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Doorway State Expansion Approach to Coupled Channels Problems and Application to Heavy Ions*

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Abstract

The doorway-expansion method is extended to Coupled Channels problems in low energy heavy ion collisions. As a test, it is applied to an exactly soluble model and the convergence problem is discussed.

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The method is then applied to heavy ion elastic scattering due to the optical potential and to a simple Coupled Channels problem. In both cases very good convergence is reached with 6 doorway states. The calculation with a single doorway is shown to be much better than the DWBA.

1 Introduction

The theoretical description of nuclear reactions presents numerical difficulties which become very serious when the reaction dynamics involves a large number of channels [1], [2]. If one wants to go beyond semiclassical [2], [3] or perturbative [2] approximations, it is necessary to look for efficient techniques to handle the coupled equations.

The Doorway Expansion Method (DEM), developed for describing pion-nucleus interactions in the region dominated by the Δ resonance [4], has proven to be very useful as a powerful calculational procedure while it affords physical insight into the important many-body dynamics. Subsequently, this approach was extended and applied to the scattering from an optical potential [5]. It was possible, from the obtained analytical results, to identify the physical parameters controlling the convergence of the doorway expansion. It was found that these parameters are simply related to the target geometry and to the interaction strength.

In this paper we further extend the DEM to the more general case of coupled channels. This extension is tested in an exactly soluble model and the convergence of the expansion is studied. The method is then applied to a typical heavy ion reaction with strong absorption and strong coupling between the elastic and one inelastic channel.

The paper is organized as follows. In section 2, a resume of the doorway state approach to optical potential scattering is presented and the generalization to coupled channels is developed, for both plane wave and distorted wave representations. The formalism is then tested in an application to an exactly soluble coupled channels problem , with a separable interaction. In section 3 we consider more realistic coupled channels problems encountered in a typical heavy ion collision. We choose the $^{16}O + ^{28}Si$ collision at $E_{cm} = 55$ MeV, populating the giant monopole mode in the ^{28}Si target. Finally, in section 4 we present a brief discussion and our concluding remarks.

2 The Doorway Expansion Method (DEM)

2.1 Scattering from a short range potential

Let us start with the scattering from a short range potential. In this case, the collision is described by the Hamiltonian

$$H = H_0 + U \ , \tag{1}$$

and the transition operator T satisfies the Lippmann-Schwinger equation

$$T = U + U\ G^{(+)}\ T \ . \tag{2}$$

Above, $H_0 = K$ is the kinetic energy operator, U is a potential vanishing faster than $1/r$ as $r \to \infty$, and $G^{(+)}$ is the free particle Green's function: $G^{(+)} = (E - H_0 + i\eta)^{-1}$. Eq (2) takes a convenient form if one writes U as a product of two factors

$$U = v \cdot \bar{v} \ . \tag{3}$$

In this case, it can be written

$$T = v\ (1 - W)^{-1}\ \bar{v} \ , \tag{4}$$

where W is the rescattering operator, defined as

$$W = \bar{v}\ G^{(+)}\ v \ . \tag{5}$$

The choice of v and $\bar{v}$ depends on the scattering problem considered. In the present case, it is convenient to set $v = \bar{v} = U^{1/2}$.

As shown in Reference [5], these equations lead to an efficient technique to derive approximations for the transition operator and the wave function. For this purpose, we define the entrance channel doorway state and its dual space counterpart

$$|D^{(0)}> = N_0\ \bar{v}\ |\mathbf{k}> \ , \qquad |\tilde{D}^{(0)}> = \tilde{N}_0\ v^\dagger\ |\mathbf{k}> \ , \tag{6}$$

where $|\mathbf{k}>$ is the incident plane wave. The normalization constants N_0 and $\tilde{N}_0$ satisfy the relation

$$N_0\ \tilde{N}_0^* = 1/ < \mathbf{k}|U|\mathbf{k} > \ . \tag{7}$$

To complete our description, we define the remaining elements of the orthonormal doorway basis;

$$\begin{aligned} |D^{(n)}> &= N_n \left\{ W|D^{(n-1)}> - \sum_{j=0}^{n-1} |D^{(j)}> < \tilde{D}^{(j)}|W|D^{(n-1)}> \right\} \\ |\tilde{D}^{(n)}> &= \tilde{N}_n \left\{ W^\dagger|\tilde{D}^{(n-1)}> - \sum_{j=0}^{n-1} |\tilde{D}^{(j)}>< D^{(j)}|W^\dagger|\tilde{D}^{(n-1)}> \right\} \ . \end{aligned} \tag{8}$$

The product of the coefficients N_n and $\tilde{N}_n$ is determined by the normalization condition

$$< \tilde{D}^{(n)}|D^{(n)} >= 1 \ , \tag{9}$$

which implies that

$$N_n^{-1} =< \tilde{D}^{(n)}|W|D^{(n-1)} > ; \qquad \tilde{N}_n^{-1} =< D^{(n)}|W^\dagger|\tilde{D}^{(n-1)} > \ . \tag{10}$$

In terms of the doorway basis, the T-operator can be written

$$T = \sum_{nm} U^{1/2}|D^{(n)}>\ \Gamma^{(n,m)}\ < \tilde{D}^{(m)}|\ U^{1/2} \ , \tag{11}$$

where $\Gamma^{(n,m)}$ are the matrix elements

$$\Gamma^{(n,m)} =< \tilde{D}^{(n)}|\ \frac{1}{1-W}\ |D^{(m)}> \ . \tag{12}$$

It is easy to show that the Lippmann-Schwinger equation for the wave function $|\Psi_{\mathbf{k}}^{(+)}>$ leads to the doorway basis expansion

$$|\Psi_{\mathbf{k}}^{(+)}>= [N_0 U^{1/2}]^{-1} \sum_n \Gamma^{(n,0)}\ |D^{(n)}> \ . \tag{13}$$

A very convenient property of this basis is that it makes the rescattering matrix tridiagonal, i.e.,

$$W^{(n,m)} \equiv< \tilde{D}^{(n)}|W|D^{(m)} >= 0, \qquad \text{if } |n-m| > 1 \ . \tag{14}$$

This property has two important consequences. The first is that the matrix element $\Gamma^{(0,0)}$, which corresponds to the diagonal element of the T-matrix, can be written as the continued fraction

$$\Gamma^{(0,0)} \equiv \left(N_0 \tilde{N}_0^*\right) < \mathbf{k} \mid T \mid \mathbf{k} >= \cfrac{1}{1 - W^{(0,0)} - \cfrac{W^{(0,1)} W^{(1,0)}}{1 - W^{(1,1)} - \cfrac{W^{(1,2)} W^{(2,1)}}{1 - W^{(2,2)} \ldots}}} \, . \tag{15}$$

The second consequence is that $\Gamma^{(n,m)}$ can be generated from $\Gamma^{(0,0)}$ through the recursion relation

$$\Gamma^{(n,m)} = [\delta_{nm} + W^{(n,n-1)} \, \Gamma^{(n-1,m)} + W^{(n,n+1)} \, \Gamma^{(n+1,m)}] \, / (1 - W^{(n,n)}) \, , \tag{16}$$

and an analogous relation with respect to the index m. These results were derived and applied to model problems in Reference [5].

2.2 Scattering by two potentials

A very useful extension of the previous section is to the case of the scattering by a potential given as a sum of two terms

$$U = V + \Delta V \, . \tag{17}$$

In this case, we include the potential V in the hamiltonian H_0, so that the Green's function $G^{(+)}$ contains effects of distortion by this potential. The transition operator can be put in the form

$$T = T_0 \; + \; \Delta T \, . \tag{18}$$

Above, T_0 is the T-matrix associated to H_0 and ΔT is a correction arising from ΔV. This correction can be written

$$\Delta T = (\Omega^{(-)})^\dagger \; t \; \Omega^{(+)} \, , \tag{19}$$

in terms of the Møller wave operators associated to H_0, $\Omega^{(\pm)}$, and a "reduced transition matrix" t, which satisfies the Lippmann-Schwinger equation

$$t = \Delta V \; + \; \Delta V \; G^{(+)} t \, . \tag{20}$$

The doorway expansion can also be applied to eq. (20). Writing

$$\Delta V = v \; \cdot \; \bar{v} \, , \tag{21}$$

one can put the Lippmann-Schwinger equation in the form

$$t = v \; (1 - W)^{-1} \; \bar{v} \, . \tag{22}$$

The rescattering operator is given by eq. (5), but now $G^{(+)}$ is distorted by V.

The matrix elements $\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})}$ are given by

$$\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})} \equiv < \mathbf{k}' | \; \Delta T \; | \mathbf{k} >=< \chi^{(-)}(\mathbf{k}') \mid t \mid \chi^{(+)}(\mathbf{k}) > \, , \tag{23}$$

where $|\chi^{(\pm)}(\mathbf{k}) >= \Omega^{(\pm)} \, |\mathbf{k} >$ are incident waves distorted by the potential V.

