \theta \) and longitude \( \phi \); radial modes correspond to the degree \( l = 0 \), whereas for nonradial modes, with \( l > 0 \), the degree measures the overall wavelength over the stellar surface. Here we are concerned only with spherically symmetric stars, whose frequencies are independent of the azimuthal order \( m \). For each \( l \) there is a sequence of modes, characterized by the radial order \( n \) whose absolute value, roughly speaking, measures the number of nodes in the radial direction. Conventionally, \( g \) and \( p \) modes are labelled by \( n < 0 \) and \( n > 0 \), respectively. The intermediate modes, with \( n = 0 \), are denoted \( f \) modes. Observations of stars other than the Sun, where there is little or no spatial resolution, are generally sensitive only to low-degree modes: at higher degree cancellation of regions in opposite phase on the star's surface largely eliminates the sensitivity of the observations to the modes. (Exceptions to this are found for rapidly rotating pulsating stars, where higher-degree modes can be isolated through Doppler imaging; e.g. Baade 1984; Kennely et al. 1993.)

A detailed description of the properties of stellar oscillations was given by Unno et al. (1989).

Pulsating stars are found throughout the Hertzsprung-Russell diagram, from the most luminous red giants to white dwarfs. An important sequence belongs to the instability strip, a steeply inclined, narrow strip in the HR diagram at spectral types around A – G which contains the classical Cepheids and, near the main sequence, the \( \delta \) Scuti stars. Among somewhat hotter stars near the

* It should be pointed out that Rosseland (1932) did in fact present an analysis of nonradial oscillations.
main sequence, at spectral type B, one finds the $\beta$ Cephei stars; additional pulsating stars in this general region are the 53 Persei stars (e.g. Smith & Buta 1979) and the slowly pulsating B (or SPB) stars (e.g. Waelkens 1991). An important example of a pulsating star is the Sun, where a very large number of modes have been detected (for a recent review see, for example, Gough & Toomre 1991). It is likely that stars similar to the Sun will show similar oscillations. The expected very small amplitudes make observation difficult, and no unambiguous detection has so far been made; however, Brown et al. (1991) found power at approximately the expected frequencies in Procyon, and very recently Kjeldsen et al. (1994) found strong evidence for oscillations, determining several frequencies, in $\eta$ Boo.

The most interesting aspects of stellar pulsation are the information provided by the observed frequencies about the stellar interiors, and the understanding of the cause of the pulsations. The information content evidently depends on the number of frequencies observed, and the accuracy with which they can be determined (note, however, that frequencies can typically be measured with far higher precision than any other aspect of a star). In those cases where the observations are restricted to a single mode, often taken to be the fundamental radial oscillation, its frequency essentially only provides a measure of the mean density of the star. The opposite extreme is provided by the Sun, where thousands of modes have been observed with very high precision; here the frequencies may be used to infer detailed properties of the solar interior, including the variation of structure with position in the Sun.

The pulsations of stars in the instability strip and, it is now believed, in the B-star region, are inherently unstable: they are driven by interaction with the flow of energy through the stellar envelope. The same is true of the pulsating red giants and white dwarfs. On the other hand, the modes of the Sun and solar-like stars appear to be stable: here the observed oscillations are probably caused by stochastic driving by the turbulent convection near the stellar surface, where convective motion reaches speeds close to, or exceeding, the local speed of sound.

The properties of stellar oscillations are intimately linked to the opacity in the stellar interior, through its effects on the mean structure of the star and on the interaction between the oscillations and the radiative energy transport. Here I concentrate on some aspects of these relationships, where major improvements in the agreement between theory and observation, or in our understanding of the origin of the pulsations, have occurred in the last few years as a result of improvements in the computation of the Rosseland mean opacity. These improvements were to a large extent motivated by a paper by Simon (1982) pointing out that discrepancies in the period ratios for double-mode Cepheids could be resolved by very substantial localized opacity increases, which in addition might explain the instability of the $\beta$ Cephei stars.* Additional evidence for the need for opacity revisions came from helioseismology: thus Christensen-Dalsgaard et al. (1985) argued that the difference between the solar sound speed inferred from the observed solar oscillation frequencies and the sound speed in solar models could result from errors in the opacity. The recalculation of opacities was carried out independently by two groups using rather different techniques: the OPAL group at Livermore (e.g. Rogers & Iglesias 1992; Iglesias, Rogers & Wilson 1992) and the Opacity Project (OP; Seaton 1987, 1993). In both cases much more detailed information than before on the effects

* The paper by Simon is one of the more remarkable examples of a prediction in astrophysics, and hence its abstract deserves to be quoted in full:

