IF - UFRJ - 14 - 92
IF - UFRJ - 92 - 14

INSTITUTO DE FÍSICA -

Fermilab Library
0 1160 0010454 1

25

IF/UFRJ/92/14

SYMPLETIC QUANTIZATION OF SUPERFIELDS

J. Barcelos-Neto and E.S. Cheb-Terrab

May/92

UNIVERSIDADE FEDERAL DO RIO DE JANEIRO
INSTITUTO DE FÍSICA
Cx. P. 68.528
21.944 - RIO DE JANEIRO
BRASIL

IF-UFRJ-14/92

Sympletic quantization of superfields

J. Barcelos-Neto* and E.S. Cheb-Terrab†

Instituto de Física
Universidade Federal do Rio de Janeiro
RJ 21945 - Caixa Postal 68528 - Brasil

Abstract

We show how the extension of the Faddeev-Jackiw sympletic quantization (including true constraints) can be used to superspace. We first deal with supersymmetric free field theory in the component language. After that, we consider the method aplied to superfields, taken as canonical variables. We also use the formalism, directly in superfield formulation, to the supersymmetric nonlinear sigma model.

PACS: 02.40.+m; 11.10.Ef; 11.30.Pb.

(*) Electronic mail: ift03001 at ufrj.

(†) Permanent address: Instituto de Física, Universidade do Estado do Rio de Janeiro, RJ 20550 - Brasil.

1. Introduction

The consistent treatment of constrained systems was given by Dirac a long time ago [1]. The Dirac method is a Hamiltonian formalism where the goal is to find out generalized Poisson brackets (Dirac brackets) which are the bridge to quantum commutators (after problems with ordering operators have been solved) [2].

In a more recent work, Faddeev and Jackiw (FJ) [3] have shown how to obtain the Dirac brackets for some systems in a geometric approach, based on sympletic structures [4]. This was achieved for systems which are constrained in the Dirac formalism but are not in the geometric point of view. As examples we mention the Dirac spinors [5] and selfdual fields in two space-time dimensions [3,6,7].

Unfortunatelly, a great part of the constrained systems is also constrained in the geometric formalism (we shall call these constraints *true constraints*). For this kind of system, direct use of the elegant method proposed by Faddeev and Jackiw cannot be made. Of course, a natural way to circumvent this problem is to try to eliminate the superfluous degrees of freedom by means of the true constraints and, after that, to use the FJ approach. Incidentally we mention that this procedure was followed in a recent work by Kulshreshtha and Müller-Kirsten [8] to deal with FJ quantization in superspace.

In a recent couple of papers, one of us and a collaborator [9] have proposed a way of consistently extending the FJ approach for systems where true constraints are involved [10]. The main results of these works are here summarized in Sec. 2.

The purpose of the present paper is to use the ideas introduced in the works of reference [9] for superfields. First we deal with supersymmetric field theory in a two-dimensional space-time for free fields. In Sec. 3 we work with component fields. This is done to muster confidence toward the results that shall be obtained in Sec. 4, where superfields are directly considered as canonical variables. We shall see that it is not possible to consistently obtain the sympletic matrix in the superfield language. This is so because the presence of Grassmannian variables makes the corresponding matrix singular. However, as was pointed out in our work mentioned in ref. [10], the brackets can be consistently obtained by just considering part of the inverse of the (singular) matrix. In Sec. 5 we

directly in superfield language, the supersymmetric nonlinear sigma model. We leave Sec. 6 for some concluding remarks. We also include an appendix with some details of the calculations.

2. Sympletic formalism

Let us consider a dynamical system evolving in a phase space and described by the canonical set of variables (q_i, p_i) $(i = 1, \ldots, N)$ $(*)$. If these variables are not constrained, they satisfy the fundamental Poisson brackets, namely

$$\begin{aligned} \{q_i, q_j\} &= 0 = \{p_i, p_j\}, \\ \{q_i, p_j\} &= \delta_{ij}. \end{aligned} \tag{2.1}$$

Considering that the brackets of some quantity, defined in the phase space, $A(q,p)$, with anything satisfies the relation

$$\{A(q,p), \cdots\} = \frac{\partial A}{\partial q_i}\{q_i, \cdots\} + \frac{\partial A}{\partial p_i}\{p_i, \cdots\}, \tag{2.2}$$

and using the fundamental brackets (2.1), one can write the usual Poisson bracket relation involving two arbitrary quantities, say $A(p,q)$ and $B(p,q)$, as

$$\begin{aligned} \{A(q,p), B(p,q)\} &= \frac{\partial A}{\partial q_i}\{q_i, q_j\}\frac{\partial B}{\partial q_j} + \frac{\partial A}{\partial q_i}\{q_i, p_j\}\frac{\partial B}{\partial p_j} \\ &+ \frac{\partial A}{\partial p_i}\{p_i, q_j\}\frac{\partial B}{\partial q_j} + \frac{\partial A}{\partial p_i}\{p_i, p_j\}\frac{\partial B}{\partial p_j}, \\ &= \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}. \end{aligned} \tag{2.3}$$

$(*)$ In this section, for the sake of simplicity, we only deal with bosonic coordinates. The extrapolation for fermionic ones and fields can be done in a straightforward way, as we are going to see in the next sections.

just one set of $2N$ generalized coordinates which we denote by y^α $(\alpha = 1, \cdots, 2N)$, in such a way that

$$\begin{aligned} y^i &= q_i, \\ y^{N+i} &= p_i. \end{aligned} \tag{2.4}$$

Now, the fundamental Poisson brackets simply read

$$\{y^\alpha, y^\beta\} = \epsilon^{\alpha\beta}, \tag{2.5}$$

where the antisymmetric tensor $\epsilon^{\alpha\beta}$ is given by the matrix

$$\left(\epsilon^{\alpha\beta}\right) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{2.6}$$

and I is the $N \times N$ identity matrix. The inverse of this matrix, $(\epsilon_{\alpha\beta})$, defines the sympletic tensor [4] characteristic of this unconstrained system. It is easily seen that the Poisson bracket involving two arbitrary quantities $A(y)$, $B(y)$ can be directly obtained in terms of the tensor $\epsilon^{\alpha\beta}$. This is given by

$$\begin{aligned} \{A(y), B(y)\} &= \frac{\partial A}{\partial y^\alpha}\{y^\alpha, y^\beta\}\frac{\partial B}{\partial y^\beta}, \\ &= \epsilon^{\alpha\beta}\frac{\partial A}{\partial y^\alpha}\frac{\partial B}{\partial y^\beta}. \end{aligned} \tag{2.7}$$

The advantage of this notation is that it permits us to directly write the general form of the brackets in the case where constraints are involved. These are given by

$$\{y^\alpha, y^\beta\} = f^{\alpha\beta}(y). \tag{2.8}$$

The important point to be emphasized is that the tensor $f^{\alpha\beta}$ is nonsingular. Its inverse is the sympletic tensor corresponding to the considered constrained system. We mention

that the sympletic tensor can be used as a metric (sympletic metric) that raises and lowers indices in the sympletic manifold.

