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MESON RELATIVISTIC SPECTROSCOPY

H.S. Carvalho[•], H.F. de Carvalho[•] and A.S. de Castro[+]

[•]UFRJ, Instituto de Física, Cidade Universitária
21944 - Rio de Janeiro - RJ - Brasil
[+]UNESP, Departamento de Física e Química, Campus de Guaratinguetá
12500 - Guaratinguetá - SP - Brasil

ABSTRACT

A two-body relativistic equation, derived from Dirac's constraint dynamics, is used for obtaining the meson mass spectra. Spin-dependent effects are not considered. Comparison with recent experimental data and with the results given by a nonrelativistic approximation is made. The leptonic and hadronic decay widths and radiative transition rates are also calculated for some mesons.

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Hadrons are usually considered as being composed of quarks. The model describing the interaction among quarks is Quantum Chromodynamics (QCD) with symmetry group given by the colour SU(3). In this model mesons are interpreted as quark-antiquark bound states belonging to the color singlet representation. The discovery of the families J/ψ $(c\bar{c})$ e Υ $(b\bar{b})$ represented a great contribution to the understanding of the quark-antiquark bound states. The ratios v/c for these systems are relatively small and consequently the nonrelativistic approximation of the Bethe-Salpeter equation can be used for the calculation of the spectra. Relativistic corrections are added through perturbative calculations. The static potential in the nonrelativistic approximation has one part related to asymptotic freedom and another part, purely phenomenological, related to confinement. Various potentials[1] have been proposed and the calculated spectra are, in general, compatible. Motivated by the success of those phenomenological descriptions, many authors have applied the nonrelativistic calculation to systems composed of light quark-antiquark is a common place. The purpose of this paper is to calculate the meson spectra using a relativistic equation and to confront the results with those obtained through a nonrelativistic approach. The relativistic equation adopted here is the one derived from Dirac's constraint formalism for two scalar particles[2], hence the spectra obtained is spin averaged.

The Breit-Fermi equation, obtained through the instantaneous approximation of the Bethe-Salpeter equation, is commonly used in the calculation of the meson mass spectroscopy. That equation provides a Schrödinger equation plus spin-dependent and spin-independent relativistic corrections:

$$\left[m_1 + m_2 + \frac{\vec{p}^2}{2\mu} + V(\vec{r}) + \text{relativistic corrections}\right]\psi = E\psi \qquad (1)$$

These corrections depend on the behaviour of the potential under Lorentz transformations. The Coulombian part of the potential, due to one gluon exchange, is vectorial. Nevertheless the confining potential does not have a Lorentz structure known from first principles, but only informations from a phenomenological point of view. The confining potential has been considered as a mixture of vectorial and scalar coupling, *i.e.*,

$$V(r) = V_{Coul}(r) + V_{conf}(r) + C(q_a\bar{q}_b) \qquad (2a)$$

$$V(r) = V_v(r) + V_s(r) + C(q_a\bar{q}_b) \qquad (2b)$$

$$V_v(r) = (1-f)\, V_{conf}(r) + V_{Coul}(r) \qquad (2c)$$

$$V_s(r) = f\, V_{conf}(r) \qquad (2d)$$

The confining potential is purely vectorial for f=0 and purely scalar for f=1. Comparing the results obtained using a nonrelativistic calculation[3] with the experimental[4] data one can conclude that the scalar contribution to the confining potential is of the order of 50% to 60%.

For light quark-antiquark systems the nonrelativistic approach is safe only if the ratio v/c is small. As a matter of fact, only the $b\bar{b}$ system fits nicely in this approach. Although the $c\bar{c}$ system can also be considered as acceptable. For an alternative approach a relativistic equation , derived through Dirac's constraint dynamics, is considered. With the equation

$$\left[\vec{p}^2 - (\varepsilon_w - A)^2 + 2\varepsilon_w \mathcal{V} - \mathcal{V}^2 + (m_w + S)^2 + \frac{1}{2}\nabla^2 \ln G + \frac{1}{4}(\nabla \ln G)^2\right]\phi = 0 \qquad (3)$$

Crater and Alstine[2] obtained the light and heavy meson mass spectra considering only time-like vector couplings (A=0, G=1 and $\mathcal{V}\neq 0$) and scalar coupling. The expression for the interactions $\mathcal{V}$ and S were obtained with the Richardson potential[1]. This potential, in the nonrelativistic approximation, describes only the heavy meson spectroscopy, not being adequate for the light meson spectroscopy. Crater and Alstine extended consistently the applicability of the Richard potential to the light meson spectra.[2]