"It is shown that increasing the opacity due to heavy elements by a factor of 2-3 leads to classical Cepheid models which reproduce observed period ratios at evolutionary masses and luminosities. Thus the mass anomalies are removed both in the double-mode and bump Cepheid regimes. The proposed increase may also serve to energize $\beta$ Cephei variables, thus solving yet another important problem in the theory of pulsating stars. We argue that opacity changes of this order are not implausible and urge further work in this important area."
of spectral lines was included; this resulted in large increases in the opacity caused in particular by lines of iron. These increases have led to the solution of a number of long-standing problems in stellar astrophysics.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{petersen_diagram.png}
\caption{Petersen diagram, plotting the ratio between the first overtone and fundamental radial period against the logarithm of the latter. The observed values are shown by crosses. The curves show the variation along the instability strip; the solid curve was based on models computed with the Cox & Tabor (1976) opacities, whereas the dashed curve used recent OPAL tables from Rogers & Iglesias (1992).}
\end{figure}

2 Period ratios for double-mode pulsators

Most classical Cepheid variables appear to pulsate in a single mode, which is generally identified with the fundamental radial mode. However, in a few cases two oscillation periods are found, identified as the periods $P_0$ and $P_1$ of the fundamental and first overtone of radial pulsation. The same behaviour is found for some large-amplitude $\delta$ Scuti stars, close to, but still above, the main sequence. Fitch (1970) analysed the properties of the pulsation constants of such stars, based on polytropic models. In a landmark paper, Petersen (1973) used realistic stellar models to show that masses and radii could be inferred from the two periods observed for such stars. For the Cepheids the resulting values of the masses were smaller by about a factor three than the evolution masses, inferred by comparing the location of the stars in the HR diagram with evolution calculations. Reviews of this and other Cepheid mass problems were given by, e.g., Cox (1980) and Simon (1987).

An equivalent, and possibly more natural, statement of the problem is that period ratios computed for the evolutionary masses are inconsistent with the observations. This is illustrated in simplified form in Figure 1, in a so-called Petersen diagram where observed period ratios for Cepheids and $\delta$ Scuti stars are plotted against the fundamental radial period $P_0$. The theoretical values (Christensen-Dalsgaard 1993) were obtained from an assumed mass-luminosity relation along the instability strip (Becker, Iben & Tuggle 1977), based on evolution calculations (and hence corresponding to the evolutionary masses). Together with the relation between effective temperature
and luminosity defined by the location of the instability strip, this provides a unique characterization of the stars in terms of their radius, and hence gives a relation between $\Pi_0$ and $\Pi_1/\Pi_0$. This relation, based on computations using the Cox & Tabor (1976) opacities, is shown as the solid curve in the figure. It is obvious that there is a substantial difference between observations and theory; also, it is striking that the computed ratios are too low for the $\delta$ Scuti stars, whereas they are too high for the Cepheids.

Many suggestions have been made to account for this discrepancy, amongst these the proposal by Simon (1982) that the opacity would have to be increased by a substantial factor in a suitable temperature interval. Although the possibility of errors of this magnitude in the then existing tables was questioned (thus Magee, Merts & Huebner (1984) argued that "such a large increase in opacity is incompatible with atomic physics"), the suggestion played a substantial role in the decision to reconsider the opacity calculations. Later Andreasen (1988) and Andreasen & Petersen (1988) made a detailed estimate of the opacity changes required to bring the computed period ratios into agreement with the observations for the double-mode Cepheids; they furthermore pointed out that similar changes would also account for the discrepancies for the $\delta$ Scuti stars.

![Figure 2.](image)

**Figure 2.** (a) The opacity increase in the OPAL tables, relative to Cox & Tabor (1976), at the conditions corresponding to the same two models. The dotted box illustrates the opacity increase proposed by Andreasen & Petersen (1988). (b) Opacity sensitivity functions (cf. equation 1); the solid curve is for a model with $\Pi_0 = 3.5$ days, corresponding to the Cepheid region in Fig. 1, whereas the dashed curve, for a model with $\Pi_0 = 0.09$ days, is representative for the $\delta$ Scutis.

New opacity calculations have indeed shown very substantial increases at temperatures between $10^5$ and $10^6$ K. Figure 2a shows differences in $\log \kappa$, against temperature in a Cepheid and a $\delta$ Scuti model, between the OPAL opacities of the Livermore group (Rogers & Iglesias 1992; Iglesias et al. 1992) and the Cox & Tabor (1976) opacities. A very similar opacity increase was found by the OP group in a second major, independent opacity calculation (Seaton 1993; Seaton et al. 1994). The figure compares the opacity changes for the OPAL calculations with the predictions by Andreasen & Petersen (1988). It is evident that the actual opacity changes resemble the predictions. In fact, models computed with the OPAL or the OP opacities have period ratios in good agreement with the observed values (e.g. Moskalik, Buchler & Marom 1992; Kanbur & Simon 1994). This is illustrated
by the dashed line in Figure 1, which shows period ratios computed for models based on the OPAL opacity tables.