As is well-known, the conventional way to reach the expression (2.8) for a constrained system is to make use of the Dirac formalism [1,2], where constraints are classified as primary, secondary, tertiary etc., or first and second-class. Also, constraints can be weak and strong.

The FJ formalism deals with first-order Lagrangians. It is opportune to mention that this is not a serious restriction because all systems we known, described by quadratical Lagrangians, can always be set in the first-order formulation. This is achieved by extending the configuration space with the introduction of auxiliary fields. These are usually the momenta, but this is not necessary [9,10].

Let us consider a system described by a first-order Lagrangian like

$$L = a_\alpha(y)\,\dot{y}^\alpha - V(y)\,, \tag{2.9}$$

where y^α is a set of $2N$ coordinates. y^{i+N} can be the momenta or other auxiliary quantities (or may even not exist) in order to render the Lagrangian the first-order condition. From the expression above, the Euler-Lagrange equation of motion reads

$$f_{\alpha\beta}\,\dot{y}^\beta = \partial_\alpha V\,, \tag{2.10}$$

where $\partial_\alpha \equiv \partial/\partial y^\alpha$ and

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha\,. \tag{2.11}$$

If $\det(f_{\alpha\beta}) \neq 0$, one can solve (2.10) for the velocities $\dot{y}^\alpha$, i.e.

$$\dot{y}^\alpha = f^{\alpha\beta}\,\partial_\beta V\,, \tag{2.12}$$

where $f^{\alpha\beta}$ is the inverse of $f_{\alpha\beta}$. This is the sympletic tensor reported earlier and, in fact, one can show that $f^{\alpha\beta}$ are the Dirac bracket between the coordinates y^α, y^β [5,7].

An interesting and instructive point occurs when the quantity $f_{\alpha\beta}$ is singular. In this case one cannot identify it as the sumpletic tensor and, consequently, the brackets

structure of the theory cannot be consistently defined. This means that the system, even in the FJ approach, has constraints. One way to solve this problem is to follow the old and standard Dirac formalism. However, this can also be achieved by working in a geometric manner. In this case, we use the constraints to conveniently deform the singular tensor quantity in order to obtain another tensor that may not be singular. If this occurs, this new quantity can be identified as the sympletic tensor of the theory. Let us briefly review the developments of the sympletic method when there are true constraints involved [9,10]

Let us denote the above mentioned singular quantity by $f^{(0)}_{\alpha\beta}$, and let us suppose that is has, say, M ($M < 2N$) zero modes $v^{(0)}_m$, $m = 1, \cdots, M$. From equation (2.10), it is easy to see that

$$\tilde{v}^{(0)}_m\,\partial_m V = 0\,. \tag{2.13}$$

This may be a constraint. Let us suppose that this actually occurs (we shall discuss the opposite case soon). Usually, constraints can be introduced in the potential part of the Lagrangian by means of Lagrange multipliers. Here, in order to get a deformation in the tensor $f^{(0)}_{\alpha\beta}$ we introduce them instead into the kinectic part. This is done by taking the time-derivative of the constraint and also by using some Lagrange multiplier to introduce them in the Lagrangian (*).

These Lagrange multipliers, which we denote by $\lambda^{(0)}_m$, enlarge the configuration space of the theory. This permit us to identify new vectors $a^{(1)}_\alpha$ and $a^{(1)}_m$ as

$$\begin{aligned} a^{(1)}_\alpha &= a^{(0)}_\alpha + \lambda^{(0)}_m\,\partial_\alpha \Omega^{(0)}_m\,, \\ a^{(1)}_m &= 0\,. \end{aligned} \tag{2.14}$$

where $\Omega^{(0)}_m$ are the constraints obtained from (2.13). In consequence, one can now introduce the tensor quantities

(*) It is well-known that constraints satisfy the consistency condition of not evolving in time, that is to say, if Ω is a constraint we have that $\dot{\Omega}$ is also a constraint. Another point is that one could, instead, take the time derivative of the Lagrange multiplier.

$$
\begin{aligned}
f^{(1)}_{\alpha\beta} &= \partial_\alpha a^{(1)}_\beta - \partial_\beta a^{(1)}_\alpha \,, \\
f^{(1)}_{\alpha m} &= \partial_\alpha a^{(1)}_m - \partial_m a^{(1)}_\alpha = -\partial_m a^{(1)}_\alpha \,, \\
f^{(1)}_{mn} &= \partial_m a^{(1)}_n - \partial_n a^{(1)}_m = 0 \,,
\end{aligned} \tag{2.15}
$$

where $\partial_m = \partial/\partial\lambda^m$. If $\det f^{(1)} \neq 0$, where $f^{(1)}$ is a matrix which also involves the Lagrange multipliers, then we have succeeded in eliminating the constraints. If not, one should repeat the procedure above as many times as necessary.

It may also occur that we arrive at a point where we still obtain a singular matrix and the corresponding zero modes do not lead to any new constraint. This is the case, for example, of gauge theories. At this point, if we want to define the sympletic tensor, we have to introduce some gauge condition. For details, see reference [9]. In the case of superfields taken as canonical variables, a new fact appears. We shall see that we find a tensor that is singular (and there is no gauge freedom to be fixed). This occurs in virtue of the Grassmannian degrees of freedom of the superspace formulation. This means that it is not possible to consistently define de sympletic matrix in this case. However, we shall see that it is possible to obtain part of the inverse and this is precisely the bracket structure of the theory. Further details shall be reported in the examples.

3. N=1, D=2 supersymmetry

Let us consider the following supersymmetric Lagrangian density for free massless fields

$$
\mathcal{L} = -\frac{1}{2}\,\bar{D}\Phi\, D\Phi \,, \tag{3.1}
$$

where Φ is a real scalar superfield whose general form is (*)

(*) The convention and notation we use throughout this paper is $[\gamma^\mu, \gamma^\nu]_+ = 2\eta^{\mu\nu}$; $\gamma^\mu\gamma^\nu = \eta^{\mu\nu}\mathbf{1} + \epsilon^{\mu\nu}\gamma_5$; $\eta^{00} = -\eta^{11} = -1$; $\eta^{01} = \eta^{10} = 0$; $\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; $\gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\gamma_5 = \gamma^0\gamma^1$; etc.

$$
\Phi(x,\theta) = \phi(x) + i\bar{\theta}\psi(x) + \frac{i}{2}\bar{\theta}\theta F(x) \,. \tag{3.2}
$$

Here, ϕ is a real field; θ, ψ are Majorana spinors and F is an auxiliary field. The covariant derivatives D_α and $\bar{D}_\alpha$ are defined by

$$
\begin{aligned}
D_\alpha &= \frac{\partial}{\partial\bar{\theta}_\alpha} - i(\gamma^\mu\theta)_\alpha \frac{\partial}{\partial x^\mu} \,, \\
\bar{D}_\alpha &= D_\lambda \gamma^0_{\lambda\alpha} \,, \\
&= -\frac{\partial}{\partial\theta_\alpha} + i(\bar{\theta}\gamma^\mu)_\alpha \frac{\partial}{\partial x^\mu} \,.
\end{aligned} \tag{3.3}
$$