The entrance channel doorway states are then defined as

$$\begin{aligned} |D^{(0)} > \; &\equiv \; N_0 \; \bar{v} \; \Omega^{(+)} \; |\mathbf{k} >= N_0 \; \bar{v} \; |\chi^{(+)}(\mathbf{k}) > \\ |\tilde{D}^{(0)} > \; &\equiv \; \tilde{N}_0 \; v^\dagger \; \Omega^{(-)} \; |\mathbf{k}' >= \tilde{N}_0 \; v^\dagger \; |\chi^{(-)}(\mathbf{k}') > \; . \end{aligned} \tag{24}$$

This definition is analogous to that of the previous section, in the sense that $|D^{(0)} >$ ($|\tilde{D}^{(0)} >$) is generated by the action of $\bar{v}$ ($v^\dagger$) on the incident wave. The remaining states of the doorway basis are given by eq. (8).

The doorway expansion for the transition operator becomes

$$\Delta T = \sum_{nm} \; (\Omega^{(-)})^\dagger \;\; v \; |D^{(n)} > \; \Gamma^{(n,m)} \; < \tilde{D}^{(m)}| \; \bar{v} \; \Omega^{(+)} \, , \tag{25}$$

and the matrix elements $\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})}$ are

$$\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})} = \sum_n \; < \chi^{(-)}(\mathbf{k}') \mid v \; |D^{(n)} > \cdot \; \frac{\Gamma^{(n,0)}}{N_0} \, , \tag{26}$$

The expression of the wave function in terms of the distorted doorway basis has the same form as that of the previous section (eq. (13)).

It is convenient to carry out partial waves expansions for the distorted waves and for the matrix elements $\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})}$, as

$$\chi^{(+)}(\mathbf{k}, \mathbf{r}) \;\; = \;\; \frac{1}{(2\pi)^{3/2}} \; \frac{4\pi}{kr} \sum_{\ell m} \; i^\ell \; e^{i\sigma_\ell} \; \chi_\ell(kr) \;\; Y_{\ell m}(\hat{r}) \; \cdot \; Y^*_{\ell m}(\hat{k}) \tag{27}$$

$$\chi^{(-)}(\mathbf{k}, \mathbf{r}) \;\; = \;\; \frac{1}{(2\pi)^{3/2}} \; \frac{4\pi}{kr} \sum_{\ell m} \; i^\ell \; e^{-i\sigma_\ell} \; \chi^*_\ell(kr) \;\; Y_{\ell m}(\hat{r}) \; \cdot \; Y^*_{\ell m}(\hat{k}) \tag{28}$$

$$\Delta T_{(\mathbf{k}' \leftarrow \mathbf{k})} \;\; = \;\; \frac{1}{kk'} \sum_{\ell m} \; \Delta T_\ell(k, k') \;\; Y^*_{\ell m}(\hat{k}) \; \cdot \; Y_{\ell m}(\hat{k}') \, . \tag{29}$$

As we are dealing with elastic scattering, we can replace $k' = k$.

If V is the point charge Coulomb interaction, as will be the case in our application to heavy ion elastic scattering (sect. (3.1)), $\chi_\ell = \chi_\ell^* = F_\ell$, i.e. the regular Coulomb wave function.

To apply the DEM, we write ΔV in the product form of eq. (21), choosing

$$\bar{v} = 1; \qquad v = \Delta V \,. \tag{30}$$

Setting $N_0 = i^{-\ell} e^{-i\sigma_\ell}$, the ℓ-projected entrance channel doorways can be written

$$\begin{aligned} d_\ell^{(0)}(r) &= \chi_\ell(k,r) \\ \tilde{d}_\ell^{(0)}(r) &= \tilde{N}_0 \, v^\dagger \, \chi_\ell^*(k,r) \,. \end{aligned} \tag{31}$$

With this choice, the radial wave function reduces to $d_\ell^{(0)}(r)$ in the limit $\Delta V \to 0$, so that the doorway expansion converges with a single doorway.

The rescattering operator for the ℓ-th partial wave can be written

$$W_\ell = g_\ell^{(+)} \, v \,, \tag{32}$$

where $g_\ell^{(+)}$ is the ℓ-projected distorted Green's function with outgoing wave boundary condition. The remaining states of the doorway basis are given by eq. (8), in terms of W_ℓ and the ℓ-projected doorways of lower order.

Using eq. (29), we get

$$\Delta T_\ell(k,k) = \frac{2\, e^{2i\sigma_\ell}}{\pi} \sum_n \, < \chi_\ell^*(k) | \, v \, | d_\ell^{(n)} > \, \Gamma_\ell^{(n,0)} \,. \tag{33}$$

The matrix-element $\Gamma_\ell^{(0,0)}$ is given by the continued fraction of eq. (15), and $\Gamma_\ell^{(n,0)}$ can be obtained by sucessively applying the recursion relation of eq. (16). However, in the present case one must use the operator W_ℓ instead of W in these equations.

To calculate the elastic cross section one uses the relation

$$S_\ell^N = 1 \; - i \left(\frac{2\pi\mu}{\hbar^2 k} \right) \, e^{-2i\sigma_\ell} \; \cdot \; \Delta T_\ell(k,k) \;, \tag{34}$$

and with S_ℓ^N we obtain the scattering amplitude.

2.3 Coupled Channels

In many cases of practical interest, the results of the previous section may be very useful. A typical situation is when the reaction dynamics requires a coupled channels treatment. In such situations, the total nuclear state can be written

$$| \, \Psi_{\mathbf{k}}^{(+)} >= \sum_\alpha | \, \psi_\alpha^{(+)}(\mathbf{k})) \cdot | \, \alpha) \;, \tag{35}$$

where $| \;)$ represents states in the intrinsic or relative spaces which generate the total Hilbert space through direct products.

It is convenient to split the total interaction U (eq. (17)) such that V is diagonal in channel-space. One can then write

$$V = \sum_\alpha | \, \alpha) \, V_\alpha \, (\alpha | \;. \tag{36}$$

Above, $V_\alpha \equiv V_{\alpha,\alpha} = (\alpha \mid V \mid \alpha)$ is the optical potential in channel-α. One can next choose $\bar{v}$ diagonal in channel space (in our heavy ion applications we set $\bar{v} = 1$), so that the channel coupling interaction becomes

$$\Delta V = \sum_{\alpha,\beta} \; | \, \alpha) \, v_{\alpha,\beta} \cdot \bar{v}_{\beta,\beta} \, (\beta | \;, \tag{37}$$

where $v_{\alpha,\beta} = (\alpha \mid v \mid \beta)$ and $\bar{v}_{\beta,\beta} = (\beta \mid \bar{v} \mid \beta)$ are operators acting in the r-space.

For the definition of the doorway basis, one needs also the solutions of Schrödinger equation in the absence of the coupling ΔV,

$$| \, \Phi_{\mathbf{k}}^{(\pm)} >= \sum_\alpha | \, \chi^{(\pm)}(\mathbf{k}_\alpha)) \cdot | \, \alpha) \;, \tag{38}$$

Above, $\chi^{(+)}(\mathbf{k}_\alpha)$ ($\chi^{(-)}(\mathbf{k}_\alpha)$) is the distorted wave in channel-α with outgoing (ingoing) boundary condition and $k_\alpha = \sqrt{2\mu(E - \varepsilon_\alpha)/\hbar^2}$ is the channel wave number, which depends on the excitation energy ε_α.

We can now define the entrance doorway state and its dual space counterpart as

$$\begin{aligned} | \, D^{(0)} > &= N_0 \, \bar{v} \; | \, \chi^{(+)}(\mathbf{k}_0)) \cdot | \, 0) \\ | \, \tilde{D}^{(0)} > &= \tilde{N}_0 \; v^\dagger \; | \, \chi^{(-)}(\mathbf{k}_1)) \cdot | \, 1) \,. \end{aligned} \tag{39}$$

The projections of these states in each channel are

$$
\begin{aligned}
\mid D_\alpha^{(0)}) &= N_0\ \bar{v}_{\alpha,0}\ \mid \chi^{(+)}(\mathbf{k}_0)) \cdot \delta_{\alpha 0} \\
\mid \tilde{D}_\alpha^{(0)}) &= \tilde{N}_0\ v_{\alpha,1}^\dagger\ \mid \chi^{(-)}(\mathbf{k}_1)) \ . \qquad (40)
\end{aligned}
$$

While $\mid D^{(0)} >$ is fully contained in the elastic channel, $\mid \tilde{D}^{(0)} >$ has also components in the excited channels, due to the non-diagonal nature of ΔV.

The generalization of the rescattering operator leads to

$$
W = \sum_{\alpha,\beta} \mid \alpha)\ W_{\alpha,\beta}\ (\beta \mid , \qquad (41)
$$

with

$$
W_{\alpha,\beta} = \bar{v}_{\alpha,\alpha} \cdot G_\alpha^{(+)} \cdot v_{\alpha,\beta} \ . \qquad (42)
$$

Above, $G_\alpha^{(+)}$ is the optical Green's function in channel-α and we used the fact that $\bar{v}$ is diagonal in channel space.

