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BRST Quantization of the Chiral Schwinger Model in the Extended Field-Antifield Space

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Abstract

It is shown that the quantization of the Chiral Schwinger Model in the Batalin-Vilkovisky framework can be carried out in an extended space of fields and antifields, where the master equation has a local solution. The Wess-Zumino term is generated in this way, avoiding the use of non local expressions. The nilpotency of the new BRST charge is proven explicitly.

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The Batalin-Vilkovisky(BV) Lagrangean BRST quantization[1] is a powerful method of quantization of field theories. It is very useful for the treatment of a wide range of theories, including those gauge theories whose constraints do not close an algebra. However, when anomalous gauge theories are concerned, the pathological problems of this kind of theories emerge troubling the construction of a gauge independent generating functional. As was shown recently by Troost, van Nieuwenhuizen and Van Proeyen, the presence of anomalies corresponds to the non existence of local solutions to the master equation[2]. This fact is the BRST reflect of those original perturbative calculations of Feynmann diagrams that gave rise to the so called anomalous Ward identities and, that from the current algebra point of view, appears as a failure of the chiral generators in closing an algebra in perturbation theory[3,4].

The Chiral Schwinger model (CSM) is a two-dimensional theory that is very useful for understanding several features of anomalous models. Jackiw and Rajaraman[5] showed that a unitary and consistent effective theory can be constructed for this model, in spite of loosing the gauge invariance. On the other hand, following the idea of Faddeev and Shatashvili[6] of introducing additional degrees of freedom through the Wess-Zumino term, in reference [7] the Faddev-Popov procedure was applied to obtain a gauge independent vacuum functional. In reference [8] we showed that a gauge independent generating functional for the CSM can be build up using the BV procedure. In this case a non local solution for the master equation was considered. Afterwards, the introduction of an auxiliary field (the so called Wess-Zumino field) makes it possible to write out a local generating functional. The same procedure is worthless in the non-Abelian case, where one is not able to build a non local solution for the master equation. The naive application of the BV procedure to this model would not work, in the sense that the equation that defines the method, the so called master equation, has no solution.

In the same spirit of the Faddeev-Shatashvili works, recently we proposed in reference [9] the introduction of extra degrees of freedom in the BV formalism. We showed that an enlargement of the field-antifield space of the Chiral QCD_2 model makes possible the construction of local solutions for the master equation. There, by the introduction of a pair field-antifield associated to the gauge symmetry group we got a gauge independent

generating functional for Chiral QCD_2, obtaining the Wess-Zumino-Witten action coupled to the gauge field as solution for the master equation at first order in $\hbar$. More recently, a generalization of this procedure to treat in a generic way anomalous gauge theories with a closed, irreducible classical gauge algebra was proposed by Gomis and Paris[10].

The aim of the present work is first to show that the quantization of the Chiral Schwinger Model(CSM) also can be held in an extended space where the master equation has a local solution. Then we will build up the BRST generator and prove, in the canonical quantization framework, that the inclusion of the new pair field-antifield associated to the gauge group leads to the nilpotency of this generator.

Let us consider the classical action for the CSM:

$$S_o = \int d^2x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}\not{p}\,\frac{(1-\gamma_5)}{2}\,\psi\} \tag{1}$$

As usual, the quantum action W in the BV scheme must satisfy the master equation

$$\frac{1}{2}(W,W) = i\hbar\Delta W \tag{3}$$

Where the antibracket is defined as $(X,Y) = \frac{\partial_r X}{\partial\phi}\frac{\partial_l Y}{\partial\phi^*} - \frac{\partial_r X}{\partial\phi^*}\frac{\partial_l Y}{\partial\phi}$ and the operator $\Delta \equiv \frac{\partial_r}{\partial\phi^A}\frac{\partial_l}{\partial\phi^*_A}$. W can be expanded in powers of $\hbar$

$$W = S + \sum_{j=1^\infty} \hbar^j M_j \tag{4}$$

The standard zero order term corresponds to S_o plus the field-antifield terms, which become the gauge fixing terms when the antifields are restricted to the gauge surface $\phi^* = \frac{\partial\Psi}{\partial\phi}$, Ψ being the gauge fermion. Thus, one gets

$$S = S_o + \int d^2x\{A^*_\mu\partial^\mu c + i\psi^*\psi c - i\overline{\psi}\,\overline{\psi^*}c\} \tag{5}$$