The difference in response between the Cepheid and the δ Scuti models can be understood by considering the opacity response function $K_{0,1}(\log T)$ (Petersen 1992). This is based on the assumption that the change in opacity $\kappa$, at given temperature and density, can be regarded as being solely a function $\delta \log \kappa(\log T)$ of temperature $T$. For sufficiently small modifications the change in the period ratio can then be expressed as

$$
\delta \left( \frac{P_1}{P_0} \right) \simeq \int K_{0,1}(\log T) \delta \log \kappa(\log T) d\log T,
$$

where the integral is over the model (see also Korzennik & Ulrich 1989, and Section 4 below). Figure 2b shows $K_{0,1}(\log T)$ for a Cepheid and a δ Scuti model. Due to differences in the shapes of the eigenfunctions between the two models, the response is shifted towards higher temperature in the δ Scuti case. Comparison with Figure 2a shows that, as a result, the change in the period ratio resulting from the opacity increase in the OPAL tables is positive for the δ Scuti stars, while it is negative for the Cepheids, precisely as required to bring the models into agreement with the observations.

3 The excitation of B-star pulsations

The β Cephei pulsators were discovered early this century through radial-velocity observations (Frost 1902). They occupy a well-defined region in the HR diagram, corresponding to stars of spectral type early B, on and just above the main sequence (for a review, see Lesh & Aizenman 1978). Their variability was interpreted by Ledoux (1951) as arising from nonradial oscillations. Spectral-line variability has been detected in stars in neighbouring regions of the HR diagram (e.g. Smith & Buta 1979), while Waelkens (1991) found intensity oscillations with periods of up to several days in stars of somewhat later spectral types.

The cause of these B-star oscillations has until fairly recently been highly uncertain, despite a substantial number of attempts to account for them. In a review of the situation, Osaki (1982) concluded that the “origin and cause of the β Cephei phenomenon remain unknown”, and continued: “These stars are pulsating at the present moment as if they wanted to tell us something about their internal structure. We, astronomers, must therefore continue to try to understand their messages”. After more than a decade, we are now getting close to this understanding.

The existence of fairly well-defined regions of pulsating stars suggests an intrinsic instability mechanism, similar to the one operating in the Cepheid instability strip. As noted by Eddington (1926), a star can function as a heat engine if the opacity varies suitably with the phase of the oscillation: a region of the star contributes to the driving of the mode if the opacity variation is such that the region absorbs additional energy during the compression phase of the pulsation and releases energy during expansion. If the star is to be unstable, such driving regions must dominate over the damping that occurs in other parts of the star. The interaction between the radiative flow of energy through the star and the pulsations is dominated by the pulsationally induced perturbations in opacity, which in turn are largely controlled by the temperature derivative $\kappa_T \equiv (\partial \log \kappa/\partial \log T)_{\rho}$ at fixed density $\rho$. To accumulate energy during compression, the perturbation in the radiative flux must be larger at the bottom of the given region than at the top; this requires that $\kappa_T$ increases as one goes out through the layer. Hence the contribution to instability is controlled essentially by the second derivative of opacity, and thus is quite sensitive to modest localized opacity features. In general, a given feature in the opacity produces regions with both increasing and decreasing $\kappa_T$, 


such that the effects on the driving of the mode largely cancel. However, as pointed out by Cox (1967), net driving may result if the opacity feature is located at the point in the star where there is a transition from almost adiabatic to strongly nonadiabatic oscillations. To see this we note that in the outer parts of the star, the heat capacity of matter is too low to affect the energy flow during a pulsation period; this part of the star, therefore, has no effect on the damping or driving of the mode. If an opacity feature is located in the transition region, the lower part of the feature may provide driving, whereas the corresponding damping from the upper part of the feature is suppressed because of the low heat capacity; hence only the driving in the lower part of the region is felt, resulting in instability. Thus the condition for instability is that the thermal time scale of that part of the star which lies outside the opacity feature must be comparable with the pulsation period.*

It follows that the depth of the transition region increases with increasing period of the pulsation; thus longer-period modes are favoured when the relevant opacity feature lies deeper in the star, as is the case for stars of lower effective temperature.

The Cepheid instability results from the operation of this mechanism in the region of second helium ionization; it was shown by Cox & Whitney (1958) that coincidence of the transition region with the ionization zone accounts approximately for the location of the instability strip. In the

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* Similar arguments were used by Rosseland (1949) to discuss the phase lag for variable stars.
B stars, however, helium ionization occurs so close to the surface as to be almost entirely within the nonadiabatic region. Thus instability requires a suitable opacity feature at substantially higher temperature. Such a feature was absent in older opacity tables, resulting in stability of B-star models. Stellingwerf (1978) and Simon (1982) pointed out that instability could be induced by including an additional bump in the opacity at a temperature of about $10^5$ K. Such a bump is in fact a consequence of the increased iron line contribution to the opacity in the new OPAL and OP opacities. It was shown by Cox et al. (1992), Kiriakidis, El Eid & Glatzel (1992) and Moskalik & Dziembowski (1992) that this leads to instability of β Cephei models.

![Graph showing evolution tracks in the Hertzsprung-Russell diagram](image)

**Figure 4.** Evolution tracks in the Hertzsprung-Russell diagram for $Z = 0.03$ (left panel) and $Z = 0.02$ (right panel). The masses in solar units are indicated. The heavy portions of the tracks correspond to models where at least one mode of degree 0 - 2 is unstable. The symbols denote the location of observed β Cephei stars: the open circles are field stars, whereas the remaining symbols correspond to stars in open clusters, as indicated. (From Dziembowski & Pamyatnykh 1993.)