The remaining states of the doorway basis can now be written in terms of their channel-projections as

$$
\mid D_\alpha^{(n)}) = N_n \left\{ \sum_\beta W_{\alpha,\beta}\ \mid D_\beta^{(n-1)}) - \sum_{j=0}^{n-1} < \tilde{D}^{(j)} \mid W \mid D^{(n-1)} > \mid D_\alpha^{(j)}) \right\} \qquad (43)
$$

and

$$
\mid \tilde{D}_\alpha^{(n)}) = \tilde{N}_n \left\{ \sum_\beta W_{\alpha,\beta}^\dagger\ \mid \tilde{D}_\beta^{(n-1)}) - \sum_{j=0}^{n-1} < D^{(j)} \mid W^\dagger \mid \tilde{D}^{(n-1)} > \mid \tilde{D}_\alpha^{(j)}) \right\} . \qquad (44)
$$

The contribution to the T-matrix arising from ΔV is given by

$$
\begin{aligned}
\Delta T_{(\mathbf{k}_\alpha;\alpha\leftarrow\mathbf{k}_0;0)} &= < \chi^{(-)}(\mathbf{k}_\alpha) \mid\ t_{\alpha,0}\ \mid \chi^{(+)}(\mathbf{k}_0) > \\
&= \sum_{n,\beta} < \chi^{(-)}(\mathbf{k}_\alpha) \mid\ v_{\alpha,\beta}\ \mid D_\beta^{(n)} > \frac{\Gamma^{(n,0)}}{N_0} \ . \qquad (45)
\end{aligned}
$$

The T-matrix for the excitation of channel-1, in terms of which the doorway $\mid \tilde{D}^{(0)} >$ was defined (eq. (39)), is related to the corresponding DWBA matrix element in a simple way. Since ΔT^{DWBA} is

$$
\Delta T^{\mathrm{DWBA}}_{(\mathbf{k}_1;1\leftarrow\mathbf{k}_0;0)} =< \chi^{(-)}(\mathbf{k}_1) \mid v_{1,0} \mid \chi^{(+)}(\mathbf{k}_0) >= (\tilde{N}_0^* N_0)^{-1} \ , \qquad (46)
$$

we can write

$$
\Delta T_{(\mathbf{k}_1;1\leftarrow\mathbf{k}_0;0)} = \Delta T^{\mathrm{DWBA}}_{(\mathbf{k}_1;1\leftarrow\mathbf{k}_0;0)} \ \cdot\ \Gamma^{(0,0)} \ . \qquad (47)
$$

For applications of the DEM to low energy coupled channels problems, one frequently needs to carry out partial wave decompositions. This task is particularly simple for scalar couplings. In this case, ΔV conserves orbital angular momentum and the partial wave projection is analogous to that of the previous section. The ℓ-projected T-matrix elements can be written

$$
\Delta T_{\ell(\alpha\leftarrow 0)} = \left(\frac{2}{\pi}\right)\ e^{i(\sigma_{\alpha,\ell}+\sigma_{0,\ell})} \sum_n < \chi_\ell^*(k_\alpha) \mid\ v_{\alpha,\beta}\ \mid d_\ell^{(n)} > \frac{\Gamma_\ell^{(n,0)}}{N_0} \ , \qquad (48)
$$

where $\sigma_{\alpha,\ell}$ is the Coulomb phase shift for the partial wave ℓ in channel α.

2.4 Study of an exactly soluble Coupled Channels model

In this section we apply the doorway expansion formalism to a soluble Coupled Channels model. We consider a short ranged scalar interaction with a separable form, and study the s-wave T-matrix elements. In this case the channel expansion of U (eqs. (36) and (37)) leads to potentials

$$
U_{\alpha,\beta}(r,r') \equiv < \mathbf{r};\alpha \mid U \mid \beta;\mathbf{r}' >= u_\alpha(r) \cdot g_{\alpha\beta} \cdot u_\beta^*(r'). \qquad (49)
$$

Above, $g_{\alpha\beta}$ are the channel-coupling strengths and $u_\alpha(r)$ and $u_\beta(r')$ are scalar functions of the relative (projectile-target) coordinate.

The Coupled Channels scattering problems with the above interaction can be solved exactly [6]. It is easy to show that the inelastic transition amplitude can be written

$$
T_{(\mathbf{k}_\alpha;\alpha\leftarrow\mathbf{k}_0;0)} =< \mathbf{k}_\alpha \mid u_\alpha > \cdot < u_0 \mid \mathbf{k}_0 > \ \cdot\ f_\alpha(E) \ , \qquad (50)
$$

where E is the collision energy and $f_\alpha(E)$ is given by the equation

$$\sum_\beta [\delta_{\alpha\beta} - g_{\alpha\beta}\, \mathcal{G}_\beta] \cdot f_\beta(E) = g_{\alpha 0} \tag{51}$$

Above, the coefficient $\mathcal{G}_\beta$ is

$$\mathcal{G}_\beta =< u_\beta \mid (E - K + i\eta)^{-1} \mid u_\beta >= \int \frac{dp\, p^2 \;\mid u_\beta(p) \mid^2}{\left(2\mu/\hbar^2\right) E_\beta - p^2 + i\eta}\,, \tag{52}$$

where $E_\beta = E - \varepsilon_\beta$ is the collision energy in channel β.

In the present case it is convenient to split the total interaction so that V is diagonal and ΔV is purely off-diagonal in channel-space. Namely,

$$V_{\alpha,\beta} = \delta_{\alpha\beta}\, U_{\alpha,\beta}, \qquad (\Delta V)_{\alpha,\beta} = (1 - \delta_{\alpha\beta})\, U_{\alpha,\beta}\,. \tag{53}$$

It is convenient to write ΔV in the product form of eq (21) such that $\bar{v}$ is diagonal in channel space. This can be accomplished with

$$\begin{aligned} v_{\alpha,\beta}(r,r') &= (1 - \delta_{\alpha\beta}) \cdot g_{\alpha\beta} \cdot u_\alpha(r) \cdot Z(r') \\ \bar{v}_{\alpha,\beta}(r,r') &= \delta_{\alpha\beta} \cdot Z(r) \cdot u^*_\beta(r')\,, \end{aligned} \tag{54}$$

where Z is an arbitrary real function, normalized to unity. As will be clear below, the transition amplitudes do not depend on the particular choice of this function.

Splitting the T-matrix into two terms, $T = T_0 + \Delta T$, as in eqs.(18) and (19), one trivially gets

$$T_{0(\mathbf{k}_\alpha;\alpha\leftarrow\mathbf{k}_0;0)} = \delta_{\alpha 0} \cdot \;< \mathbf{k}_0 \mid u_0 > \cdot < u_0 \mid \mathbf{k}_0 > \left[\frac{g_{00}}{1 - g_{00}\mathcal{G}_0}\right]. \tag{55}$$

To get the second term one has do introduce the doorway basis. Following eq. (39) we write

$$\begin{aligned} D^{(0)}_\alpha(\mathbf{r}) &= N_0 \cdot \delta_{\alpha 0} \cdot \left[\frac{< u_0 \mid \mathbf{k}_0 >}{1 - g_{00}\, \mathcal{G}_0}\right] \cdot Z(r) \\ \tilde{D}^{(0)}_\alpha(\mathbf{r}) &= \tilde{N}_0 \cdot (1 - \delta_{\alpha 1}) \cdot \left[\frac{< u_1 \mid \mathbf{k}_1 >\; g_{1\alpha}}{1 - g_{11}\, \mathcal{G}_1}\right] \cdot Z(r)\,. \end{aligned} \tag{56}$$

As in the previous section, the remaining states of the doorway basis are defined by eq. (8) and $\Delta T_{(\mathbf{k}_\alpha;\alpha\leftarrow\mathbf{k}_0;0)}$ is given by eq. (45). However, the separable nature of the interaction leads to the analytical expression

$$\Delta T_{(\mathbf{k}_\alpha;\alpha\leftarrow\mathbf{k}_0;0)} = \left[\frac{< \mathbf{k}_\alpha \mid u_\alpha >}{1 - g_{\alpha\alpha}\, \mathcal{G}_\alpha}\right] \cdot \left\{\sum_{n,\beta} (1 - \delta_{\alpha\beta})\, \mathcal{C}^{(n)}_\beta\, \Gamma^{(n,0)}\right\} \cdot \left[\frac{< u_0 \mid \mathbf{k}_0 >}{1 - g_{00}\, \mathcal{G}_0}\right], \tag{57}$$

where $\mathcal{C}^{(n)}_\beta =< Z \mid D^{(n)}_\beta >$. ¿From the definitions of eq. (56) we see that

$$\mathcal{C}^{(0)}_\beta = \delta_{\beta 0}; \qquad \tilde{\mathcal{C}}^{(0)}_\beta =< \tilde{D}^{(0)}_\beta \mid Z >= 1 \;-\; \delta_{\beta 1}\,. \tag{58}$$

The remaining coefficients $\mathcal{C}^{(n)}_\beta$ can be obtained from eq. (43). One gets,

$$\begin{aligned} \mathcal{C}^{(n)}_\beta &= N_n \left\{\sum_{\beta'} (1 - \delta_{\beta\beta'})\, \mathcal{G}_\beta \cdot g_{\beta\beta'} \cdot \mathcal{C}^{(n-1)}_{\beta'} - \sum_{j=0}^{n-1} W^{(j,n-1)}\, \mathcal{C}^{(j)}_\beta\right\} \\ \tilde{\mathcal{C}}^{(n)}_\beta &= \tilde{N}^*_n \left\{\sum_{\beta'} (1 - \delta_{\beta\beta'})\, \mathcal{G}_{\beta'} \cdot g_{\beta\beta'} \cdot \tilde{\mathcal{C}}^{(n-1)}_{\beta'} - \sum_{j=0}^{n-1} W^{(n-1,j)}\, \tilde{\mathcal{C}}^{(j)}_\beta\right\}. \end{aligned} \tag{59}$$

The matrix-elements of the rescattering operator now become

$$W^{(n,m)} = \sum_{\alpha,\beta} \tilde{\mathcal{C}}^{(n)}_\alpha \cdot \mathcal{C}^{(m)}_\beta\, (1 - \delta_{\alpha\beta})\, \mathcal{G}_\alpha \cdot g_{\alpha\beta}\,. \tag{60}$$

It is important to notice that neither the entrance doorways nor the quantities appearing in the recursion relation for the basis (eq. (59)) depends on $Z(r)$. Therefore, the transition matrix is independent of the particular choice of this function.