Another potential, which has been used for describing the light and heavy meson spectra in a nonrelativistic description[3], is used in this work in the relativistic equation for two scalar particles. The potentials which appear in Eq.(1) are defined by:

$$\mathcal{A} = V_{Coul}(r) = -\frac{4\alpha_s}{3r} \tag{4a}$$

$$G = \left(1 - \frac{2\mathcal{A}}{W}\right)^{-1/2} \tag{4b}$$

$$\mathcal{V}(r) = (1 - f)\, V_{conf}(r) = (1 - f)\, Kr^{1/2} \tag{4c}$$

$$S(r) = f\, V_{conf}(r) + C(q_a\bar{q}_b) = f\, Kr^{1/2} + C(q_a\bar{q}_b) \tag{4d}$$

With these values Eq.(1) takes the form

$$\left[\vec{p}^{\,2} - 2\alpha_s \frac{\varepsilon_w}{r} - \frac{\alpha_s^2}{r^2} + \frac{5\alpha_s^2}{4r^2(Wr + 2\alpha_s)^2} + 2K\left(\varepsilon_w(1 - f) + fm_w + fC\right)r^{1/2}\right.$$
$$\left. + K^2(2f - 1)r + 2m_wC + C^2\right]\psi = b^2\psi \tag{5}$$

The parameters of the potential and quark masses are fitted in a way slightly different from that one of the nonrelativistic case. Instead of given as inputs, the quarks masses are fitted together with the potential parameters. Again the parameter K is universal for all pairs of quark-antiquark. The strong coupling is not constant but depends of the transferred momentum and the number of flavors of the quarks:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda^2)} \tag{6}$$

The scale parameter Λ is fixed by the bottomonium spectrum and N_f assumes the values 5,4,3 and 2, for the systems $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, $q\bar{q}$, respectively. Here q represents a light quark u or d. Using Eq.(5) one can obtain f=1, Λ=.118 GeV, K=.740 $GeV^{3/2}$, and b, c, s, and u quark masses of 4.5, 1.41, .337 and .16 GeV, respectively, whereas $\alpha_s(b)$=.187, $\alpha_s(c)$=.231, $\alpha_s(s)$=.324 and $\alpha_s(u)$=.346. $C(q_a\bar{q}_a)$ take the values -.275, -.800, -1.129 and -1.207 GeV, for the $b\bar{b}$, $c\bar{c}$, $s\bar{s}$ and $u\bar{u}$, respectively. $C(q_a\bar{q}_b)$ is fitted as

$$C(q_a\bar{q}_b) = 0.004x^2 + 0.099x - 0.894 \tag{7a}$$

where

$$x = \ln\left[m_{q_a}^2 m_{\bar{q}_b} + m_{q_a} m_{\bar{q}_b}^2\right] \tag{7b}$$

The resulting spectra are shown in tables Ia and Ib. The experimental results for the states $u\bar{b}$ and $s\bar{c}$ have a structure $^1S_0$ and are only shown as a hint to the values of the $^3S_1$ states. The masses of the bound states $u\bar{b}$, $s\bar{c}$, $c\bar{b}$ and $s\bar{b}$ were calculated by using (7). In table II we compare our results with those obtained by Crater and Alstine, and also with those obtained by using a nonrelativistic approach.

The expressions for the leptonic and hadronic decays of the n^3S_1 states, including perturbative corrections of QCD are given by[5]

$$\Gamma_{q\bar{q}\to e^+e^-} = 16\pi\alpha^2 e_q |\psi(0)/M_{q\bar{q}}|^2 \left[1 - \frac{16\alpha_s}{3\pi}\right] \quad (8a)$$

$$\Gamma_{\psi\to hadrons} = \Gamma^{(0)}\left[1 + (4.9\pm0.5)\frac{\alpha_s}{\pi}\right] \quad (8b)$$

$$\Gamma_{\Upsilon\to hadrons} = \Gamma^{(0)}\left[1 + (3.8\pm0.5)\frac{\alpha_s}{\pi}\right] \quad (8c)$$

where

$$\Gamma^{(0)} = \frac{160}{81}(\pi^2 - 9)\,\alpha_s^3\,|\psi(0)/M_{q\bar{q}}|^2 \quad (8d)$$

Our results are shown in Table III. For light mesons, the hadronic decay width was calculated without perturbative corrections from QCD.

The expressions for the electromagnetic transitions in the electric dipole approximations are given by[6]

$$\Gamma_{E1}(2^3S_1 \to \gamma + 1^3P_J) = \frac{4}{3}\,\frac{(2J+1)}{9}\,\alpha e_q^2\omega^3 \left|\int_{-\infty}^{+\infty} dr\, R_{1P}(r) r^3 R_{2S}(r)\right|^2 \quad (9a)$$

$$\Gamma_{E1}(1^3P_3 \to \gamma + 1^3S_1) = \frac{4}{9}\,\alpha e_q^2\omega^3 \left|\int_{-\infty}^{+\infty} dr\, R_{1S}(r) r^3 R_{1P}(r)\right|^2 \quad (9b)$$

where ω is the emitted photon energy and $R(r)$ is the normalized radial wave function. The results for charmonium and bottomonium families are shown in Table IV.