In this section we shall work with component fields. Using (3.2) and (3.3), we obtain from (3.1) that the Lagrangian density in components reads

$$
\mathcal{L} = -\frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\,\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\,F^2 \,. \tag{3.4}
$$

In order to use the sympletic formalism, we transform this Lagrangian to the first-order notation. This is achieved by extending the configuration space as

$$
\begin{aligned}
-\frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi &= \frac{1}{2}\,\dot{\phi}\dot{\phi} - \frac{1}{2}\,\phi'\phi' \,, \\
&\longrightarrow -\frac{1}{2}p^2 + p\dot{\phi} - \frac{1}{2}\phi'\phi' \,,
\end{aligned} \tag{3.5}
$$

where $\phi' \equiv \partial_1\phi$. Here, the auxiliary quantity p is the momentum conjugate to ϕ. Introducing this result into (3.4), we get

$$
\mathcal{L}^{(0)} = \dot{\phi}p + \frac{i}{2}\dot{\psi}\psi + V(\phi, p, \psi, F) \,, \tag{3.6}
$$

where

$$
V = \frac{1}{2}\,p^2 + \frac{1}{2}\,\phi'\phi' - \frac{i}{2}\,\psi\gamma_5\psi' - \frac{1}{2}\,F^2 \,. \tag{3.7}
$$

The relative position of the velocities in (3.6) is due to the adopting of the left-derivative convention for fermionic fields. The Lagrangian density (3.6) permits us to identify

$$
\begin{aligned}
a_{\phi}^{(0)} &= p\,, \\
a_{p}^{(0)} &= 0\,, \\
a_{\psi}^{(0)} &= \frac{i}{2}\psi\,, \\
a_{F}^{(0)} &= 0\,.
\end{aligned}
\tag{3.8}
$$

The nonvanishing elements of the matrix $f^{(0)}$ are

$$
\begin{aligned}
f_{\phi p}^{(0)}(x,x') &= \frac{\delta a_p(x')}{\delta\phi(x)} - \frac{\delta a_\phi(x)}{\delta p(x')} = -\delta(x-x')\,, \\
f_{\psi\psi}^{(0)}(x,x') &= -\frac{\delta a_\psi(x')}{\delta\psi(x)} - \frac{\delta a_\psi(x)}{\delta\psi(x')} = -i\delta(x-x')\,.
\end{aligned}
\tag{3.9}
$$

The quantities above, considered to be at $x_0 = x_0' = t$, are the natural extension of the coordinate case that we have seen in Sec. 2. In the case of $f_{\psi\psi}$, both signs are equal in virtue of its fermionic nature and are negative in virtue of the left-derivative convention.

Thus the general form of the matrix $f^{(0)}$ is

$$
f^{(0)} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \delta(x-x')\,,
\tag{3.10}
$$

where the elements of rows and columns follow the order ϕ, p, ψ, F.

This matrix is obviously singular. This means that the theory described by the Lagrangian (3.6) is constrained in the FJ formalism (there are true constraints). We shall see that this constraint is related to the equation of motion of the auxiliary field. It is opportune to mention that there are no constraints related to the fields ψ_α and p, as would occur in the Dirac formalism.

From the matrix (3.10), one immediately obtains the following zero-mode

$$
v^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\tag{3.11}
$$

and a contraint may be obtained from

$$
\int dx\, \frac{\delta}{\delta F(x,t)} \int dx'\, V(x',t) = 0\,,
\tag{3.12}
$$

which is the generalization for fields of expression (2.13). Considering (3.7), we obtain the well-known auxiliary field constraint of supersymmetric theories, namely

$$
F = 0\,.
\tag{3.13}
$$

Following the procedure mentioned in Sec. 2, we now take the time derivative of this constraint and introduce the result into the previous Lagrangian by means of a Lagrange multiplier λ. The result is

$$
\mathcal{L}^{(1)} = \dot{\phi}p + \frac{i}{2}\dot{\psi}\psi + \dot{F}\lambda - V\,.
\tag{3.14}
$$

The Lagrangian (3.14) permits us to identify the quantities

$$
\begin{aligned}
a_{\phi}^{(1)} &= p\,, \\
a_{p}^{(1)} &= 0\,, \\
a_{\psi}^{(1)} &= \frac{i}{2}\psi\,, \\
a_{F}^{(1)} &= \lambda\,, \\
a_{\lambda}^{(1)} &= 0\,,
\end{aligned}
\tag{3.15}
$$

and besides (3.9) we have the following nonvanishing element

$$
f_{\lambda F}(x,x') = \frac{\delta a_F(x')}{\delta\lambda(x)} - \frac{\delta a_\lambda(x)}{\delta F(x')} = \delta(x-x')\,.
\tag{3.16}
$$

The matrix $f^{(1)}$ then reads (the elements follow the order ϕ, p, ψ, F, λ)

$$
f^{(1)} = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \delta(x-x')\,.
\tag{3.17}
$$

This matrix is not singular. This means that there are no more constraints. We can thus identify $f^{(1)}$ as the sympletic matrix of the theory. Its inverse reads

$$f^{(1)^{-1}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix} \delta(x - x'), \tag{3.18}$$

and considering (2.8) we immediately identify the nonvanishing brackets

$$\begin{aligned} \{\phi(x), \dot{\phi}(x')\} &= \delta(x - x'), \\ \{\psi(x), \psi(x')\} &= i\,\delta(x - x'), \\ \{F(x), \lambda(x')\} &= \delta(x - x'). \end{aligned} \tag{3.19}$$

In terms of the original fields of the theory, we just have

$$\begin{aligned} \{\phi(x), \dot{\phi}(x')\} &= \delta(x - x'), \\ \{\psi(x), \psi(x')\} &= i\,\delta(x - x'). \end{aligned} \tag{3.20}$$

These are the same brackets which we would have obtained if the Dirac formalism had been used. We mention that the bracket involving F and λ is not a Dirac one (this would be zero). It just appears to render the matrix $f^{(1)}$ the condition of nonsingularity.

