IEKP–KA/92–11
August 8, 1992
IEKP-KA-92-11

Determination of α_s from the Scaling Violation in the Fragmentation Functions of the Process $e^+e^- \to h + X$*

W. de Boer† and T. Kußmaul‡
Institut für Experimentelle Kernphysik
Universität Karlsruhe, W-7500 Karlsruhe 1

Abstract

The QCD analysis of the fragmentation functions obtained from the process of inclusive hadron production $e^+e^- \to h + X$ shows a strong scaling violation. The data from PETRA and LEP have been analysed, which implies data at large Q^2 and large x: $196 \leq Q^2 \leq 8312\ GeV^2$ and $z(= E^h/E_{beam}) > 0.2$. This data have been used to extract the strong coupling constant in second order QCD, which was found to be: $\alpha_s(M_Z)$=0.119±0.006. The corresponding QCD scale for five quark flavours is: $\Lambda^{(5)}_{MS} = 240^{+90}_{-70}$ MeV.

*Talk given by W. de Boer at the workshop "QCD–20 Years Later", Aachen, June 9-13, 1992
†Bitnet: DEBOERW@CERNVM
‡Bitnet: BD41@DKAUNI2

1 Introduction

In the Quark-Parton model one expects scaling, i.e. the momentum distributions expressed as fraction of the maximum kinematical allowed value are independent of energy. In Quantum Chromodynamics (QCD) gluon radiation changes the kinematics, which leads to logarithmic scaling violations. They have been observed, both in the space-like regime in deep inelastic nucleon scattering[1]–[3] and in the time-like regime, e.g. in e^+e^- annihilation[4]. From the scaling violations the QCD coupling constant can be extracted. For example, in deep inelastic scattering one finds[5] –[6]: $\alpha_s(M_Z) = 0.113 \pm 0.005$, which is somewhat lower, but not in disagreement with direct α_s measurements at the Z^0 mass from shape variables, jet rates and total cross sections at LEP[7].
Until now α_s has not been determined from the scaling violations in the timelike region, since this requires precise data at two energies far enough apart to observe a significant scaling violation. At PETRA energies[4] the data were only precise at a single energy ($\sqrt{s} = 35$ GeV).
In this paper we present preliminary data of the inclusive hadron spectra, as measured with the DELPHI detector at LEP and present the first α_s determination from the scaling violations in the fragmentation functions by combining the data at LEP with data from PETRA, i.e. data for $196 \leq Q^2 \leq M_z^2 (\simeq 8312)$ GeV^2. The fragmentation functions $\bar{D}(x, Q^2)$ are directly related to the scaled hadron momentum distributions:

$$\bar{D}(x, Q^2) \equiv \sum_{i=1}^{5} W_i(Q^2) \bar{D}_i(x, Q^2) \equiv \frac{1}{\sigma_{tot}} \frac{d\sigma}{dx}(e^+e^- \rightarrow h + X), \tag{1}$$

where h is any charged hadron and $x = p_h/E_b$ is the hadron momentum divided by the beam energy. $\bar{D}(x, Q^2)$ is the sum of fragmentation functions $\bar{D}_i(x, Q^2)$ of all five flavours, each having a weight $W_i(Q^2)$ given by the electroweak theory. Note that the range of Q^2 studied here is much larger than the one studied in deep-inelastic scattering, which makes the so-called higher twist effects negligible.

2 Differential cross sections for inclusive multihadron production

The data have been collected during 1991 at energies near the Z^0 mass. The good multihadronic events inside the acceptance of the detector were selected according to the criteria given in Ref.[8]. The selection requires that there were at least 5 charged tracks with momenta above 0.2 GeV/c, the total charged energy seen in both hemispheres ($z > 0$ and $z < 0$) together exceeded 15 GeV (pion mass assumed),

each of the two hemisphere contained a total charged energy larger than 3 GeV, and the polar angle of the sphericity axis was in the OD range. In addition we selected "momentum balanced" events which have the absolute sum of the three-momenta of charged particles less than 20 GeV/c.
After these selection criteria about 195000 events at a mean center of mass energy of 91.2 GeV are kept. The background due to beam-gas scattering and $\gamma\gamma$-interactions is less than 0.1% and the $\tau^+\tau^-$ events contribute to 0.2% of the selected samples.
The scaled inclusive momentum spectrum has been obtained by correcting the x distribution of all charged particles for initial state radiation, particle decays, detector effects, and selection cuts. This was achieved by multiplying the data in each bin of the histogram by a correction factor defined as