To illustrate the relation between the opacity and the driving, Figure 3 shows results of a stability calculation by Dziembowski & Pamyatnykh (1993). The dashed curves were based on an early version of the OPAL tables, whereas the continuous curves used tables including additional effects and a revised composition. As shown in the top panel, this had a moderate effect on the opacity in the bump near $\log T \approx 5.3$, caused by the iron-line contribution. The effect is substantially more noticeable in $\kappa T$, shown in the middle panel. Finally, the bottom panel shows the local contribution to the driving. It is evident that strong positive driving is associated with an increase in $\kappa T$ towards lower temperature, as discussed above. Furthermore, the modest difference between the two opacity tables clearly has a significant effect on the stability properties. Note that the bump in the opacity near $\log T \approx 4.6$, which is caused by second helium ionization, has no effect on the stability: this occurs in a region where the mode is strongly nonadiabatic and where therefore there is little coupling between the oscillation and the radiative energy transport.

It might be noted that Stellingwerf proposed the enhancement to be associated with an opacity feature occurring at the point where the second ionization of helium is almost complete.
In Figure 4 the location in the HR diagram of stars with unstable modes, for two different values of the heavy-element abundance $Z$, is compared with the observed location of $\beta$ Cephei stars. Given that stars are most likely to be observed in the phase of core hydrogen burning, corresponding to the initial part of the evolution tracks, there is evidently reasonable agreement between the theoretical instability region and the location of the pulsating stars. It should also be noted that since the opacity feature responsible for the instability is caused by the iron contribution, the stability properties are quite sensitive to the heavy-element abundance. For $Z = 0.01$ no models in the $\beta$ Cephei region are found to be unstable. This is consistent with the fact that $\beta$ Cephei pulsators appear to be largely absent in the Magellanic Clouds where $Z$ is typically a factor of 2 – 4 lower than for the Sun (Balona 1992, 1993; Kjeldsen & Baade 1994).

As noted above, the condition for instability favours longer-period modes in stars of lower effective temperature and hence lower mass on the main sequence. This is consistent with the fact that the slowly pulsating B stars generally are found at somewhat later spectral types than the $\beta$ Cephei stars. Indeed, Dziembowski, Moskalik, & Pamyatnykh (1993) found instability towards $g$ modes of low degree and with periods between 0.5 and 3 days, for main-sequence models of mass between 3 and 7 $M_\odot$. In Figure 5 the region of instability is compared with the observed location of some SPB stars; although the number of observed stars is modest, it is evident that there is good agreement between observations and theory.

An extensive survey of B-star instability was also carried out by Gautschy & Saio (1993).

The oscillation properties of evolving B stars are strongly affected by the presence of steep gradients in the hydrogen abundance outside shrinking convective cores. This is illustrated in

**Figure 5.** The region of slowly pulsating B stars in the HR diagram. The masses in solar units are indicated at the evolution tracks. The heavy lines indicate the region of unstable $g$ modes for $l = 1$ (dashed line) and $l = 2$ (continuous line). The symbols show the location of observed SPB stars. (From Dziembowski et al. 1994.)
Figure 6. Properties of a 4 $M_\odot$ star, as a function of the fractional radius $r/R$. The top panel indicates schematically the abundance $X$ by mass of hydrogen. The next panel shows the buoyancy frequency $N$ and the Lamb frequency $S_l$ for degrees $l = 1$ and 2, in units of $(3GM/R^3)^{1/2}$, where $G$ is the gravitational constant, and $M$ and $R$ are the mass and radius of the star. The remaining panels show results for the most unstable mode of degree 1: the kinetic energy density $\mathcal{E}$, and, at the bottom, the differential work integral $dW/dr$ defined such that positive values correspond to instability. (Adapted from Dziembowski et al. 1993.)

Figure 6 for a 4 $M_\odot$ model of an SPB star. The top panel gives a schematic indication of the abundance $X$ by mass of hydrogen. In the convective core, matter is effectively mixed and the composition is homogeneous. The size of the core decreases during evolution, leaving behind a steep gradient in $X$, as indicated. Here the buoyancy frequency $N$ (i.e., the characteristic frequency of internal gravity waves) reaches very large values, as shown in the second panel. This allows trapping of high-order $g$ modes in this region. The third panel shows the kinetic energy density for a $g$ mode of period 1.5 days; although the mode has significant amplitude throughout the radiative interior,
it is clearly strongly enhanced in the region of changing composition. However, as shown in the
bottom panel the contribution to the driving of the mode is still concentrated in the outer layers
of the star, the mode being rendered unstable by the bump in the metal opacity in the same way
as the $p$ mode illustrated in Figure 3.