Comparison of the results obtained in this work with more recent experimental results[4] show that this model describes well the meson mass spectra. The greatest discrepancy takes place in the $u\bar{u}$ and $u\bar{s}$ systems. For the system $u\bar{u}$, the resonances $\rho(1450)$ and $\rho(1700)$ are interpreted as 2S and 3S states, respectively, whereas for the $u\bar{s}$ system the resonances $K^*(1370)$ and $K^*(1680)$ are interpreted as 2S and 3S, respectively. Until 1988 the experimental results had furnished evidence that the resonance $\rho(1600)$ was a 2S state of the $u\bar{u}$ system. Since then, it is believed that this resonance is a superposition of two others, E(1450) and E(1700). Table II shows that our results are compatible with some other approaches.

For the bottomonium, the leptonic decay widths present a better agreement with the experimental results than in the nonrelativistic case. The same do not happen with the results for charmonium.

We do not obtain compatible results for the hadronic decays. Our results become worse than those obtained with the relativistic approach.

The radiative transition rates, in the eletric dipole approximation, furnish pratically the same results for the bottomonium when compared with the nonrelativistic case. Nevertheless for the charmonium the results obtained with the relativistic description are improved. This fact shows that the relativistic effects become more important for the charmonium.

REFERENCES:

1. E. Eichten at al., Phys.Rev. Lett. 34, 369(1975); H.F. de Carvalho et al., Lett. Nuovo Cim. 22, 679(1978); H.F. de Carvalho and A.B. d'Oliveira, Lett. Nuovo Cim. 33, 572 (1982);

   J. L. Richardson, Phys. Lett. B 82, 272 (1979).
2. H.W. Crater and P. Van Alstine, Phys. Lett B 100, 166 (1981); H.W. Crater and P. Van Alstine, Phys. Rev. D 36, 10 (1987).
3. A.S. de Castro et al., Lett. Nuovo Cim. 43, 161 (1985)
4. J.J.Hernández (Particle Data Group), Phys. Lett. B 239 ,1 (1990).
5. R. Barbieri et al., Phys. Lett. B, 57, 455 (1975); P.B. Mackenzie and G. P. Lepage, Phys. Rev. Lett. 47, 1244 (1981).
6. J.D. Jackson, SLAC report no. 198 (1976).

RESUMO

Uma equação relalivistica de dois corpos, derivada da dinâmica de vinculo de Dirac é usada para obter os espectros das massas dos mesons. Efeitos dependentes de spin não são considerados. Fazemos a comparação com os resultados experimentais recentes e com os resultados obtidos por uma aproximação não relativistica. As larguras dos decaimentos leptônico e hadrônico e as taxas das transições radiativas também são calculadas para alguns mesons.

	$b\bar{b}$	$c\bar{c}$	$s\bar{s}$	$u\bar{u}$
$C(q_a\bar{q}_a)$	-0.275	-0.800	-1.129	-1.207
1S teoria	9.463	3.097	1.020	0.768
exp.	Υ(9.460)	ψ(3.097)	ϕ(1.020)	ρ(0.768)
2S teoria	10.010	3.686	1.679	1.451
exp.	Υ(10.023)	ψ(3.686)	ϕ(1.680)	ρ(1.450)
3s teoria	10.353	4.092	2.171	1.953
exp.	Υ(10.355)	ψ(4.040)		ρ(1.712)
4S teoria	10.617	4.418	2.567	2.360
exp.	Υ(10.580)	ψ(4.415)		
5S teoria	10.838	4.696	2.906	2.705
exp.	Υ(10.865)			
6S teoria	11.030	4.941	3.201	3.006
exp.	Υ(11.019)			
1P teoria	9.838	3.510	1.431	1.176
exp.	(9.900)	(3.525)	(1.476)	(1.262)
2P teoria	10.220	3.956	1.996	1.774
exp.				
1D teoria	10.138	3.795	1.761	1.516
exp.		ψ(3.769)		ρ(1.691)
2D teoria	10.437	4.171	2.249	2.030
exp.		ψ(4.159)		

Table Ia. Mass spectra of the light and heavy mesons, in GeV.

	$u\bar{b}$	$s\bar{c}$	$u\bar{c}$	$u\bar{s}$
$C(q_a\bar{q}_b)$	-0.511	-0.926	-0.928	-1.224
1S teoria	5.156	2.117	2.010	0.892
exp.	B(5.271)	F^*(2.140)	D^*(2.010)	K^*(0.892)
2S teoria	5.661	2.712	2.600	1.565
exp.				K^*(1.370)
3S teoria	6.011	3.143	3.034	2.066
exp.				K^*(1.678)
1P teoria	5.493	2.509	2.392	1.309
exp.			D_2^*(2.459)	K_2^*(1.430)
2P teoria	5.887	2.990	2.879	1.893
exp.				
1D teoria	5.861	2.811	2.684	1.648
				K^*(1.780)

Table IIb. Mass spectra of the light and heavy mesons, in GeV.