To study the convergence of the DEM, we have evaluated the s-wave components of the above discussed T-matrix. In this schematic calculation, we choose units such that $\hbar = m = 1$. For the functions $u_\alpha(r)$, we took Yamaguchi [6] form factors, which led to

$$\mathcal{G}_\alpha = \left[\frac{\gamma}{\Delta - i\,\sqrt{E_\alpha}}\right]^2 \tag{61}$$

where γ and Δ are parameters and $E_\alpha = E - \varepsilon_\alpha$ is the collision energy in channel-α. For the coupling strength, we took the expression

$$g_{\alpha\beta} = \exp\left[-\left(\frac{\alpha-\beta}{a}\right)^2\right] . \tag{62}$$

Thus, the parameter a measures the extent to which the strength of the coupling is distributed among distant channels.

In table I we study the convergence of the DEM. The calculation of $T_{\ell=0(\alpha\leftarrow 0)}$ has been performed for 8 channels, with $N = 6, 7$ and 8 doorway states. For a comparison, we also present the corresponding DWBA amplitudes (last column). We used the parameters: $\Delta = 0.1$, $\gamma = 2.094$, $a = 2$ and assumed an equally spaced spectrum given by $\varepsilon_\alpha = 0.1\ \alpha$. The collision energy was $E = 20\ \alpha$. As one can see, the convergence is very good, if the number of doorways, N, is large enough. One notices that the convergence of each excitation amplitude requires a different number of doorways. The relation between the channel label α and the minimal N for convergence is roughly $N \approx \alpha$. This rule is an immediate consequence of our choice for $\tilde{D}^{(0)}$, which is obtained through the action of the coupling interaction on the first excited channel. It is important to notice that, even with a few doorways, the DEM is much more accurate than the DWBA. Whereas in the previous study of the one-channel scattering problem [5] the parameters that decide upon the convergence proprieties of the doorway method were identified clearly, it has not proved to make such a clear identification here. The underlying reason is the multiplicity of parameters. Nevertheless, this model problem clearly has very strong channel coupling effects; indeed, the DWBA result is in serious disagreement with the exact results. Thus, the $N \approx \alpha$ result is quite encouraging. In any case, the DEM is a constructive procedure which can be truncated when convergence is achieved.

3 Application to heavy ion collisions

In this section we apply the Doorway Expansion Method to low energy heavy ion collisions. Such collisions are influenced by the long ranged Coulomb interaction and by the short ranged potentials, arising from nuclear forces. One should split the interaction in two parts and use the results of section (2.2). As examples we study the elastic scattering due to the optical potential and a simple coupled channels problem.

3.1 Elastic scattering by the optical potential

We take $V = Z_p Z_t e^2/r$ and write ΔV as the sum

$$\Delta V = V_{\rm opt} + \Delta V_c . \tag{63}$$

Above, $V_{\rm opt}$ is the optical potential and ΔV_c is a correction to the point charge Coulomb interaction,

$$\Delta V_c = \begin{cases} (Z_p\ Z_t\ e^2/2R_c)\ \left[3-(r/R_c)^2\right]\ -\ V\ ; & r < R_c \\ 0\ ; & r \geq R_c \end{cases} , \tag{64}$$

where R_c is the Coulomb radius parameter.

We now apply the DEM to the elastic scattering of ^{16}O from ^{28}Si, at the collision energy $E \equiv E_{\rm cm} = 55$ MeV. For $V_{\rm opt}$ (eq. (63)), we use the E18 strong absorption potential [7], which has the usual Saxon-Woods form with the parameters $V_0 = 10$ MeV, $r_r = 1.35$ fm, $a_r = 0.618$ fm; $W_0 = 23.4$ MeV, $r_i = 1.23$ fm and $a_i = 0.552$ fm (the subscripts r and i stand for real the imaginary parts). The Coulomb radius parameter was taken as $R_c = 1.2(A_1^{1/3} + A_2^{1/3})$ fm.

In fig. 1, we illustrate the behavior of the first two states of the doorway basis and their dual space counterparts, for the peripheral partial wave $\ell = 37$. One firstly notices that, due to the choice of eq. (31), $d_\ell^{(0)}$ is real. One notices also that $d_\ell^{(n)}$ oscillates as $r \to \infty$, while the states of the dual space are confined. This is a consequence of the fact that they are obtained through the action of the short range potential $v^\dagger$.

In fig. 2 we show the absolute value (fig. 2a) and the phase (fig. 2b) of the nuclear S-matrix, as functions of the angular momentum ℓ. The solid line corresponds to the DEM calculation and the solid circles are the "exact results" of a standard optical model code. In fig. 3, the comparison if for the $\ell = 37$ wave function. We conclude that the DEM leads to very accurate wave functions and S-matrices.

The convergence of the DEM calculation is studied in fig. 4. We plot the deviation $\Delta S_\ell(n) = \mid (S_\ell(n) - S_\ell)/S_\ell \mid$, where $S_\ell(n)$ is the DEM result obtained with n doorways and S_ℓ is "exact" nuclear S-matrix obtained with the optical model code. We see that the convergence is rapidly reached. With a few doorways it is accurate to within a few percent.

3.2 Coupled Channels problems in heavy ion collisions

Is this section, we illustrate the use of the DEM in coupled channels problems, applying it to the excitation of monopole vibrations. In this case, the scalar nature of the coupling leads to conservation of orbital angular momentum. We consider the excitation of the ^{28}Si target in the collision considered in the previous section. The total interaction U is the same strong absorption potential of the previous section, but now with an oscillating radius. The static target radius is modified as

$$R_T \rightarrow R_T(x) = R_T \ [1 + x \ (b/R_T)] \ , \tag{65}$$

Above, x is the dimensionless intrinsic coordinate, which is normalized with respect to the oscillator length b.

With the target vibration, the optical potential radii become x-dependent, in the form

$$R_{r,i} = r_{r,i} \ \left[A_P^{1/3} \ + \ A_T^{1/3}(1 + x \ (b/R_T))\right] \ . \tag{66}$$

As before, the subscripts r and i stand for real and imaginary parts, respectively. The x-dependence of the optical potential leads to the coupling between the elastic and vibrational excited channels. For simplicity, we will restrict our calculation to two channels, corresponding to the ground state and to the excitation of a single phonon. Their intrinsic wave functions are

$$(x \mid 0) = \left(\frac{1}{\sqrt{\pi}}\right)^{1/2} \ e^{-x^2/2}; \qquad (x \mid 1) = \left(\frac{2}{\sqrt{\pi}}\right)^{1/2} \ x \ \cdot e^{-x^2/2} \ . \tag{67}$$

The vibrating potential U is split into two terms, as in eq. (17): the distorting potential, diagonal in channel space,

$$V(r) = U(r, x = 0) \tag{68}$$

and the channel coupling term

$$\Delta V(r, x) = U(r, x) - V(r) \ . \tag{69}$$

In the present case, it is convenient to write ΔV as in eq. (21) and choose $\bar{v} = 1$ and $v = \Delta V$.

The intrinsic energy of the one-phonon state, $\mid 1)$, should be $\varepsilon_1 = 17.2$ MeV, according to the systematics of the monopole vibration energies [8]. However, the corresponding reaction Q-value ($Q = -\varepsilon_1$) is too large and the energy mismatch between the distorted waves $\chi^{\pm}(\mathbf{k}_0)$ and $\chi^{\pm}(\mathbf{k}_1)$ leads to very weak excitation amplitudes. We then chose to use the smaller value $Q = -5$ MeV. As one can see from eq. (66), the effect of the harmonic vibration on the optical potential grows with the oscillator length b. One can, therefore, use the parameter b as a measure of the channel coupling strength.

In fig. 5 we show the elastic (5a) and inelastic (5b) S-matrices as functions of the angular momentum ℓ. The collision is ^{16}O + ^{28}Si and the energy $E =$ 55 MeV, as in the previous section. The coupling parameter is $b = 1.5$ fm. The figures include results of DEM calculations with $n = 1$ (dashed-line) and $n = 6$ (solid lines), which gives convergence within a few percent. They include also results of a DWBA calculation. In the elastic case (fig. 5a), the three curves are similar. In the inelastic case (fig. 5b), however, the DWBA S-matrix differs appreciably from those of DEM calculations. We see also that the calculation with a single doorway is already quite close to the converging result.