The properties of the boundaries of convective cores in stars of intermediate and high mass are
highly uncertain. It is likely that there is vigorous penetration beyond the unstable region, possibly
followed by weaker mixing even further out. This would have substantial effects on the evolution
of the stars, by bringing more hydrogen into the nuclear-burning region. Furthermore, the structure
of the core may be affected by the poorly understood process of semiconvection. Some evidence
for overshoot has been found from studies of the HR diagram of stellar clusters (e.g. Maeder &
Meynet 1989) or from analysis of binary stars (for a review, see Andersen 1993). Observation of
B-star pulsations offers the hope of additional information about the properties of this region. The
distribution of $g$-mode energy shown in Figure 6 indicates that the period of this mode is sensitive
to the structure at the edge of the convective core. Furthermore, Dziembowski et al. (1993) found
instability towards a large number of modes of this nature; if a substantial number of modes can in
fact be observed, one would obtain detailed information about the structure of the deep interior of
these stars. Indeed, Dziembowski (1994) showed that sufficient data may become available to allow
inversion of the periods to determine local conditions in the star. The observational difficulties of
separating dense spectra of long-period modes are certainly very substantial; however, the potential
rewards in terms of understanding of stellar evolution on the main sequence and beyond may well
justify the effort required.

4 Helioseismic investigation of solar structure

By far the most extensive sets of oscillation data have been obtained for the Sun: at frequencies
between roughly 1 and 5 mHz, modes have been observed at all degrees between 0 and more than
1000. The relative accuracy of many of the observed frequencies exceeds $10^{-5}$. As a result, it is
possible to probe the solar interior in great detail.

The relation between solar structure and the observed frequencies is determined by the properties
of the modes of oscillation, which can be understood from the fact that they result from the
interference of acoustic waves. Figure 7 shows ray paths traced by acoustic waves in a cross-
section of a star. It is evident that waves that start obliquely at the stellar surface penetrate to a
comparatively shallow depth before reaching a turning point where the wave undergoes total inter­
nal refraction; on the other hand, waves whose initial direction is nearly vertical reach the stellar
core. The former waves have short horizontal wavelength and hence correspond to modes of large
degree, whereas the latter waves form low-degree modes. The radial modes correspond to waves
that propagate in the radial direction and hence pass through the centre of the star. This relation
between the degree, which is observable, and the depth of penetration underlies the determina­
tion of local properties of the solar interior: very roughly, by combining data corresponding to modes
penetrating to slightly different depths one obtains information about the region lying between the
turning points.

An elaboration of this argument leads to an inversion procedure whereby the sound speed in
the solar interior can be inferred from the observed frequencies, without the use of a solar model
(Gough 1984). Figure 8 shows early results of such an analysis (Christensen-Dalsgaard et al. 1985),
in the form of the difference between the solar sound speed and the sound speed of a solar model,
based on the Cox & Tabor (1976) opacities. It is evident that the solar sound speed is systemati­
cally higher than the sound speed in the model in much of the radiative region. Christensen-Dalsgaard
et al. argued that this discrepancy could be eliminated by modest increases, of order 20 per cent, in
Figure 7. Rays illustrating the propagation of acoustic waves in a star. Interference between such rays gives rise to p-mode oscillations. In each case, the dotted line indicates the sphere of avoidance, into which the modes do not penetrate. The rays going through the centre correspond to spherically symmetric modes, with $i = 0$.

Figure 8. Relative difference between the squared sound speed $c^2$ obtained by inverting observed frequencies of solar oscillations and $c^2$ obtained from inversion of the corresponding frequencies in a solar model. The shaded area indicates $1-\sigma$ errors in the inversion inferred from the random errors on the observed data. (From Christensen-Dalsgaard et al. 1985.)

the opacity. The possible need for an opacity increase was also pointed out by Korzennik & Ulrich (1989) and Cox, Guzik & Kidman (1989) on the basis of analysis of the observed solar frequencies.

The effects of modifying opacity on the results of the sound-speed inversion are illustrated
Figure 9. Relative sound-speed differences, in the sense (Sun) - (model), inferred from differential asymptotic inversion of differences between the observed solar frequencies and the frequencies of several reference models. The models are distinguished by the opacities and the possible inclusion of diffusion, as indicated by the line styles: Cox & Tabor (1976); LAOL: OPAL: OPAL, including helium settling and diffusion: The thin continuous lines indicated 1-σ error limits, inferred from the random errors in the observed frequencies, on the inversion relative to the non-diffusive OPAL model.

in Figure 9. Here the sound-speed differences were obtained from a more extensive set of data (Libbrecht, Woodard & Kaufman 1990) than used for Figure 8, by means of a differential asymptotic inversion technique (Christensen-Dalsgaard, Gough & Thompson 1989). The inversions were carried out relative to reference models using the same physics as those of Christensen-Dalsgaard, Proffitt & Thompson (1993), apart from the opacities. In addition to the Cox & Tabor (1976) opacities, results are shown for models based on opacities from the Los Alamos Opacity Library (LAOL; Huebner et al. 1977), and the OPAL opacities. It is evident that the sound-speed differences are very small in the convection zone, for \( r \geq 0.7R \), where the structure does not depend directly on the opacity. Also, the agreement between the models and the observations generally improve for the more recent opacities, although there remain significant differences even for the model based on the OPAL opacity.