4. Using superfields as coordinates

Let us now work in the superfield language by directly using the Lagrangian (3.1). Considering the expressions of the covariant derivatives, we rewrite (3.1) as

$$\mathcal{L} = \frac{1}{2}\,\bar{\theta}\theta\,\dot{\Phi}\dot{\Phi} - i\bar{\theta}\,\frac{\partial\Phi}{\partial\theta}\,\dot{\Phi} + \frac{1}{2}\,\frac{\partial\Phi}{\partial\theta}\,\frac{\partial\Phi}{\partial\bar{\theta}} - i\bar{\theta}\gamma_5\,\frac{\partial\Phi}{\partial\theta}\,\Phi' - \frac{1}{2}\,\bar{\theta}\theta\,\Phi'\Phi'. \tag{4.1}$$

In order to have a first-order Lagrangian, we linearize the first term of the equation above by setting

$$\frac{1}{2}\,\bar{\theta}\theta\,\dot{\Phi}\dot{\Phi} \longrightarrow \bar{\theta}\theta\Big(\Pi\dot{\Phi} - \frac{1}{2}\Pi\Pi\Big). \tag{4.2}$$

It is opportune to mention that here Π is not the momentum conjugate to Φ. The first-order Lagrangian then reads

$$\mathcal{L}^{(0)} = \Big(\bar{\theta}\theta\Pi - i\bar{\theta}\frac{\partial\Phi}{\partial\theta}\Big)\,\dot{\Phi} - V(\Phi, \Pi), \tag{4.3}$$

where

$$V = -\frac{1}{2}\,\frac{\partial\Phi}{\partial\theta}\,\frac{\partial\Phi}{\partial\bar{\theta}} + i\bar{\theta}\gamma_5\,\frac{\partial\Phi}{\partial\theta}\,\Phi' + \frac{1}{2}\,\bar{\theta}\theta\,(\Phi'\Phi' + \Pi\Pi). \tag{4.4}$$

From the Lagrangian above one identifies

$$\begin{aligned} a_{\Phi}^{(0)} &= \bar{\theta}\theta\,\Pi - i\bar{\theta}\,\frac{\partial\Phi}{\partial\theta}, \\ a_{\Pi}^{(0)} &= 0. \end{aligned} \tag{4.5}$$

The nonvanishing elements of the matrix $f^{(0)}$ are directly calculated

$$\begin{aligned} f^{(0)}_{\Phi\Phi}(\theta, x; \theta', x') &= \frac{\delta a_{\Phi}^{(0)}(\theta', x')}{\delta\Phi(\theta, x)} - \frac{\delta a_{\Phi}^{(0)}(\theta, x)}{\delta\Phi(\theta', x')}, \\ &= 4i\,\theta\theta'\,\delta(x - x'), \end{aligned}$$

$$\begin{aligned} f^{(0)}_{\Phi\Pi}(\theta, x; \theta', x') &= \frac{\delta a_{\Pi}^{(0)}(\theta', x')}{\delta\Phi(\theta, x)} - \frac{\delta a_{\Phi}^{(0)}(\theta, x)}{\delta\Pi(\theta', x')}, \\ &= -\,\bar{\theta}\theta\bar{\theta}'\theta'\,\delta(x - x'). \end{aligned} \tag{4.6}$$

The matrix $f^{(0)}$ reads

$$f^{(0)} = \begin{pmatrix} 4i\theta\theta' & -\bar{\theta}\theta\bar{\theta}'\theta' \\ \bar{\theta}\theta\bar{\theta}'\theta' & 0 \end{pmatrix} \delta(x - x'). \tag{4.7}$$

This matrix is singular. It is easy to see that there are two zero-modes

$$v_1^{(0)} = \begin{pmatrix} \bar{\theta}\theta \\ 0 \end{pmatrix}, \qquad v_2^{(0)} = \begin{pmatrix} 0 \\ \bar{b}\theta \end{pmatrix}, \tag{4.8}$$

where b is some constant spinor. To calculate the corresponding constraints, we proceed as usual. Considering $v_1^{(0)}$ we have

$$\int d^2\theta\, \bar{\theta}\theta \frac{\delta}{\delta\Phi(x,\theta} \int d^2\theta'\, V = 0\,,$$
$$\Longrightarrow \qquad \frac{\partial^2}{\partial\theta\partial\bar{\theta}}\,\Phi = 0\,. \tag{4.9}$$

This in components is nothing more than the equation of motion for the auxiliary field. The use of $v_2^{(0)}$ gives

$$\int d^2\theta\, \bar{b}\theta \frac{\delta}{\delta\Pi(x,\theta)} \int d^2\theta'\, V = 0\,,$$
$$\Longrightarrow \qquad 0 = 0\,. \tag{4.10}$$

As one observes, the second zero-mode does not lead to any constraint, it just gives an identity.

The next step is to introduce the constraint (4.9) into the kinectic part of the Lagrangian (4.3). Here, we choose to take the time derivative of the Lagrange multiplier. The new Lagrangian $\mathcal{L}^{(1)}$ then reads

$$\mathcal{L}^{(1)} = \left(\bar{\theta}\theta\,\Pi - i\bar{\theta}\frac{\partial\Phi}{\partial\theta}\right)\dot{\Phi} + \frac{\partial^2}{\partial\theta\partial\bar{\theta}}\,\Phi\dot{\Lambda} - V\,. \tag{4.11}$$

Now, besides (4.6) we also have

$$f^{(1)}_{\Lambda\Phi}(x,\theta;x'\theta') = \frac{\delta a^{(1)}_{\Phi}(x',\theta')}{\delta\Lambda(x,\theta)} - \frac{\delta a^{(1)}_{\Lambda}(x,\theta)}{\delta\Phi(x',\theta')}\,,$$
$$= -4\,\delta(x-x')\,. \tag{4.12}$$

The matrix $f^{(1)}$ is

$$f^{(1)} = \begin{pmatrix} 4i\theta\theta' & -\bar{\theta}\theta\bar{\theta}'\theta' & 4 \\ \bar{\theta}\theta\bar{\theta}'\theta' & 0 & 0 \\ -4 & 0 & 0 \end{pmatrix} \delta(x-x')\,. \tag{4.13}$$

This matrix is again singular. The corresponding zero-mode is

$$v^{(1)} = \begin{pmatrix} 0 \\ \bar{c}\theta \\ 0 \end{pmatrix}, \tag{4.14}$$

where c is another constant spinor. One can easily see that the use of this eigenvector in the expression (2.13) just leads to the identity $0 = 0$. No more constraints can be obtained in this way. On the other hand, since this is not a gauge theory there are no constraints arising by gauge-fixing. This means that the sympletic tensor cannot be consistently defined when we work in the superfield language. Incidentally we mention that this is not a particular problem of the FJ formulation. In the case of the Dirac treatment we also have problems when we try to obtain the inverse of the matrix formed by the Poisson brackets of constraints. For details of the use of the Dirac formalism with superfields, see references [11,12,13].