The master equation (3) at first order in $\hbar$ is

$$(M_1, S) = i\Delta S \tag{6}$$

where $e^{\Delta S}$ is the BRST Jacobian, thus bearing the anomalous properties of the path integral measure. So, the computation of ΔS requires the introduction of a regularization process, as is explained in detail in ref.[2] where the Pauli-Villars scheme was used. We use here the computation of reference [8], where we used a point splitting scheme, obtaining

$$\Delta S = \frac{i}{4\pi}\int d^2x\ c\,[(1-a)\partial_\mu A^\mu - \epsilon^{\mu\nu}\partial_\mu A_\nu] \tag{7}$$

As already mentioned, the master equation (6) with the action (5) will not admit local solutions.

In order to remove this obstacle we propose to enlarge the field-antifield configuration space including besides the fields that are present at the classical level, also the field θ associated to the Lie algebra of gauge group U(1) together with the corresponding anti-field θ^*. The θ field will become dynamical only at the quantum level. Since the classical action S_o is independent of θ, the model is invariant under arbitrary variations of θ:

$$\theta \to \theta + \lambda \tag{8}$$

besides to the usual gauge symmetry of this model. Now, the generator of this extra symmetry must be included in the Hessian matrix of the extended action, solution of the master equation at the classical level. This is attained simply adding to the action (5) the corresponding term:

$$\overline{S} = S_o + \int d^2x\{A^*_\mu\partial^\mu c + i\psi^*\psi c - i\overline{\psi}\,\overline{\psi^*}c + \theta^*\,c\} \tag{9}$$

Observe that now the action $\overline{S}$ depends on θ^* through the inclusion of the term $\theta^* c$, and this extra term play a fundamental role modifying the master equation (6), in such a way that one can construct a local solutions, depending also on the field θ .

The modified master equation at order $\hbar$ is

$$(M_1, \overline{S}) = \int \{ \frac{\partial_r M_1}{\partial A^\mu} \frac{\partial_l \overline{S}}{\partial A^*_\mu} + \frac{\partial_r M_1}{\partial \theta} \frac{\partial_l \overline{S}}{\partial \theta^*} \} = i\Delta\overline{S} \tag{10}$$

It is worth remarking the inclusion of θ field leaves $\Delta S = \Delta\overline{S}$, as can easily be seen from (5). Now, in this framework, the master equation at first order in $\hbar$ admits local solutions. It is easy to verify that the usual Wess-Zumino term for the Chiral Schwinger model:

$$M_1 = -\frac{1}{4\pi} \int d^2x \left\{ \frac{(a-1)}{2} \partial_\mu \theta \, \partial^\mu \theta + \theta \left[(a-1)\partial_\mu A^\mu + \epsilon^{\mu\nu} \partial_\mu A_\nu \right] \right\} \tag{11}$$

satisfy the master equation (10). The higher order contributions to the master equation (3) vanish, so the quantum action is just

$$W = \overline{S} + \hbar M_1 \tag{12}$$

This result is the same as that of reference [6]. Thus showing that it is possible to build up an action that leads to a gauge independent vacuum functional for the Chiral Schwinger Model, without making use of the Faddeev-Popov trick as in reference [7] or using non local expressions as in [3].

Let us now study the above procedure from the canonical point of view. It is well known that the presence of anomalies breaks down the nilpotency of Q. We will now investigate the operator Q^2 to understand the effect of the enlargement in the configuration space of fields and antifields. What we are really going to calculate is the anticommutator $\{Q, Q\} = 2Q^2$.