The models discussed so far were traditional "standard" solar models. In particular, they ignored effects of diffusion and gravitational settling. It has been shown that such effects may have a significant influence on the structure of the models (e.g. Noerdlinger 1977; Wambsganss 1988; Cox et al. 1989). In Figure 9 I also show results of inversions relative to a reference model that includes diffusion and settling of helium (Christensen-Dalsgaard et al. 1993). This evidently leads to a very substantial improvement in the model, the remaining relative differences in sound speed being below about \( 2 \times 10^{-3} \). It should be emphasized that this agreement was achieved simply by
improving the treatment of the physics of the model, but without attempting to fit the data. Thus it represents a major, and perhaps surprising, triumph of standard stellar evolution theory.

![Figure 10. Neutrino fluxes (panel a) and frequency separations between low-degree modes (panel b) for standard and non-standard solar models. The crosses indicate older standard models with varying physics, whereas the filled circle and diamond correspond to the OPAL models without and with diffusion, respectively, already illustrated in Figure 9. The filled square is a model where effects of WIMPs have been modelled through a localized reduction in the core opacity, and the filled triangles pointing down and up are for partially and fully mixed models, respectively. The error boxes show observed values with their 1-σ error bars. In panel (a) the flux of high-energy $^8$B neutrinos, as detected by the electron scattering experiment, is shown against the capture rate in the $^{37}$Cl experiment in Solar Neutrino Units (SNU), where one SNU corresponds to $10^{-36}$ captures per target atom per second. In panel (b) the abscissa is the average value $\bar{\nu}_0$ and the ordinate is the slope $s_0$ of the frequency separation $\delta \nu_{n_0}$ (cf. equation 3), with a reference order $n_0 = 17$.]

A possible indication that all is not well comes from the discrepancy between measured fluxes of neutrinos from the nuclear reactions in the solar core and the predictions of the standard solar models (for reviews, see Bahcall 1989; Bahcall, these proceedings). Three different types of neutrino experiments have been performed so far: a long-running experiment to measure neutrino capture in $^{37}$Cl, a measurement of neutrino scattering on electrons in water (which confirms that the neutrinos do indeed come from the direction of the Sun), and two experiments measuring neutrino capture in $^{71}$Ga. Of these, the first two are sensitive mainly to high-energy neutrinos from relative rare reactions which contribute little to the total energy generation, particularly the decay of $^8$Be to $^8$B; the $^{71}$Ga experiments, on the other hand, detect also neutrinos from the basic $^1$H + $^1$H reaction which controls the total rate of energy generation. As an example, Figure 10a shows results on the capture rate in $^{37}$Cl and the electron scattering neutrino experiments. A number of "standard" models are included, differing in equation of state, opacities and nuclear parameters (for details, see Christensen-Dalsgaard 1991), and including the OPAL models shown in Figure 9. For these models the predicted rates are larger by factors 2 - 3 than the measurements.
This clearly represents a serious discrepancy which might indicate errors in the calculation of solar models. Many proposals to reduce the predicted neutrino flux have been made; these are generally based on a reduction of the temperature in the core of the model which reduces the number of reactions producing high-energy neutrinos. Two such proposals are illustrated in Figure 10a. The first involves a contribution to the energy transport from hypothetical “Weakly Interacting Massive Particles” (WIMPs; e.g. Spergel & Press 1985; Faulkner & Gilliland 1985), modelled here by a localized reduction in the opacity in the core of the model (Christensen-Dalsgaard 1992); this reduces the temperature gradient required for energy transport and hence the central temperature. In the second proposal partial or complete mixing of the solar interior brings additional hydrogen to the core and hence reduces the temperature required for energy production. In both cases the predicted neutrino fluxes are reduced; it should be noticed, however, that the reductions occur close to a line in the diagram which does not pass the error box for the measurements (see also Bahcall & Bethe 1993).

These non-standard models may obviously be tested by comparing their oscillation frequencies with those observed. As argued above, the innermost parts of the Sun are probed by the lowest-degree modes. A convenient measure of conditions in the core is the small frequency separation

$$\delta \nu_{nl} = \nu_{nl} - \nu_{n-1,l+2},$$  \hspace{1cm} (2)

where $\nu_{nl}$ is the frequency of a mode of degree $l$ and radial order $n$. As indicated by Figure 7, the properties of low-degree modes differ only in the core of the model. As a result, the contributions from the outer parts of the star to $\delta \nu_{nl}$ are largely suppressed. A more careful analysis confirms that for small $l$ $\delta \nu_{nl}$ is in fact predominantly determined by the properties of the core. Elsworth et al. (1990) introduced a convenient representation of $\delta \nu_{nl}$ in terms of a linear fit

$$\delta \nu_{nl} \approx \overline{\delta \nu}_l + s_l(n - n_0),$$  \hspace{1cm} (3)

where $n_0$ is a suitable reference order. In Figure 10b observed values for $l = 0$ are compared with results for a number of “standard” models, as well as for the model simulating WIMPs and the partially mixed model. It is evident that the normal models are in reasonable agreement with the observations, particularly when helium settling is taken into account; on the other hand, the models computed to reduce the neutrino flux are entirely inconsistent with the data (see also Elsworth et al. 1990; Cox, Guzik & Raby 1990).