The way of circumventing this problem was given in [10], and we briefly review it here. We first mention that the inverse of $f^{(1)}$ would have to satisfy the relation

$$\int d^2\theta' dx'\, f^{(1)}(x,\theta;x',\theta')\, f^{(1)^{-1}}(x',\theta';x'',\theta'') = \delta^2(\theta-\theta'')\,\delta(x-x')\,, \tag{4.15}$$

The impossibility of obtaining $f^{(1)^{-1}}$ is because some terms of $\delta^2(\theta-\theta'')$ cannot be generated. What we have to do is to calculate a matrix, which we call $\tilde{f}$, in order to generate the maximum possible number of terms of $\delta^2(\theta-\theta'')$. To do this, we consider that the matrix $\tilde{f}$ has the following general form

$$\tilde{f} = \begin{pmatrix} A & B & C \\ -\underline{B} & D & E \\ -\underline{C} & -\underline{E} & F \end{pmatrix}, \tag{4.16}$$

where $\underline{B}(x,\theta;x',\theta') = B(x',\theta';x,\theta)$ etc. We thus obtain the equations

$$
\begin{aligned}
&\int d^2\theta' \left(4i\theta\theta' A + \bar\theta\theta\bar\theta'\theta' \underline{B} - 4\underline{C}\right) = \delta^2(\theta - \theta''),\\
&\int d^2\theta' \left(4i\theta\theta' B + \bar\theta\theta\bar\theta'\theta' D - 4\underline{E}\right) = 0,\\
&\int d^2\theta' \left(4i\theta\theta' C + \bar\theta\theta\bar\theta'\theta' E + 4F = 0,\right.\\
&\int d^2\theta' \bar\theta\theta\bar\theta'\theta' A = 0,\\
&\int d^2\theta' \bar\theta\theta\bar\theta'\theta' B = \bar\theta\theta,\\
&\int d^2\theta' \bar\theta\theta\bar\theta'\theta' C = 0,\\
&\int d^2\theta' A = 0,\\
&\int d^2\theta' B = 0,\\
&\int d^2\theta' C = -\frac{1}{4}\bar\theta''\theta'',
\end{aligned}
\tag{4.17}
$$

whose solution is easy to obtain and the matrix $\tilde f$ reads

$$
\tilde f(x,\theta;x',\theta') = \begin{pmatrix} i\theta\theta' & 1 & -\frac{1}{4}\bar\theta\theta\bar\theta'\theta' \\ -1 & 0 & 0 \\ \frac{1}{4}\bar\theta\theta\bar\theta'\theta' & 0 & 0 \end{pmatrix} \delta(x - x').
\tag{4.18}
$$

This matrix permits us to identify the following nonvanishing brackets

$$
\begin{aligned}
\left\{\Phi(x,\theta),\, \Phi(x',\theta')\right\} &= i\theta\theta'\,\delta(x - x'),\\
\left\{\Phi(x,\theta),\, \Pi(x',\theta')\right\} &= \delta(x - x'),\\
\left\{\Phi(x,\theta),\, \Lambda(x',\theta')\right\} &= -\frac{1}{4}\,\bar\theta\theta\bar\theta'\theta'\,\delta(x - x').
\end{aligned}
\tag{4.19}
$$

If one writes these brackets in components one obtains the same results of the previous section.

5. Supersymmetric nonlinear sigma-model

The main purpose of this section is to check the ideas presented before in a more subtle system. Let us then consider the supersymmetric $O(N)$ nonlinear σ-model in $1+1$ spacetime dimensions. The Lagrangian density for this theory reads

$$
\mathcal{L} = -\frac{1}{2}\,\bar D\Phi^i D\Phi^i + \frac{1}{2}\,\Lambda\left(\Phi^i\Phi^i - 1\right),
\tag{5.1}
$$

where $i = 1, \cdots, N$ is the index related to the $O(N)$ symmetry group.

In this section we shall directly work in the superfield language. Using the same development to obtain (4.3), we get

$$
\mathcal{L}^{(0)} = \left(\bar\theta\theta\,\Pi^i - i\bar\theta\,\frac{\partial\Phi^i}{\partial\theta}\right)\dot\Phi^i - V(\Phi,\Pi,\Lambda).
\tag{5.2}
$$

Here, V is given by

$$
V = -\frac{1}{2}\,\frac{\partial\Phi^i}{\partial\theta}\,\frac{\partial\Phi^i}{\partial\bar\theta} + i\bar\theta\gamma_5\frac{\partial\Phi^i}{\partial\theta}\,\Phi'^i + \frac{1}{2}\,\bar\theta\theta\left(\Phi'^i\Phi'^i + \Pi^i\Pi^i\right) - \frac{\Lambda}{2}\left(\Phi^i\Phi^i - 1\right).
\tag{5.3}
$$

From the Lagrangian (5.2) one identifies

$$
\begin{aligned}
a_\Phi^{(0)i} &= \bar\theta\theta\,\Pi^i - i\bar\theta\,\frac{\partial\Phi^i}{\partial\theta},\\
a_\Pi^{(0)i} &= 0,\\
a_\Lambda^{(0)} &= 0.
\end{aligned}
\tag{5.4}
$$

The elements of the matrix $f^{(0)}$ are obtained as before and we have

$$
f^{(0)} = \begin{pmatrix} 4i\theta\theta'\delta^{ij} & -\bar\theta\theta\bar\theta'\theta'\delta^{ij} & 0 \\ \bar\theta\theta\bar\theta'\theta'\delta^{ij} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \delta(x - x').
\tag{5.5}
$$

This matrix has three zero-modes which are

$$
v_1^{(0)i} = \begin{pmatrix} c^i\bar\theta\theta \\ 0 \\ 0 \end{pmatrix}, \qquad v_2^{(0)i} = \begin{pmatrix} 0 \\ \bar b^i\theta \\ 0 \end{pmatrix}, \qquad v_3^{(0)i} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.
\tag{5.6}
$$

The use of the eigenvector $v_1^{(0)i}$ into (2.13) gives

$$
\begin{aligned}
& \int d^2\theta\, \bar\theta\theta \frac{\delta}{\delta\Phi^i} \int d^2\theta'\, V = 0\,, \\
\Longrightarrow \quad & \int d^2\theta\, \bar\theta\theta \left(\frac{\partial^2 \Phi^i}{\partial\theta\partial\bar\theta} - \Lambda\Phi^i \right) = 0\,, \\
\Longrightarrow \quad & \frac{\partial^2 \Phi^i}{\partial\theta\partial\bar\theta} \left[\bar\theta\theta \left(\frac{\partial^2 \Phi^i}{\partial\theta\partial\bar\theta} - \Lambda\Phi^i \right) \right] = 0\,, \\
\Longrightarrow \quad & \frac{\partial^2 \Phi^i}{\partial\theta\partial\bar\theta} - \Lambda(0)\Phi^i(0) = 0\,.
\end{aligned} \tag{5.7}
$$

The use of $v_2^{(0)i}$ leads to the identity $0 = 0$ and it is easily seen that $v_3^{(0)i}$ gives the characteristic constraint of the nonlinear σ-model theory, namely

$$
\Phi^i\Phi^i - 1 = 0\,. \tag{5.8}
$$

Introducing both constraints (5.7) and (5.8) into the kinectic part of the Lagrangian (5.2) we get

$$
\mathcal{L}^{(1)} = \left(\bar\theta\theta\Pi^i - i\bar\theta \frac{\partial\Phi^i}{\partial\theta} + \eta\Phi^i \right) \dot\Phi^i + \left(\frac{\partial^2\Phi^i}{\partial\theta\partial\bar\theta} - \Lambda(0)\Phi^i(0) \right) \dot\rho^i - V\,. \tag{5.9}
$$