Both actions (5) and (12) represent, from the Hamiltonian point of view, constrained systems. Their quantization can be obtained, when there is no operator ordering problem, calculating the Dirac (anti)brackets for the classical theory, and then associating them with the (anti)commutators of the quantum fields as usual: $\{A, B\}_{Dirac} \to -i[A, B]$. There is a pair of constraints that are specially important in our analysis of the effect of the inclusion of the θ field in the canonical algebra: the primary constraint associated to the mo tum conjugate to A_o and the secondary constraint that comes from the time evolution of the former, namely the Gauss law.

For action (5), where θ and θ^* are not present, these constraints read:

$$\begin{aligned} \Omega_0 &\equiv \Pi_0 \approx 0 \\ \Omega_1 &\equiv \Pi_1' + i\overline{\psi}\gamma_0 \frac{(1-\gamma_5)}{2} \psi \approx 0 \end{aligned} \tag{13}$$

where the prime means ∂_1. The BRST transformations of the fields and antifields are can be obtained from the antibracket of the fields with the action (5):

$$\begin{aligned} \delta_o A^\mu &= \partial^\mu c\rho \\ \delta_o \psi &= i\psi c\rho \\ \delta_o \overline{\psi} &= -i\overline{\psi} c\rho \\ \delta_o c &= 0 \\ \delta_o A^*_\mu &= \partial^\nu F_{\mu\nu}\rho - \overline{\psi}\gamma_\mu \frac{(1-\gamma_5)}{2}\psi \\ \delta_o \psi^* &= \frac{1}{2}\overline{\psi}\overline{\not{\partial}}(1-\gamma_5)\rho - i\psi^* c\rho \\ \delta\overline{\psi}^*_o &= -\frac{1}{2}\not{\partial}(1-\gamma_5)\psi\rho + i\overline{\psi}^* c\rho \\ \delta_o c^* &= A^*_\mu \partial^\mu \rho + \psi^*\psi\rho - \overline{\psi\psi}^*\rho \end{aligned} \tag{14}$$

Here ρ is the parameter of the transformation.

The Noether's current associated to these transformations is easily calculated:

$$J_\mu = -\partial^\nu F_{\mu\nu} c + \overline{\psi}\gamma_\mu \frac{(1-\gamma_5)}{2}\psi c \tag{15}$$

So, the generator Q_o of transformations (14) is thus

$$Q_o = \int dx_1 \left[-\Pi_1' \, c + \overline{\psi}\gamma_o \frac{(1-\gamma_5)}{2}\psi c \right] \tag{16}$$

The Dirac brackets are such that the first term in Q_o will not contribute to $\{Q_o, Q_o\}$.

Thus

$$\{Q_o, Q_o\} = \int dx_1 \int dx_1' \{c(x)[\overline{\psi}(x)\gamma_0 \frac{(1-\gamma_5)}{2}\psi(x), \overline{\psi}(x')\gamma_0 \frac{(1-\gamma_5)}{2}\psi(x')]c(x')\}_{x_o=x_o'} \tag{17}$$

So, Q_o^2 is essentially proportional to the commutator of the chiral current

$$J_0^5 = \overline{\psi}(x)\gamma_0 \frac{(1-\gamma_5)}{2}\psi(x)$$

This current algebra requires a careful management at the quantum level, since it involves the product of operators at the same point . As it is well known, the calculation of this commutator generates the Schwinger term, transforming the current algebra in a Kac-Moody one. As in $[J^5, J^5]$ the appearance of the Schwinger term means the breakdown of the gauge symmetry, the lost of the Q_o nilpotency reflects the same fact. Following the standard calculations of the chiral current algebra [11], we obtain:

$$\{Q_o, Q_o\} = i \int dx_1 c(x)\partial_1 c(x) \tag{18}$$

Now, we will analyze in the same way what happens with the extended action (12). The addition of the Wess-Zumino term M_1, obtained as solution of the $\hbar$ order master equation, comes mainly to modify the constraints (13), leaving

$$\begin{aligned} \overline{\Omega}_0 &\equiv \Pi_0 + \frac{\hbar}{4\pi}(a-1)\theta \approx 0 \\ \overline{\Omega}_1 &\equiv \Pi_1' + i\overline{\psi}\gamma_0 \frac{(1-\gamma_5)}{2}\psi - \Pi_\theta \approx 0 \end{aligned} \tag{19}$$

Π_θ is the momentum conjugate to the field θ. Observe that the zero component of the chiral current J_0^5 is now related to Π_θ, and this will be of fundamental importance in the calculation of the new Q^2.