$$c^{(i)} = \frac{N^{(i)}_{gen}}{N^{(i)}_{sim}}, \tag{2}$$

where $N^{(i)}$ is the content of the ith histogram bin either on the generator level (i.e. charged hadrons) or after the detailed detector simulation. In N_{gen} we kept all primary produced particles with an averaged lifetime smaller than 3.10^{-10} s. The corrected distribution is simply:

$$N^{(i)}_{corr} = c^{(i)} \cdot N^{(i)}_{Data}. \tag{3}$$

The corrections are less than 35% for x below 0.8 (see Fig. 1a). For larger x values the momentum measurement errors (momentum resolution of the detector $\delta p/p = 0.002p$ for muons) cause a smearing towards lower momenta, which results in a correction factor of 0.4 near x=1. The corrected spectrum has been displayed in Fig. 1b and tabulated in Table 1.
The integral under the x distribution is equal to the total charged multiplicity, since for each event one has N_{ch} entries, so a systematic error in the multiplicity after detector simulation translates in an uncertainty on the normalization of the x distribution. The resulting overall normalization errors is estimated to be 2%. In principle, the uncertainty could be larger at small values of x, because of photon conversions and nuclear interactions in the beam pipe. However, the amount of material before the charged particle detectors is small and the number of photon conversions is well described by the Monte Carlo. Therefore, each point was assigned the same systematic point-to-point error of 2%, which was added in quadrature to the statistical error.
If we compare the spectrum at 91 GeV with the data from TASSO[4] at lower center of mass energies, one observes a significant scaling violation (see Fig. 1b). The data at lower energies have been measured precisely only at a center of mass energy of 35 GeV. As shown in Fig. 2, this data deviates about 30 % from the data at 91

GeV for the small - as well as the large x range. The horizontal line corresponds to the expectation of the Quark-Parton Model without gluon radiation, i.e. without scaling violation. The deviation of the data from the horizontal line in each x bin is proportional to the strong coupling constant α_s, so each data point provides an independent measurement of α_s.
In Fig. 3 we plot the Q^2 dependence for several x intervals from various experiments[4, 9, 10, 11]. Clearly, at small x values the fragmentation function increases about 30%, while at high x values it decreases by roughly the same amount. This is exactly what one expects: the higher the energy, the more phase space one has for gluon radiation. Since the primary quarks lose now more energy due to radiation, this depopulates the high x region. The radiated gluons tend to populate the small x region, thus increasing the spectrum here.

3 Determination of the strong coupling constant

The inclusive production of charged hadrons in the reaction p) $e^+e^- \rightarrow h + X$ can be described by two kinematical variables:

- The square of the momentum transferred from the leptonic side (e^+e^-) to the hadronic one $(h + X)$, i.e. $Q^2 = q^2 = (k_{e^+} + k_{e^-})^2 \equiv s(= 4E^2_{beam}$ in c.m.s.)
- The fraction x of the beam energy E_b carried by the hadron h , i.e. $x = 2pq/Q^2(= E_h/E_b$ in c.m.s.). Instead of E_h the momentum p_h will be used, which is experimentally easier accessible and contains the same information concerning the scaling violation.

The scaling violations in the fragmentation functions, defined by Eq. 1, are described by the coupled integro–differential evolution equations[13], which in leading order can be written as:

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} \bar{D}_q(x, Q^2) \\ \bar{D}_G(x, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} P_{qq} & P_{Gq} \\ 2\sum_{i=1}^{5} P_{qG} & P_{GG} \end{pmatrix} \otimes \begin{pmatrix} \bar{D}_q(x, Q^2) \\ \bar{D}_G(x, Q^2) \end{pmatrix} \qquad (4)$$

The splitting functions $P_{ij}(z)$ are the probabilities of finding parton i with momentum fraction x from a parent parton j with momentum fraction y, where $x = zy$. These probabilities can be obtained by integrating the exact QCD matrix element. In order to obtain the probability of finding a hadron with momentum fraction x, one has to integrate $P_{ij}(z)$ between x and 1 and convolute this with the probability

$D_i^h(x/z, Q^2)$ that the parton with energy fraction z fragments into a hadron with momentum fraction x. This is meant by the symbol $\otimes$:

$$P(z) \otimes \bar{D}(x, Q^2) = \int_x^1 \frac{dz}{z} P(z) \cdot \bar{D}(\frac{x}{z}, Q^2) \tag{5}$$