		Nonrelativ.[3]	Relativ.	Relativ.[2]	Exp.[4]
(1S)	$b\bar{b}$	9.467	9.463	9.460	9.460
	$c\bar{c}$	3.094	3.097	3.097	3.097
	$u\bar{c}$	2.008	2.010	1.990	2.010
	$s\bar{s}$	1.020	1.020	1.020	1.020
	$u\bar{s}$	0.892	0.892	0.892	0.892
	$u\bar{u}$	0.770	0.768	0.759	0.768
	$u\bar{b}$	5.271	5.156	5.311	5.271*
	$c\bar{b}$	6.329	6.330	6.337	
	$s\bar{b}$	5.383	5.318	5.414	
	$s\bar{c}$	2.140	2.117	2.140	2.140*
(2S)	$b\bar{b}$	10.012	10.010	10.021	10.023
	$c\bar{c}$	3.696	3.686	3.661	3.686
	$u\bar{c}$	2.694	2.600	2.575	
	$s\bar{s}$	1.727	1.679	1.706	1.680
	$u\bar{s}$	1.616	1.565	1.606	1.370
	$u\bar{u}$	1.511	1.451	1.509	1.450
	$u\bar{b}$	5.941	5.661	5.830	
	$c\bar{b}$	6.904	6.887	6.879	
	$s\bar{b}$	6.028	5.845	5.939	
	$s\bar{c}$	2.804	2.712	2.685	
(3S)	$b\bar{b}$	10.352	10.353	10.349	10.353
	$c\bar{c}$	4.093	4.092	4.055	4.040
	$s\bar{s}$	2.208	2.171		1.680
	$u\bar{u}$	2.015	1.953		1.712
(4S)	$b\bar{b}$	10.614	10.617	10.604	10.580
	$c\bar{c}$	4.406	4.418	4.383	4.415
(1P)	$b\bar{b}$	9.875	9.883	9.935	9.900
	$c\bar{c}$	3.516	3.510	3.556	3.525
	$u\bar{c}$	2.475	2.392	2.457	2.459
	$s\bar{s}$	1.499	1.431	1.564	1.476
	$u\bar{s}$	1.381	1.309	1.456	1.434
	$u\bar{u}$	1.269	1.176	1.353	1.262
	$u\bar{b}$	5.729	5.493	5.573	
	$c\bar{b}$	6.740	6.478	6.786	
	$s\bar{b}$	5.827	5.681	5.840	
	$s\bar{c}$	2.594	2.509	2.569	

TABLE II - Mass spectra of the light and heavy mesons, in GeV. Values with a asterisk are $^1S_0$ states.

	Nonrelativ.[3]		Relativ.		Exp.[4]	
	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$
$\Gamma(1S \to e^+e^-)$	0.88	5.26	0.99	3.70	1.34	4.72
$\Gamma(2S \to e^+e^-)$	0.44	2.37	0.54	2.57	0.59	2.14
$\Gamma(3S \to e^+e^-)$	0.31	1.56	0.41	2.10	0.44	0.75
$\Gamma(4S \to e^+e^-)$	0.25	1.16	0.35	1.85	0.24	0.47
$\Gamma(1S \to \text{hadrons})$	59.51	93.60	68.00	150.00	32.00	58.00

Table III - Leptonic and hadronic decay widhts for the bottomonium and charmonium S-states, in KeV.

	Nonrelativ.[3]		Relativ.		Exp.[4]	
	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$
$\Gamma_{E1}(2^3S_1 \to \gamma 1^3P_2)$	2.0	40.0	1.872	25.83	0.7±0.9	17±5
$\Gamma_{E1}(2^3S_1 \to \gamma 1^3P_1)$	2.0	57.8	1.898	38.46	1.6±0.8	19±5
$\Gamma_{E1}(2^3S_1 \to \gamma 1^3P_0)$	1.1	64.7	1.218	46.81	1.0±0.7	21±6
$\Gamma_{E1}(1^3P_2 \to \gamma 1^3S_1)$	49.5	601.7	40.763	493.83		330±170
$\Gamma_{E1}(1^3P_1 \to \gamma 1^3S_1)$	43.2	436.7	35.353	359.00		<700
$\Gamma_{E1}(1^3P_0 \to \gamma 1^3S_1)$	36.9	206.7	28.064	162.69		97±38

TABLE IV - Radiative transitions in bottomonium and charmonium, in KeV.