In fig. 6 we show the angular distributions for elastic (6a) and inelastic (6b) scattering. The details of the collision and the convention for the curves shown are the same as in fig. 5. In the case of elastic scattering, both the DWBA and the 1-doorway DEM (DEM1) fall below the converging DEM calculation. However, the DEM1 is closer to the DEM, in the sense that it does not show the pronounced oscillations of the DWBA results. In the case of inelastic scattering, the DWBA approximation is clearly much poorer then the DEM1. We see that the Doorway expansion method converges very fast and that even the calculation with a single doorway is quite accurate.

In fig. 7, we study the dependence of the above discussion on the strength of the channel coupling. Again, the details of the collision and the convention for the curves shown are the same as in fig. 5. In figs. 7a and 7b, we show results for the absolute value and for the phase of the inelastic S-matrix. We see that one gets a good approximation with a single doorway, even for very strong coupling ($b > 2.5$ fm) where the DWBA S-matrix violates unitarity.

Finally, in fig. 8, we show the relative deviation $\Delta S(n) = |(S_\ell(n) - S_\ell)/S_\ell|$, where S_ℓ stands for the "exact" converging DEM value and $S_\ell(n)$ for the

calculation with n doorways, as a function of n. At $n = 0$, we plot the result of the DWBA calculation. We conclude that the convergence is attained very rapidly. Finally, we remark that we have compared our converged result to the results obtained with a standard couple channels calculation [9] and found agreement within a few percent.

4 Conclusions

The doorway expansion method (DEM) has been extended to coupled channels problems and to typical situations in heavy ion collisions. The theory has been tested in an exactly soluble coupled channels problem and the convergence of the method investigated. It has been shown that very good convergence is obtained when the number of doorways reaches the number of coupled channels.

An application of the method to a typical coupled channels problem commonly encountered in strongly absorptive heavy ion systems has been made. The example worked out in detail is the elastic scattering and the excitation of the giant monopole mode in ^{28}Si as a target in the collision ^{16}O + ^{28}Si at $E_{cm} = 55$ MeV. We have shown that the DEM converges very rapidly. With 6 doorway states, the DEM inelastic cross section reaches the asymptotic value within a few percent and with a single doorway the approximation is already much better than the DWBA.

One reason for the rapid convergence could be the strongly absorptive nature of the collision. However, we are unable to pin down a simple a convergence parameter. We expect it to be a combination of several factors: the ratio of the imaginary to the real parts of the optical potential, the degree of inelasticity (Q-value), the nature of the coupling potential and the choice of the entrance-channel doorway states. Clearly, such a convergence parameter will depend on the multiplicity of parameters characterizing non-trivial coupled channels problems.

Acknowledgement
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Table Captions.

- **Table I:** The convergence of the DEM in an exactly soluble Coupled Channels problem. The table shows the real A_r and imaginary A_i parts of the transition amplitudes $T_{\ell=0(\alpha\leftarrow 0)}$, for the channels $\alpha = 0, 1, ...7$. For each channel, the table shows also the exact and the DWBA values.

Figure Captions.

- **Fig. 1:** The first two states of the doorway basis and their dual space counterparts in the collision $^{16}O + ^{28}Si$ at $E = 55$ MeV, for the partial wave $\ell = 37$.
- **Fig. 2:** Absolute values (a) and phases (b) of the nuclear S-matrix. The system and scattering conditions are the same as in fig. 1. The solid line represents the the DEM calculation and the solid circles are "exact results" of an optical model code.
- **Fig. 3:** Comparison between the "exact" (optical model code) and the DEM wave functions for the same collision as in the previous figures.
- **Fig. 4:** Study of the convergence of the DEM calculation. The collision is the same as in fig. 1. For details see the text.
- **Fig. 5:** Absolute values of the elastic (a) and inelastic (b) S-matrices obtained with the DEM and the DWBA. The results are plotted as functions of the angular momentum. For details see the text.
- **Fig. 6:** Elastic (a) and inelastic angular distributions obtained with the DEM and with the DWBA. For details see the text.
- **Fig. 7:** Comparison of the absolute values (a) and the phases (b) of the inelastic S-matrix as obtained with the DEM, DEM1 and DWBA. The results are plotted as functions of the coupling strength.
- **Fig. 8:** Study of the convergence of the DEM. The inset shows the the deviation $\Delta S(n)$ in a smaller scale. At $n = 0$ we plot the deviation for the DWBA calculation. For further details see the text.

TABLE I

Channel / amplitude		Number of Doorways 6	7	8	Exact	DWBA
0	A_r	−0.3263	−0.3272	−0.3273	−0.3272	−0.48
	A_i	0.3826	0.3827	0.3827	0.3827	−0.50
1	A_r	−0.1043	−0.1043	−0.1043	−0.1043	0.015
	A_i	0.2809	0.2809	0.2809	0.2809	0.35
2	A_r	$0.6348\text{x}10^{-1}$	$0.6414\text{x}10^{-1}$	$0.6422\text{x}10^{-1}$	$0.6417\text{x}10^{-1}$	$0.55\text{x}10^{-2}$
	A_i	$0.8267\text{x}10^{-1}$	$0.8257\text{x}10^{-1}$	$0.8253\text{x}10^{-1}$	$0.8259\text{x}10^{-1}$	0.16
3	A_r	$0.3072\text{x}10^{-1}$	$0.5143\text{x}10^{-1}$	$0.5137\text{x}10^{-1}$	$0.5141\text{x}10^{-1}$	$0.89\text{x}10^{-3}$
	A_i	$-0.3029\text{x}10^{-1}$	$-0.2986\text{x}10^{-1}$	$-0.2968\text{x}10^{-1}$	$-0.2981\text{x}10^{-1}$	$0.42\text{x}10^{-1}$
4	A_r	$-0.5410\text{x}10^{-2}$	$-0.6247\text{x}10^{-2}$	$-0.6043\text{x}10^{-2}$	$-0.6191\text{x}10^{-2}$	$0.96\text{x}10^{-7}$
	A_i	$-0.2709\text{x}10^{-1}$	$-0.2788\text{x}10^{-1}$	$-0.2818\text{x}10^{-1}$	$-0.2794\text{x}10^{-1}$	$0.69\text{x}10^{-2}$
5	A_r	$-0.1404\text{x}10^{-1}$	$-0.1464\text{x}10^{-1}$	$-0.1511\text{x}10^{-1}$	$-0.1475\text{x}10^{-1}$	$-0.21\text{x}10^{-4}$
	A_i	$0.7992\text{x}10^{-3}$	$0.1034\text{x}10^{-2}$	$0.1264\text{x}10^{-2}$	$0.9830\text{x}10^{-3}$	$0.68\text{x}10^{-3}$
6	A_r	$-0.1733\text{x}10^{-2}$	$-0.1142\text{x}10^{-2}$	$-0.7112\text{x}10^{-3}$	$-0.1086\text{x}10^{-2}$	$-0.29\text{x}10^{-5}$
	A_i	$0.6149\text{x}10^{-2}$	$0.6780\text{x}10^{-2}$	$0.6698\text{x}10^{-2}$	$0.6873\text{x}10^{-2}$	$0.40\text{x}10^{-4}$
7	A_r	$0.3469\text{x}10^{-2}$	$0.3656\text{x}10^{-2}$	$0.3497\text{x}10^{-2}$	$0.3661\text{x}10^{-2}$	$-0.19\text{x}10^{-6}$
	A_i	$0.6924\text{x}10^{-3}$	$0.2095\text{x}10^{-3}$	$0.2192\text{x}10^{-3}$	$0.1775\text{x}10^{-3}$	$0.14\text{x}10^{-5}$