From the Lagrangian above we obtain

$$
\begin{aligned}
a_\Phi^{(1)i} &= \bar\theta\theta\,\Pi^i - i\bar\theta \frac{\partial\Phi^i}{\partial\theta} + \eta\Phi^i\,, \\
a_\Pi^{(1)i} &= 0\,, \\
a_\Lambda^{(1)} &= 0\,, \\
a_\eta^{(1)} &= 0\,, \\
a_\rho^{(1)i} &= \frac{\partial^2\Phi^i}{\partial\theta\partial\bar\theta} - \Lambda(0)\,\Phi^i(0)\,.
\end{aligned} \tag{5.10}
$$

The matrix $f^{(1)}$, whose rows and columns are displayed in the order Φ, Π, η, ρ, Λ, is given by

$$
f^{(1)} = \begin{pmatrix}
4i\theta\theta'\delta^{ij} & -\bar\theta\theta\bar\theta'\theta'\delta^{ij} & -\Phi^i\delta^2(\theta-\theta') & (4-\bar\theta\theta\Lambda)\delta^{ij} & 0 \\
\bar\theta\theta\bar\theta'\theta'\delta^{ij} & 0 & 0 & 0 & 0 \\
\Phi^j\delta^2(\theta-\theta') & 0 & 0 & 0 & 0 \\
(-4+\bar\theta\theta\Lambda)\delta^{ij} & 0 & 0 & 0 & \bar\theta'\theta'\Phi^i \\
0 & 0 & 0 & -\bar\theta\theta\Phi^j & 0
\end{pmatrix} \delta(x-x')\,. \tag{5.11}
$$

The only zero-mode which may lead to a new constraint is

$$
v^{(1)} = \begin{pmatrix} 0 \\ \Phi^i \\ -\bar\theta\theta \\ 0 \\ 0 \end{pmatrix}. \tag{5.12}
$$

Using this eigenvector in the equation (2.13) we get

$$
\Phi^i(0)\Pi(0) = 0\,. \tag{5.13}
$$

We now incorporate this constraint into the kinectic part of the Lagrangian (5.9) by means of a Lagrange multiplier ζ. The result is

$$
\begin{aligned}
\mathcal{L}^{(2)} = & \left(\bar\theta\theta\,\Pi^i - i\bar\theta \frac{\partial\Phi^i}{\partial\theta} + \eta\Phi^i \right) \dot\Phi^i + \left(\frac{\partial^2\Phi^i}{\partial\theta\partial\bar\theta} - \Lambda(0)\Phi^i(0) \right) \dot\rho^i \\
& + \Phi^i(0)\Pi^i(0)\,\dot\zeta - V\,.
\end{aligned} \tag{5.14}
$$

Identifying the new quantities $a^{(2)}$, we construct the matrix $f^{(2)}$ (whose order of elements is Φ, Π, η, ρ, Λ, ζ)

$$
f^{(2)} = \begin{pmatrix}
4i\theta\theta'\delta^{ij} & -\bar\theta\theta\bar\theta'\theta'\delta^{ij} & -\Phi^i\delta^2(\theta-\theta') & (4-\bar\theta\theta\Lambda)\delta^{ij} & 0 & \bar\theta\theta\Pi^i \\
\bar\theta\theta\bar\theta'\theta'\delta^{ij} & 0 & 0 & 0 & 0 & \bar\theta\theta\Phi^i \\
\Phi^j\delta^2(\theta-\theta') & 0 & 0 & 0 & 0 & 0 \\
(-4+\bar\theta\theta\Lambda)\delta^{ij} & 0 & 0 & 0 & \bar\theta'\theta'\Phi^i & 0 \\
0 & 0 & 0 & -\bar\theta\theta\Phi^j & 0 & 0 \\
-\bar\theta'\theta'\,\Pi^j & -\bar\theta'\theta'\,\Phi^j & 0 & 0 & 0 & 0
\end{pmatrix}
.\delta(x-x') . \tag{5.15}
$$

This matrix is again singular. However, one can verify that its zero-modes do not lead to any new constraint. Following the same procedure of the previous section we also construct a matrix $\tilde f$ which gives the maximum number of the elements of $\delta^2(\theta-\theta')$ (see appendix for details). The elements of $\tilde f$ permit us to identify the following nonvanishing brackets

$$
\begin{aligned}
\{\Phi^i(x,\theta),\,\Phi^j(x',\theta')\} =& i\Big[\theta\theta'(\delta^{ij}-\Phi^i(0)\Phi^j(0)) \\
&+\frac{1}{2}\Big(\bar\theta'\theta'\theta\frac{\partial\Phi^i}{\partial\bar\theta}\Phi^j(0)-\bar\theta\theta\theta'\frac{\partial\Phi^j}{\partial\bar\theta'}\Phi^i(0)\Big)\Big]\,\delta(x-x'), \\
\{\Phi^i(x,\theta),\,\Pi^j(x',\theta')\} =& \Big[(1+\frac{\bar\theta\theta}{4})\Big(\delta^{ij}-\Phi^i(0)\Phi^j(0)\Big)+\bar\theta\frac{\partial\Phi^j}{\partial\theta}\Phi^i(0)\Big]\,\delta(x-x'), \\
\{\Phi^i(x,\theta),\,\eta(x',\theta')\} =& \Big(\frac{1}{4}\bar\theta\theta\bar\theta'\theta'\,\frac{\partial\Phi^j}{\partial\theta}\,\frac{\partial\Phi^j}{\partial\bar\theta}+\delta^2(\theta-\theta')\Big)\,\Phi^i(0)\,\delta(x-x'). \\
\{\Phi^i(x,\theta),\,\rho^j(x',\theta')\} =& -\frac{1}{4}\,\bar\theta\theta\bar\theta'\theta'\,\Big(\delta^{ij}-\Phi^i(0)\Phi^j(0)\Big)\,\delta(x-x'), \\
\{\Pi^i(x,\theta),\,\Pi^j(x',\theta')\} =& \Big(-\Phi^i(0)\Pi^j(0)+\Phi^j(0)\Pi^i(0)+i\frac{\partial\Phi^i}{\partial\theta}\,\frac{\partial\Phi^j}{\partial\theta}\Big)\delta(x-x'), \\
\{\Pi^i(x,\theta),\,\eta(x',\theta')\} =& -\Big(2i\bar\theta'\,\frac{\partial\Phi^i}{\partial\theta}+\bar\theta'\theta\Pi^i(0)\Big)\,\delta(x-x'), \\
\{\zeta(x,\theta),\,\Pi^i(x',\theta')\} =& -\bar\theta'\theta'\,\Phi^i(0)\,\delta(x-x'), \\
\{\eta(x,\theta),\,\eta(x',\theta')\} =& 4i\,\theta\theta'\,\delta(x-x'), \\
\{\Lambda(x,\theta),\,\eta(x',\theta')\} =& -4\,\delta(x-x'), \\
\{\zeta(x,\theta),\,\eta(x',\theta')\} =& \bar\theta\theta\bar\theta'\theta'\,\delta(x-x'), \\
\{\Lambda(x,\theta),\,\rho^i(x',\theta')\} =& -\,\theta\theta\Phi^i(0)\,\delta(x-x').
\end{aligned} \tag{5.16}
$$