The results shown in Figure 10 indicate the difficulty of reducing the neutrino flux while maintaining the agreement between the observed and computed $\overline{\delta \nu}_l$. This may be analyzed in more detail by considering response functions analogous to $K_{0,1}(\log T)$ for the period ratio which was introduced in equation (1). In the solar case, it is convenient to modify the definition somewhat, however. The current solar luminosity and radius are known with high precision. Models of the Sun are therefore required to have the correct luminosity and radius; this is achieved by adjusting the initial helium abundance $Y_0$ and the mixing-length parameter $\alpha_C$ which characterizes the convective efficiency. When considering the effect of an opacity modification on the model one therefore makes corresponding changes in $Y_0$ and $\alpha_C$ to keep the luminosity and radius unchanged. For small opacity changes depending only on temperature the change $\delta F$ in any property $F$ of the model can then be expressed in terms of a kernel $K_F(\log T)$ as

$$\frac{\delta F}{F} \approx \int K_F(\log T) \delta \log \kappa(\log T) d \log T.$$  \hspace{1cm} (4)
The kernels $K_F$ may be computed by evaluating the effects of localized changes $\delta \log \kappa$ centred on points $\log T_0$ which are varied through the model (Korzennik & Ulrich 1989; Tripathy & Christensen-Dalsgaard 1994). In Figure 11, panels (a) and (b) show results for neutrino capture rates and the average frequency separation $\delta \nu_0$, respectively. It is interesting to note that the $^{71}\text{Ga}$ capture rate is far less sensitive to opacity changes than is the capture rate for $^{37}\text{Cl}$: since the $^{71}\text{Ga}$ experiment is dominated by the neutrinos from the basic $^1\text{H} + ^1\text{H}$ reaction, which controls the total energy production, the condition that the total luminosity is unchanged means that the $^{71}\text{Ga}$ capture rate changes little. In contrast, the $^{37}\text{Cl}$ rate is dominated by rare reactions which contribute little to the energy generation and depend strongly on temperature; consequently it is highly sensitive to opacity changes. It is striking that opacity increases near $\log T = 7$ lead to decreases in the
neutrino capture rates. Such opacity increases cause an increase in the temperature and hence in
the energy generation rate in the outer parts of the core; the condition of fixed luminosity leads to
compensating changes in the innermost parts of the core which reduce the overall neutrino fluxes.
The behaviour of the kernel for \( \delta \nu_0 \) probably reflects the fact that the frequency separation results
from differences between radial modes penetrating essentially to the centre and \( l = 2 \) modes which
avoid the inner core: thus opacity increases very near the centre affect predominantly the radial
modes, leading to frequency increases; opacity increases at slightly lower temperature and hence
larger distance from the centre causes an increase in the \( l = 2 \) frequencies and hence a decrease
in \( \delta \nu_{m1} \). The small oscillations in the kernel may reflect interference between the location of the
opacity change and the average shape of the eigenfunctions.

The kernels for the neutrino rates and the frequency separation are somewhat similar; thus
opacity changes that reduce the neutrino capture rate would in general also reduce \( \delta \nu_0 \) and hence
destroy the agreement between the standard models and the observations, in accordance with Figure
10. It would be possible to design a \( \delta \log \kappa(\log T) \) such as to leave \( \delta \nu_0 \) unchanged while reducing
the neutrino rates; but such an opacity change would have to be rather contrived.

The results presented here indicate that it may be difficult to construct solar models that are
consistent with both the observed oscillation frequencies and the neutrino data. Given the generally
excellent agreement between the models and the frequencies, this suggests that the origin of the
neutrino problem is not to be found in the theory of stellar evolution. One possible solution is
the so-called MSW effect (Wolfenstein 1978; Mikheyev & Smirnov 1985): if neutrinos have small
but finite masses interaction between the neutrinos and matter in the Sun may lead to transitions
such that some of the electron neutrinos generated in the solar core are transformed into muon or
tau neutrinos which are not recorded by the current detectors. It is possible to choose masses and
interaction parameters such that standard solar models are consistent with the existing neutrino
measurements (e.g. Shi, Schramm & Bahcall 1992; Krauss, Gates & White 1993), although the
parameter values are fairly tightly constrained already by the existing data.