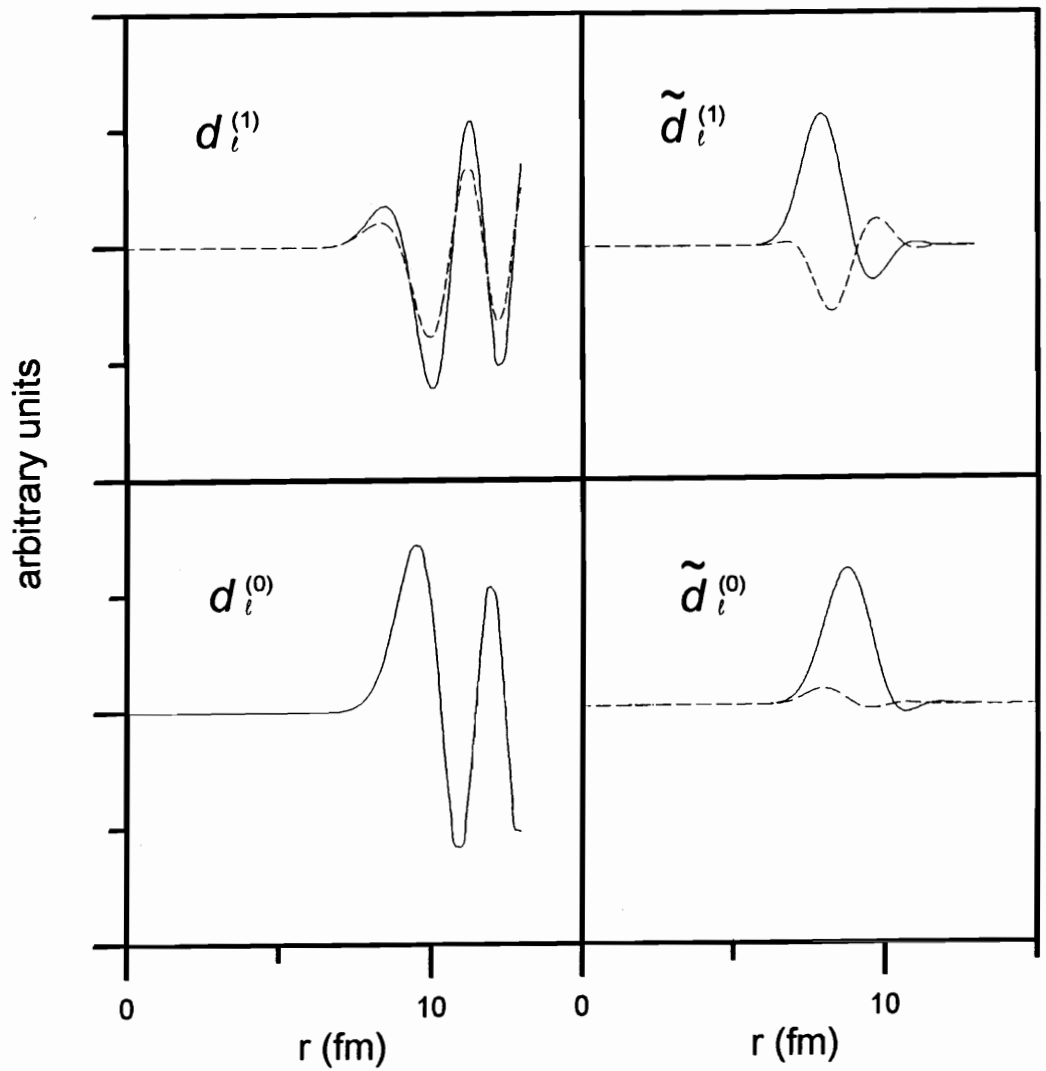
$d_\ell^{(1)}$
$\tilde{d}_\ell^{(1)}$
$d_\ell^{(0)}$
$\tilde{d}_\ell^{(0)}$
arbitrary units
0
10
r (fm)
0
10
r (fm)

Fig. 1

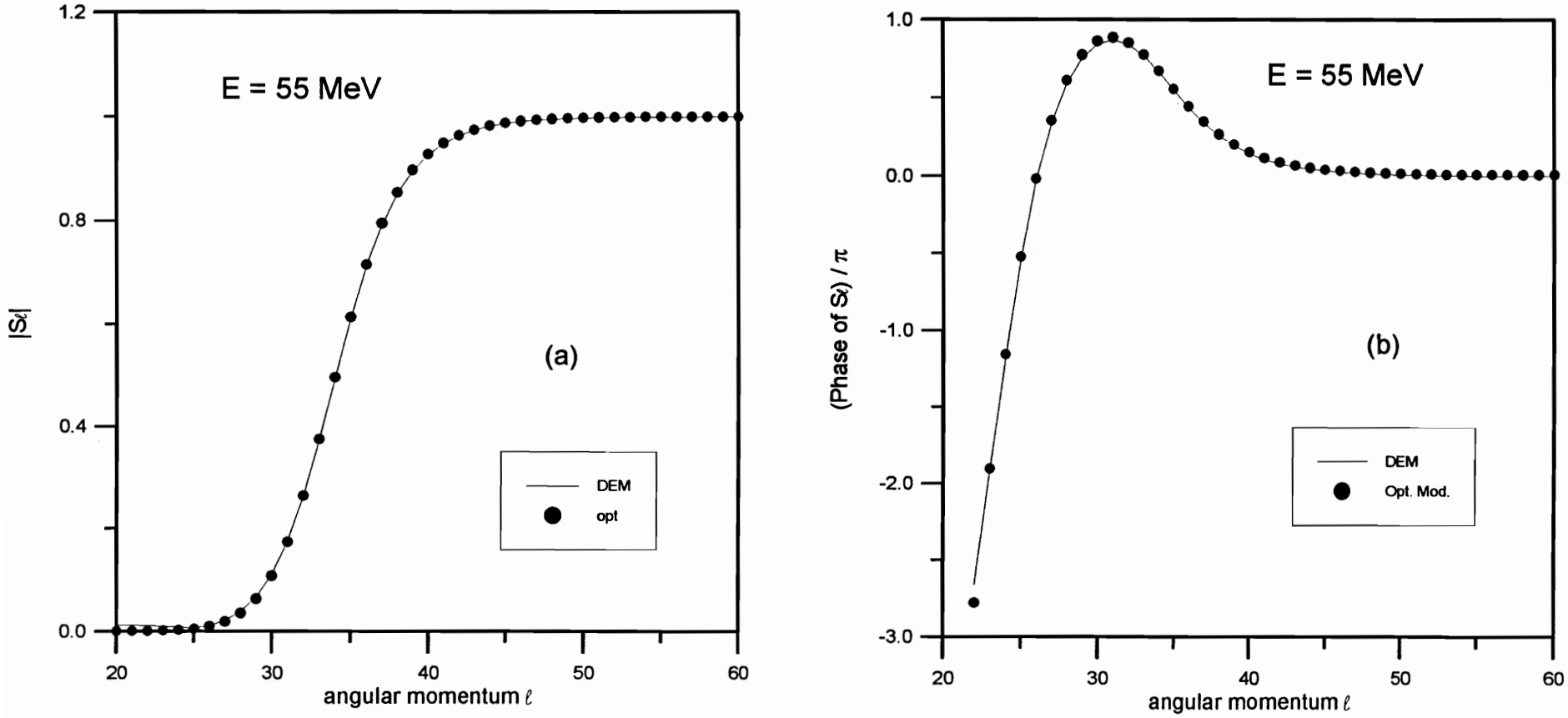
E = 55 MeV
(a)
DEM
opt
|Sℓ|
angular momentum ℓ
E = 55 MeV
(b)
DEM
Opt. Mod.
(Phase of Sℓ) / π
angular momentum ℓ

Fig. 2

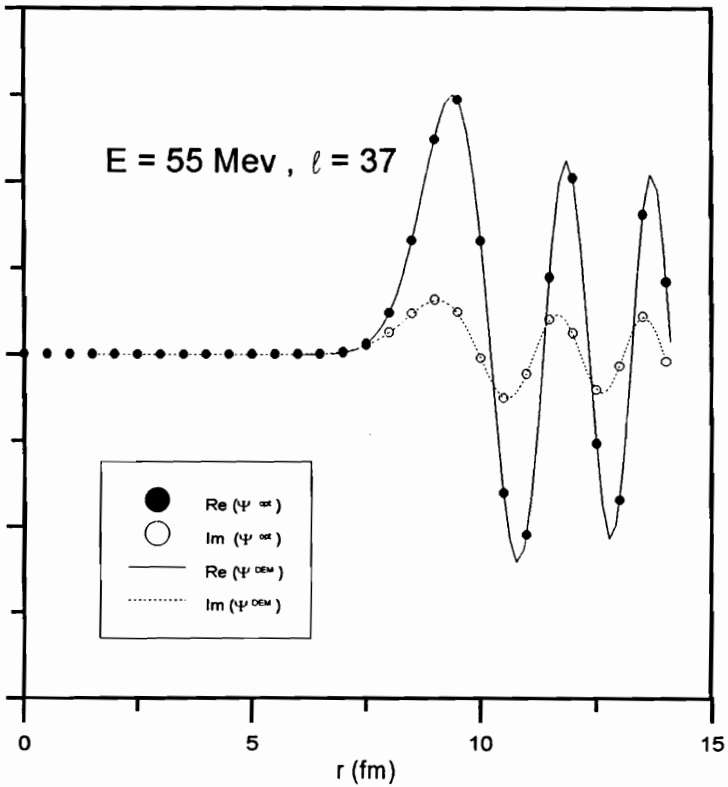
E = 55 Mev , ℓ = 37
Re (Ψ opt)
Im (Ψ opt)
Re (Ψ DEM)
Im (Ψ DEM)
0
5
10
15
r (fm)