In terms of the initial field Φ^i, we just have

$$
\begin{aligned}
\{\Phi^i(x,\theta),\,\Phi^j(x',\theta')\} =& i\Big[\theta\theta'(\delta^{ij}-\Phi^i(0)\Phi^j(0) \\
&+\frac{1}{2}\Big(\bar\theta'\theta'\theta\frac{\partial\Phi^i}{\partial\bar\theta}\Phi^j(0)-\bar\theta\theta\theta'\frac{\partial\Phi^j}{\partial\bar\theta'}\Phi^i(0)\Big)\Big]\,\delta(x-x'), \\
\bar\theta'\theta'\,\{\Phi^i(x,\theta),\,\Phi^j(x',\theta')\} =& \bar\theta'\theta'\Big(1+\frac{\bar\theta\theta}{4}\Big)\,(\delta^{ij}-\Phi^i(0)\Phi^j(0))\,\delta(x-x').
\end{aligned} \tag{5.17}
$$

Here also, if we write the brackets above in components we will obtain the usual brackets of the supersymmetric nonlinear sigma-model theory.

6. Conclusion

we have used the extension of the Faddeev-Jackiw sympletic formalism to superspace. First we have considered a supersymmetric field theory in a two dimensional space-time for free fields, both in components and in superfield formulations. In this last case, we have seen that it is not possible to define a sympletic matrix because the final matrix is singular. We have shown that the brackets can be consistently obtained by just considering part of the singular matrix. Finally, we have studied the supersymmetric nonlinear sigma-model directly in superfield formulation and the results are also in agreement with those in components, by using the Dirac formalism.

Acknowledgments

This work was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Brazilian Reserach Agency).

Appendix

Due to the presence of the fermionic variables θ_α, it is not possible to obtain $f^{(2)^{-1}}$ which would have to satisfy the relation

$$\int d^2\theta' dx'\, f^{(2)}(x,\theta;x',\theta')\, f^{(2)^{-1}}(x',\theta';x'',\theta'') = \delta^2(\theta-\theta'')\delta(x-x')\,. \qquad (A.1)$$

As was done in Sec. 4, we consider instead of $f^{(2)^{-1}}$ a matrix $\tilde{f}$ in order to generate the maximum possible number of terms of $\delta^2(\theta-\theta'')$. We consider that $\tilde{f}$ has the form

$$\tilde{f} = \begin{pmatrix} A^{jk} & B^{jk} & C^j & D^{jk} & E^j & F^j \\ -\underline{B}^{jk} & G^{jk} & H^j & I^{jk} & J^j & K^j \\ -\underline{C}^k & -\underline{H}^k & L & M^k & N & O \\ -\underline{D}^{jk} & -\underline{I}^{jk} & -\underline{M}^j & P^{jk} & Q^j & R^j \\ -\underline{E}^k & -\underline{J}^k & -\underline{N} & \underline{Q}^k & S & T \\ -\underline{F}^k & -\underline{K}^k & -\underline{O} & -\underline{R}^k & -\underline{T} & U \end{pmatrix}, \qquad (A.2)$$

where rows and columns follow the same order of $f^{(2)}$. Introducing this matrix in the place of $f^{(2)^{-1}}$ and using the arguments given above we obtain the following sets of equations

$$\int d^2\theta' \Big(4i\theta\theta' A^{ik} + \bar{\theta}\theta\bar{\theta}'\theta' \underline{B}^{ik} + \delta^2(\theta-\theta')\,\Phi^i(\theta)\underline{C}^k - (4-\bar{\theta}\theta\Lambda(0))\underline{D}^{ik} - \bar{\theta}\theta\,\Pi^i(0)\underline{F}^k\Big) = \delta^{ik}\delta^2(\theta-\theta'') \qquad (A.3a)$$

$$\int d^2\theta' \Big(4i\theta\theta' B^{ik} - \bar{\theta}\theta\bar{\theta}'\theta' \underline{G}^{ik} + \delta^2(\theta-\theta')\,\Phi^i(\theta)\underline{H}^k - (4-\bar{\theta}\theta\Lambda(0))\underline{I}^{ik} - \bar{\theta}\theta\,\Pi^i(0)\underline{K}^k\Big) = 0 \qquad (A.3b)$$

$$\int d^2\theta' \Big(4i\theta\theta' C^i - \bar{\theta}\theta\bar{\theta}'\theta' \underline{H}^i - \delta^2(\theta-\theta')\,\Phi^i(\theta)\,L - (4-\bar{\theta}\theta\Lambda(0))\underline{M}^{ik} - \bar{\theta}\theta\,\Pi^i(0)\underline{O}\Big) = 0 \qquad (A.3c)$$

$$\int d^2\theta' \Big(4i\theta\theta' D^{ik} - \bar{\theta}\theta\bar{\theta}'\theta' I^{ik} - \delta^2(\theta-\theta')\,\Phi^i(\theta)M^k + (4-\bar{\theta}\theta\Lambda(0))P^{ik} - \bar{\theta}\theta\,\Pi^i(0)\underline{R}^k\Big) = 0 \qquad (A.3d)$$

$$\int d^2\theta' \Big(4i\theta\theta' E^i - \bar{\theta}\theta\theta'\theta' J^i - \delta^2(\theta-\theta')\,\Phi^i(\theta)N + (4-\theta\theta\Lambda(0))Q^i - \bar{\theta}\theta\,\Pi^i(0)\underline{T}\Big) = 0 \qquad (A.3e)$$

$$\int d^2\theta' \Big(4i\theta\theta' F^i - \bar{\theta}\theta\bar{\theta}'\theta' K^i - \delta^2(\theta-\theta')\,\Phi^i(\theta)O + (4-\bar{\theta}\theta\Lambda(0))\,R^i + \bar{\theta}\theta\,\Pi^i(0)\,U\Big) = 0 \qquad (A.3f)$$

$$\int d^2\theta' \Big(\theta\theta\theta'\theta'\, A^{ik} - \bar{\theta}\theta\,\Phi^i(0)\underline{F}^k\Big) = 0 \qquad (A.4a)$$

$$\int d^2\theta' \Big(\bar{\theta}\theta\bar{\theta}'\theta'\, B^{ik} - \bar{\theta}\theta\,\Phi^i(0)\underline{K}^k\Big) = \bar{\theta}\theta\,\delta^{ik} \qquad (A.4b)$$

$$\int d^2\theta' \Big(\bar{\theta}\theta\bar{\theta}'\theta'\, C^i - \bar{\theta}\theta\,\Phi^i(0)\underline{O}\Big) = 0 \qquad (A.4c)$$