It must be pointed out that observations of solar oscillations cannot by themselves rule out
an astrophysical solution to the neutrino problem: the oscillation frequencies depend on the sound
speed and density in the solar interior, but not directly on temperature which on the other hand
largely controls the neutrino flux. It is true that there is a close relation between sound speed and
temperature, but this also involves composition which cannot be determined separately. Thus,
for example, Figure 10 suggests that a judicious combination of mixing and opacity modifications
might produce a model with the correct \( \delta \nu_0 \) and reduced neutrino capture rates; such a solution
would, however, be rather unsatisfactory. To constrain the composition profile additional physics
must be brought in, such as opacities or nuclear energy generation. Then it is in principle possible
to obtain predictions of the neutrino flux from inversions of the observed oscillation frequencies;
the results obtained so far are somewhat uncertain, however, and depend on the choice of observed
data (Gough & Kosovichev 1990; Dziembowski, Pamyatnykh & Sienkiewicz 1990).

5 Conclusion

The results discussed here provide a striking confirmation of the prediction by Simon (1982) of
the need for opacity increases; also they show that by treating the physics of stellar interiors
in more detail, the agreement with the observed properties may be improved. This evidently
provides ample justification for the large amount of effort that has been put into the new opacity
calculations. Additional confirmation has come from, for example, studies of apsidal motion in
binary stars (e.g. Claret & Giménez 1993). Thus the new opacities provide a more secure basis for
further investigations of stellar evolution.
From a physical point of view, the most interesting aspect of the results on solar and stellar oscillations is that they test the description of the very complex properties of a plasma and its interaction with radiation. By far the most extensive information results from the helioseismic analysis of solar oscillations. However, the solar data sample only a limited part of the \((\log \rho, \log T)\) plane, where the opacity modifications have been relatively modest. Thus solar structure is largely insensitive to the opacity at temperatures below about \(2 \times 10^6\) K which in the Sun correspond to the convection zone whose structure is independent of opacity. In contrast, the periods of double-mode Cepheids and \(\delta\) Scuti stars, and the excitation of B stars, are very sensitive to the opacity at temperatures between \(10^5\) and \(10^6\) K where the largest opacity increases have occurred. Thus here, as in other astrophysical problems, there is complementarity between having very detailed information about a single object, i.e., the Sun, and more superficial information about a number of objects which span a range of parameters. It should also be noted that, in addition to the opacity, observations of solar oscillations have provided information on the equation of state of matter in the Sun (e.g. Christensen-Dalsgaard, Däppen & Lebreton 1988; Vorontsov, Baturin & Pamyatnykh 1992; Pérez Hernández & Christensen-Dalsgaard 1994; for a review, see also Christensen-Dalsgaard & Däppen 1992). One may hope in future to be able to extend this type of work to other stars.

In the solar case the information is sufficiently extensive that one can attempt a complete inversion for opacity corrections (Korzennik & Ulrich 1989; Saio 1992), by means of sensitivity functions similar to the one shown in Figure 11b. Thus, for example, it is tempting to eliminate the remaining small sound-speed differences in Figure 9 by postulating suitable modifications to the opacity; the required changes would only be a few per cent, certainly well within the remaining likely errors in the opacity calculations. Such an analysis, however, assumes that the opacity dominates the remaining error in the solar models. This may well be reasonable within the framework of "classical" solar models, defined entirely in terms of the microphysics, i.e., the opacity, equation of state and nuclear reaction rates: since matter is generally assumed to be almost completely ionized in the radiative interior of the Sun it is plausible that there are only insignificant uncertainties in the equation of state in this region; also, errors in the reaction rates would only affect the core of the model. However, the diffusion calculation should be improved to include also settling of heavy elements (e.g. Proffitt 1994). Also, it was shown by Christensen-Dalsgaard et al. (1993) that uncertainties in turbulent mixing beneath the convection zone can cause changes in the sound speed of a magnitude comparable with the remaining differences between the Sun and the model. Such effects cannot be investigated by helioseismic observations alone. Additional information will be required from, for example studies of lithium, beryllium and boron depletion in the Sun and other stars which may constrain the degree of mixing beyond the convectively unstable envelope.

The data on solar and stellar oscillations will increase dramatically over the coming years. A six-station network to observe solar oscillations will become operational during 1995 in the GONG project (e.g. Harvey et al. 1993) and the SOHO spacecraft, scheduled for launch in July 1995, will carry three instruments for helioseismic observations (e.g. Gabriel et al. 1991; Andersen 1991; Scherrer, Hoeksema & Busch 1991). Also, several groups are carrying out coordinated observations of stellar oscillations from several sites, reducing the effects of daily gaps in the data and providing increasingly detailed frequency spectra for different types of stars. Further work in this direction is likely to be very valuable; in particular, detailed observations of g-mode oscillations in the Slowly Pulsating B stars may provide unique information about conditions at the outer edges of convective cores. Observation of solar-like oscillations in other stars is also being pursued. Direct photometric detection seems improbably, even when a substantial number of large telescopes is used (Gilliland et al. 1993). However, the recent observations by Kjeldsen et al. (1994) offer hope for ground-based observations of solar-like oscillations in bright stars. Space observations are likely to be required to obtain detailed data on fainter stars. A very promising project is the STARS proposal which is currently undergoing Phase-A evaluation by ESA. This is expected to allow study of oscillations of
stars in several open clusters; when combined with the other data available for such clusters the results would provide stringent constraints on stellar evolution theory.

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**References**