Fig. 3

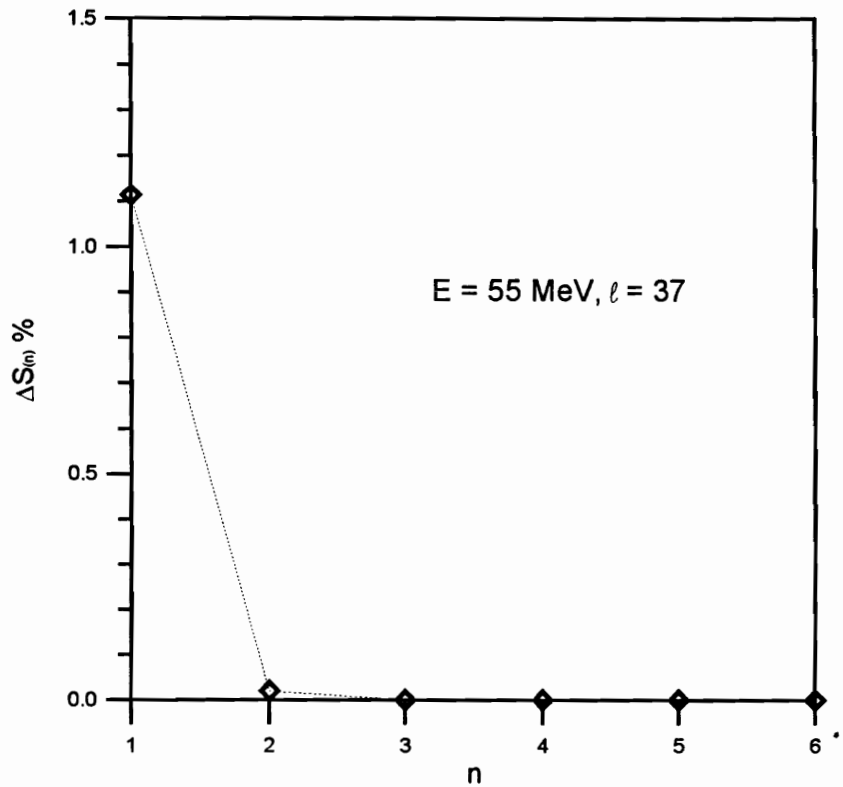
E = 55 MeV, ℓ = 37
ΔS(n) %
1.5
1.0
0.5
0.0
1
2
3
4
5
6
n

Fig. 4

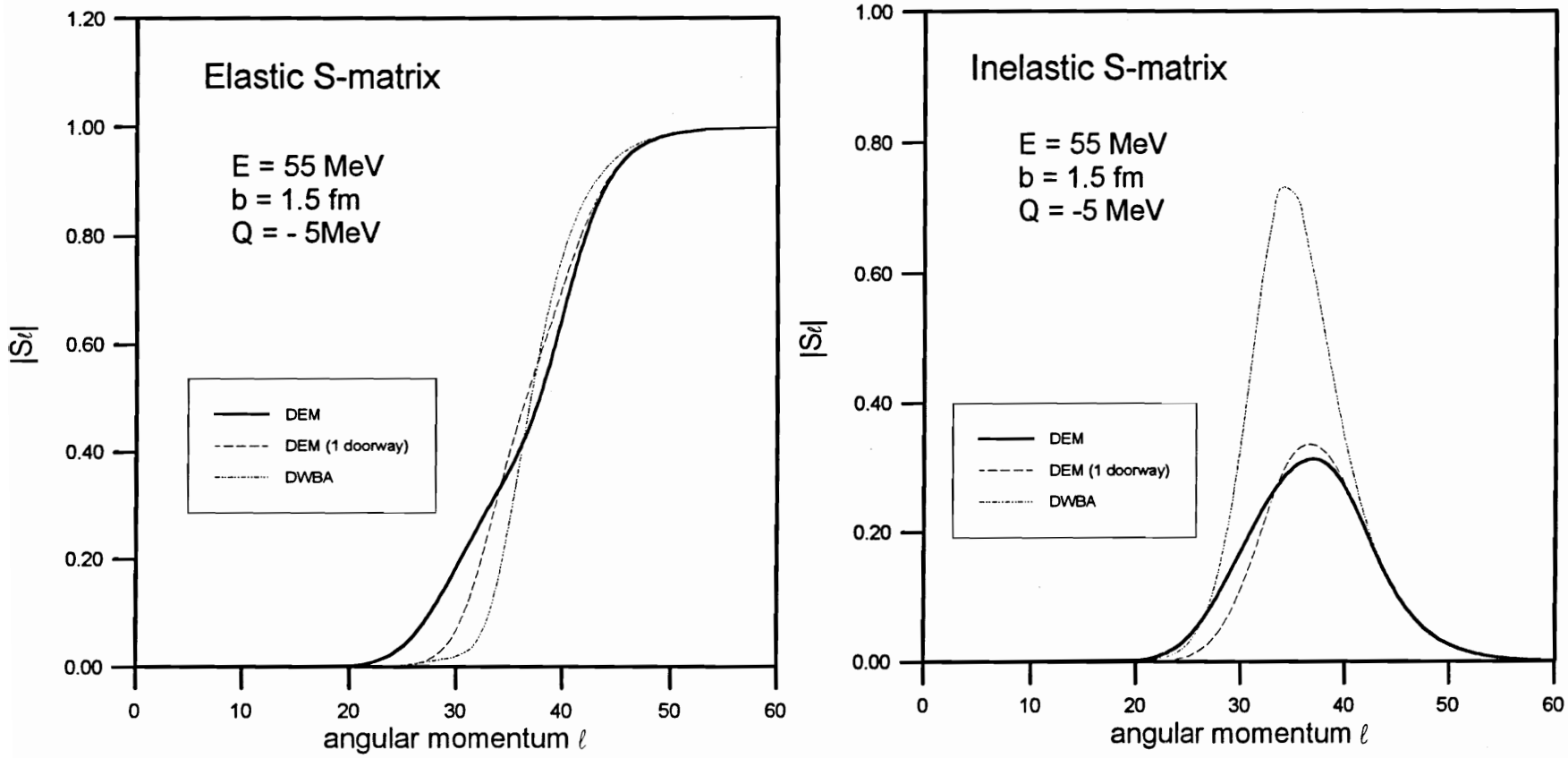
Elastic S-matrix
E = 55 MeV
b = 1.5 fm
Q = - 5MeV
DEM
DEM (1 doorway)
DWBA
|Sℓ|
angular momentum ℓ
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60
Inelastic S-matrix
E = 55 MeV
b = 1.5 fm
Q = -5 MeV
DEM
DEM (1 doorway)
DWBA
|Sℓ|
angular momentum ℓ
1.00
0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60

Fig. 5

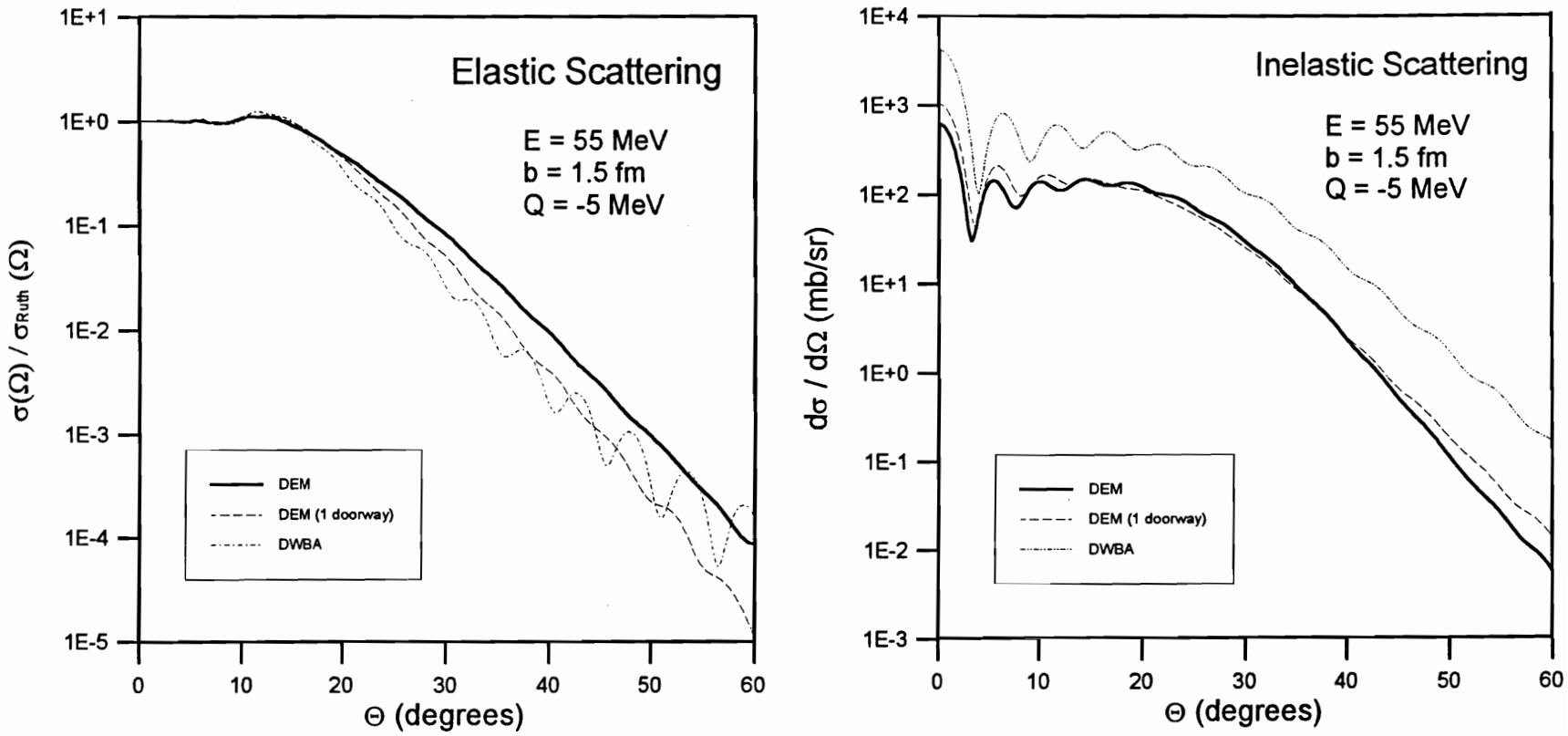
Elastic Scattering
E = 55 MeV
b = 1.5 fm
Q = -5 MeV
DEM
DEM (1 doorway)
DWBA
σ(Ω) / σRuth (Ω)
1E+1
1E+0
1E-1
1E-2
1E-3
1E-4
1E-5
0
10
20
30
40
50
60
Θ (degrees)
Inelastic Scattering
E = 55 MeV
b = 1.5 fm
Q = -5 MeV
DEM
DEM (1 doorway)
DWBA
dσ / dΩ (mb/sr)
1E+4
1E+3
1E+2
1E+1
1E+0
1E-1
1E-2
1E-3
0
10
20
30
40
50
60
Θ (degrees)