$$\int d^2\theta' \Big(\bar{\theta}\theta\bar{\theta}'\theta'\, D^{ik} - \bar{\theta}\theta\,\Phi^i(0)\underline{R}^k\Big) = 0 \qquad (A.4d)$$

$$\int d^2\theta' \Big(\bar{\theta}\theta\bar{\theta}'\theta'\, E^i - \bar{\theta}\theta\,\Phi^i(0)\underline{T}\Big) = 0 \qquad (A.4e)$$

$$\int d^2\theta' \Big(\bar{\theta}\theta\bar{\theta}'\theta'\, F^i - \bar{\theta}\theta\,\Phi^i(0)\,U\Big) = 0 \qquad (A.4f)$$

$$\Phi^j(\theta)\,A^{jk}(\theta,\theta'') = 0 \qquad (A.5a)$$

$$\Phi^j(\theta)\,B^{jk}(\theta,\theta'') = 0 \qquad (A.5b)$$

$$\Phi^j(\theta)\,C^j(\theta,\theta'') = \delta^2(\theta-\theta'') \qquad (A.5c)$$

$$\Phi^j(\theta)\,D^{jk}(\theta,\theta'') = 0 \qquad (A.5d)$$

$$\Phi^j(\theta)\,E^j(\theta,\theta'') = 0 \qquad (A.5e)$$

$$\Phi^j(\theta)\,F^j(\theta,\theta'') = 0 \qquad (A.5f)$$

$$\int d^2\theta' \Big[\theta'\theta'\,(\Lambda(0)\,A^{ik} - \Phi^i(0)\underline{E}^k) - 4\,A^{ik}\Big] = 0 \qquad (A.6a)$$

$$\int d^2\theta' \Big[\theta'\theta'\,(\Lambda(0)\,B^{ik} - \Phi^i(0)\underline{J}^k) - 4\,B^{ik}\Big] = 0 \qquad (A.6b)$$

$$\int d^2\theta' \Big[\theta'\theta'\,(\Lambda(0)\,C^i - \Phi^i(0)\underline{N}) - 4\,C^i\Big] = 0 \qquad (A.6c)$$

$$\int d^2\theta' \Big[\theta'\theta'\,(\Lambda(0)\,D^{ik} - \Phi^i(0)\underline{Q}^k) - 4\,D^{ik}\Big] = \delta^{ik}\bar{\theta}''\theta'' \qquad (A.6d)$$

$$\int d^2\theta' \Big[\theta'\theta'\,(\Lambda(0)\,E^i - \Phi^i(0)S) - 4\,E^i\Big] = 0 \qquad (A.6e)$$

$$\int d^2\theta' \left[\bar{\theta}'\theta' \left(\Lambda(0)\, F^i - \Phi^i(0)\, T\right) - 4\, F^i\right] = 0 \qquad (A.6f)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)\underline{D}^{jk} = 0 \qquad (A.7a)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)\underline{I}^{jk} = 0 \qquad (A.7b)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)\underline{M}^j = 0 \qquad (A.7c)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)P^{jk} = 0 \qquad (A.7d)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)Q^j = \bar{\theta}\theta \qquad (A.7e)$$

$$\int d^2\theta'\, \bar{\theta}\theta\, \Phi^j(0)R^j = 0 \qquad (A.7f)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(-\Pi^j(0)A^{jk} + \Phi^j(0)\underline{B}^{jk}\right) = 0 \qquad (A.8a)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(\Pi^j(0)B^{jk} + \Phi^j(0)G^{jk}\right) = 0 \qquad (A.8b)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(\Pi^j(0)C^j + \Phi^j(0)H^j\right) = 0 \qquad (A.8c)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(\Pi^j(0)D^{jk} + \Phi^j(0)I^{jk}\right) = 0 \qquad (A.8d)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(\Pi^j(0)E^j + \Phi^j(0)J^j\right) = 0 \qquad (A.8e)$$

$$\int d^2\theta'\, \bar{\theta}'\theta' \left(-\Pi^j(0)F^j + \Phi^j(0)K^j\right) = \bar{\theta}''\theta'' \qquad (A.8f)$$

A carefull solution of these equations leads to the brackets (5.16).

References

1. P.A.M. Dirac, Can. J. Math. **2**, 129 (1950).
2. For more recent reviews and applications, see P.A.M. Dirac *Lectures on quantum mechanics* (Yeshiva University, New York, 1964); A. Hanson, T. Regge and C. Teitelboim, *Constrained Hamiltonian systems* (Academia Nazionale dei Lincei, Rome, 1976); K. Sundermeyer, *Constrained Dynamics*, Lectures Notes on Physics (Springer, New York, 1982), Vol. 169.
3. L. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988).
4. For a general review on this subject see, for example, N. Woodhouse, *Geometric quantization* (Clarendon Press, Oxford, 1980). See also A. Das, *Integrable models* (World Scientific, Singapore, 1989), Chapter 1.
5. For a nice review of the FJ formalism and extension to Grassmannian variables, see J. Govaerts, Int. J. Mod. Phys. **A5**, 3625 (1990).
6. R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).
7. M.E.V. Costa and H.O. Girotti, Phys. Rev. Lett. **60**, 1771 (1988).
8. D.S. Kulshreshtha and H.J.W. Müller-Kirsten, Phys. Rev. **D43**, 3376 (1991).
9. J. Barcelos-Neto and C. Wotzasek, *Faddeev-Jackiw quantization and contraints*, to appear in Int. J. Mod. Phys. A; *Sympletic quantization of constrained systems*, to appear in Mod. Phys. Lett. A.
10. Futher applications of the FJ method with true constraints can be found in J. Barcelos-Neto and E.S. Cheb-Terrab, *Faddeev-Jackiw quantization in superspace*, to appear in Z. Phys. C; M.M. Horta-Barreira and C. Wotzasek, Phys. Rev. D45 (1992) 1410; C. Wotzasek, *Faddeev-Jackiw quantization of Siegel's chiral oscillator*, Preprint IF-UFRJ-08/92.
11. J. Barcelos-Neto and A. Das, Phys. Rev. **D33**, 2863 (1986).
12. J. Barcelos-Neto, A. Das and W. Scherer, Phys. Rev. **D34**, 1108 (1986); M.A. Almeida da Silva and J. Barcelos-Neto, Z. Phys. **C33**, 525 (1987); M.A. Almeida da Silva, J. Barcelos-Neto and N.R.F. Braga, Z. Phys. **C38**, 619 (1988); J. Barcelos-Neto and N.R.F. Braga, Phys. Rev. **D39**, 494 (1989).

13 H.J.W. Müller-Kirsten and A. Wiedemann, Z. Phys. **C35**, 471 (1987); J.A. de Azcárraga, J. Lukierski and P. Vindel, Phys. Lett. **B195**, 195 (1987); R.A. Marques Pereira, Class. Quantum Grav. **4**, 411 (1987).