The introduction of the $\theta^*\theta$ term in the action $\overline{S}$, equation (9), imply a non-trivial BRST transformation for the field θ:

$$\delta\theta = c\rho \tag{20}$$

also adding some extra terms to the BRST transformation of the antifields:

$$\begin{aligned} \delta A_\mu^* &= \delta_o A_\mu^* + \frac{\hbar}{4\pi}[(1-a)\partial_\mu\theta + \epsilon^{\mu\nu}\partial_\nu\theta]\rho \\ \delta c^* &= \delta_o c^* + \theta^*\rho \\ \delta\theta^* &= \frac{\hbar}{4\pi}\{(1-a)\partial_\mu\partial^\mu\theta - [(1-a)\partial_\mu A^\mu + \epsilon^{\mu\nu}\partial_\mu A_\nu]\} \end{aligned} \tag{21}$$

The transformations for the other fields remain as in (14).

It is worth remarking that the BRST variation of the antifields are proportional to the equations of motion thus from the canonical point of view they can be considered as being zero. So,the generator of these transformations is $Q = Q_o + Q'$ with

$$Q' = \int dx_1 [c(\Pi_\theta - \frac{\hbar(a-1)}{4\pi}\theta')] \tag{22}$$

The canonical commutation relations for the model described by the action (12) can be calculated by the Dirac's procedure. The nonvanishing commutator that will contribute to the anticommutator $\{Q', Q'\}$ is:

$$[\Pi_\theta(x), \Pi_\theta(x')] = -i\,\partial_1\delta(x_1 - x_1') \tag{23}$$

This Schwinger like term stems from the relation of Π_θ with the chiral current J_0^5, coming to cancel the analogous contribution of Q_o^2. Thus:

$$\{Q', Q'\} = -i \int dx_1 c(x)\,\partial_1 c(x) \tag{24}$$

Finally, since $\{Q', Q_o\}$ vanishes, the BRST generator Q is nilpotent:

$$Q^2 = \frac{1}{2}\{Q, Q\} = 0 \tag{25}$$

Thus, the physical states of the theory are defined by the cohomology class of the generator of BRST transformations

$$Q = c(-\partial^\nu F_{o\nu} + \overline{\psi}\gamma_o \frac{(1-\gamma_5)}{2}\psi + \Pi_\theta - \frac{h(a-1)}{4\pi}\theta') \tag{26}$$

Observe that since the physical state must be annihilated by this BRST operator, the new term added in order to restore the nilpotency impose the chiral constraint $\Pi_\theta = \frac{h(a-1)}{4\pi}\theta'$, on the enlarged Hilbert space.

Concluding, we saw that it is possible to generalize the Batalin- Vilkovisky Lagrangean method, including a field associated to the gauge group element (the Wess-Zumino field).Thus, a gauge independent formulation for an anomalous theory can be build up without using any non-local expressions. As can be easily realized, for non anomalous theories, this extension in the space of fields and antifields will not affect the theory, as the Wess-Zumino field would not become dynamical, since $M_1 = 0$ is a solution of the master equation in this case. From the canonical point of view, it was shown that the BRST charge has it's nilpotency restored in the extended space. Also, a relevant fact is the presence of the chiral constraint in the new BRST generator. This means that the physical sector of the extended Hilbert space includes only one chiral sector of the extra bosonic field. It's non gauge invariance comes to compensate the anomalous behavior of the fermionic measure.

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