Fig. 6

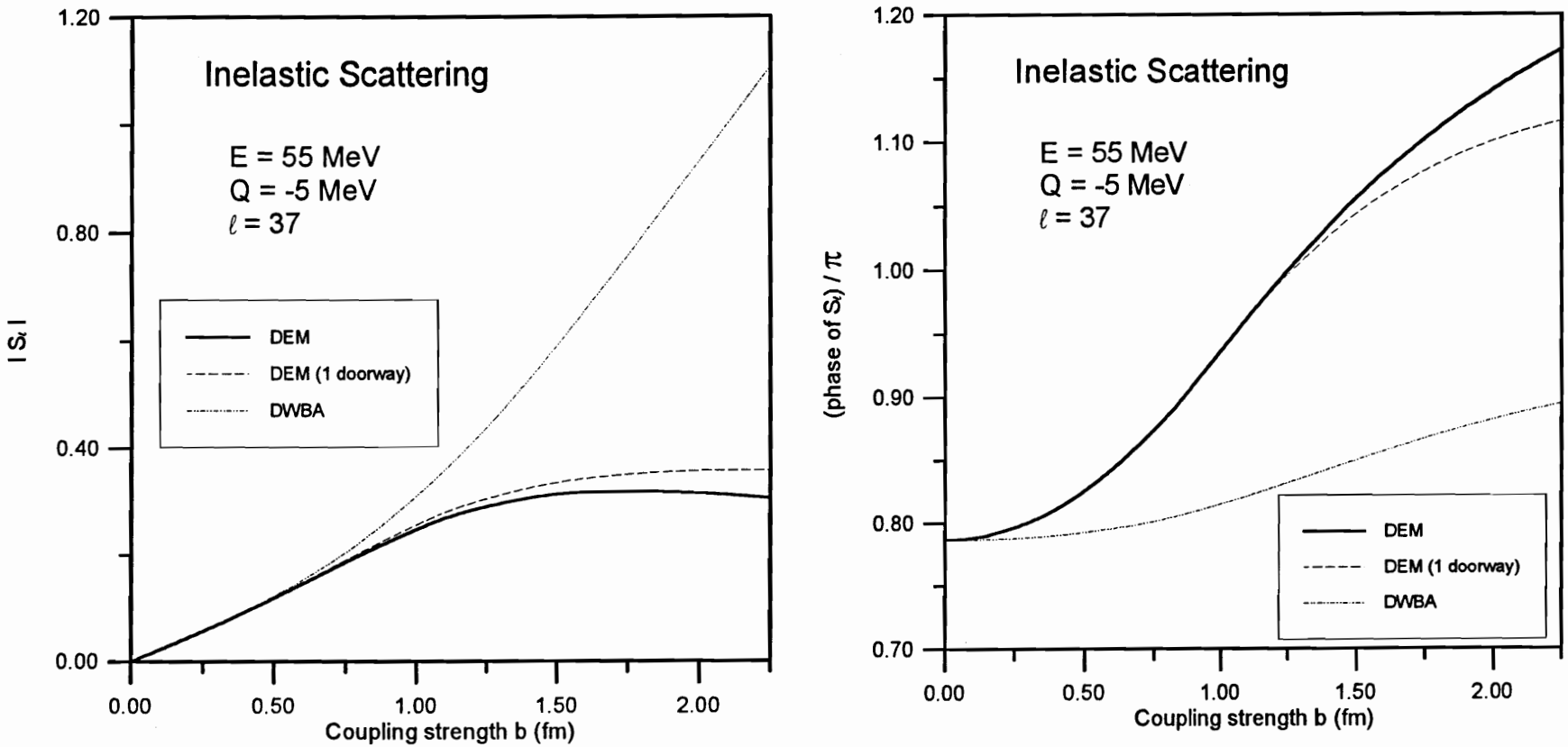
Inelastic Scattering
E = 55 MeV
Q = -5 MeV
ℓ = 37
DEM
DEM (1 doorway)
DWBA
| S_ℓ |
1.20
0.80
0.40
0.00
0.00
0.50
1.00
1.50
2.00
Coupling strength b (fm)
Inelastic Scattering
E = 55 MeV
Q = -5 MeV
ℓ = 37
DEM
DEM (1 doorway)
DWBA
(phase of S_ℓ) / π
1.20
1.10
1.00
0.90
0.80
0.70
0.00
0.50
1.00
1.50
2.00
Coupling strength b (fm)

Fig. 7

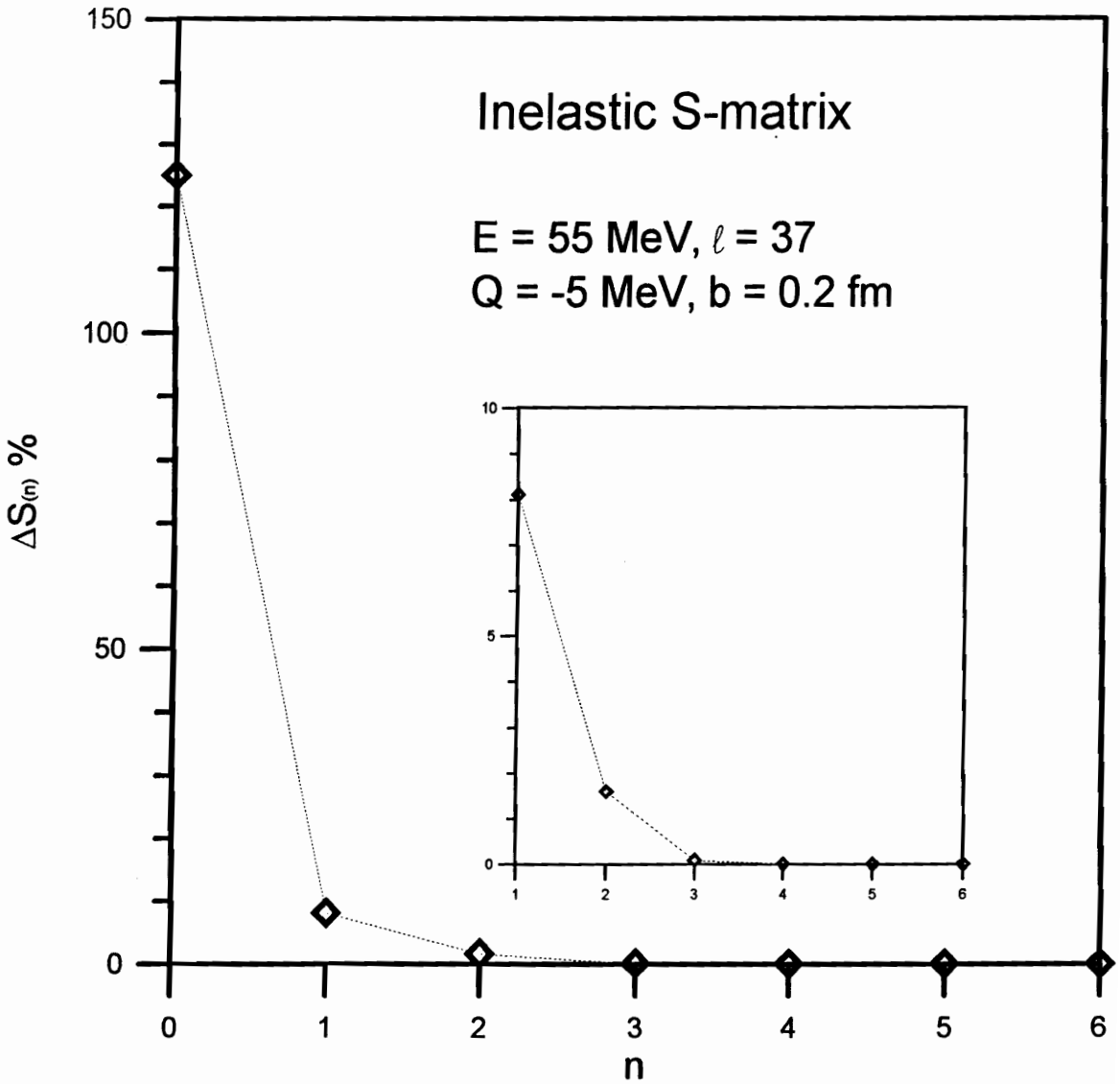
Inelastic S-matrix
E = 55 MeV, ℓ = 37
Q = -5 MeV, b = 0.2 fm
ΔS(n) %
n
150
100
50
0
0
1
2
3
4
5
6
10
5
0
1
2
3
4
5
6

Fig. 8