In the evolution equations the subscribt G refers to the fragmentation function of gluons and i to the fragmentation function of quark flavour i. Note that a gluon can split into a quark–antiquark pair of any flavour, hence the summation and the factor two in front of P_{qG}.
The evolution equations describe the energy dependence of the fragmentation functions. In principle , for any function $\bar{D}_i^h(x, Q_0^2)$ chosen as the boundary condition at a reference point $Q^2 = Q_0^2$ they define its evolution up to the point $Q^2 > Q_0^2$. The solutions have not yet been found in an explicit analytical form. The methods of their numerical solution, which allow to take into account the second order corrections to the splitting functions or to the anomalous dimensions have been developed in papers[14]-[17].
In principle, the extraction of α_s from the observed scaling violations is straightforward: one parametrizes the x dependence at a given reference energy and the evolution to higher energies is predicted from the Altarelli-Parisi equations and compared with the observed fragmentation functions at higher energies. This can be done for every value of x, so one obtains a value of α_s for every x interval, which of course all should be consistent, if the procedure works.
We will first discuss the parametrization of the x dependence of the fragmentation function and then study the Q^2 dependence. The fragmentation functions have been studied in great detail in e^+e^- annihilation. Even such fine effects as the "string effect", predicted in QCD by the interference effects of multiple gluon emission, have been observed[18] and can be well described by the string fragmentation model[19]. Therefore we have used the fragmentation function inside the string model to parametrize the x dependence. Although any other parametrization of the x dependence at a given Q^2 would do as well, we have choosen the string model for the following reasons:

- The quark and gluon fragmentation functions are described by the *same* string with the same parameters, thus reducing the number of free parameters considerably.

- The model is infrared safe in the sense that soft gluons are automatically "absorbed" in the string. Else the fragmentation of soft gluons is problematic and one would not expect the functions $D_G^h(x, Q^2)$ to be independent of the incoming gluon energy.

- Quark mass effects, e.g. in the splitting of gluons into $q\bar{q}$ pairs, are taken into account in the string model.

- The concept of independent fragmentation is somewhat questionable in QCD, where one expects the hadrons to be produced along colour strings. These "distortions" of the independent fragmentation functions $D_i^h(x, Q^2)$ are taken into account in the string model. For the light quarks the LUND symmetric fragmentation function[22] was used:

$$f(z) = \frac{(1-z)^a}{z} \exp\left(-\frac{b \cdot m_\perp^2}{z}\right), \tag{6}$$

  where $m_\perp = \sqrt{m^2 + p_\perp^2}$ is the transverse mass of the hadron, $z = (E + p_l)_h/(E + p_l)_p$ determines the fraction of the primordial parton energy taken by the hadron and a and b are two free parameters. The longitudinal spectrum depends mainly on $a - b \propto N$, where N is the total multiplicity, so one effectively has one free parameter. For the heavy quarks (b and c) the Peterson fragmentation function[23] was used:

$$f(z) = \frac{1}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_i}{1-z}\right]^{-2} \tag{7}$$

  Here one expects the free parameter ϵ_i to vary as $1/m_q^2$, so we have taken $\epsilon_c/\epsilon_b = 9.4$. We are then left with only two free parameters to tune the momentum spectrum (a and ϵ_b). The parameters determining the transverse momenta, the fraction of strange quarks picked up from the vacuum, the ratio of vector - to pseudoscalar mesons, the fraction of baryons, as well as the decay parameters were left at their default values, since we are only interested to get a good description at the reference energy Q_0^2 and this was possible with the default values.

In principle the fragmentation of quarks involves an infinite amount of soft and collinear gluons. In order to avoid these infrared divergencies and separate the fragmentation of quarks and gluons, we require the minimum invariant mass between quarks and gluons to be above 9.1 GeV. Below this cut the quarks and gluons are considered to fragment into a single jet, i.e. they are considered to belong to the non-perturbative regime in the model and are recombined beforehand. This cut is a rather arbitrary definition of quark and gluon jets, but it has to be made in any analysis of the scaling violations and will be varied, if we study the systematic uncertainties.
Such a cut defines the cross section at Q=35 GeV as consisting of 24% 3-jet events ($q\bar{q}g$) and 76% 2-jet events. The fraction of 4-jet events is negligible. At 91 GeV

the larger phase space for gluon radiation yields 74% 3-jet events, 16% 2-jet events and 10% 4-jet events, if the same cut of 9.1 GeV in invariant mass is applied to separate the perturbative and nonperturbative regions and if the evolution of α_s is taken into account. Note that the running of α_s to lower values at higher energies has the opposite effect on the parton densities as the change in phase space. These numbers have been calculated for a renormalization scale $Q = M_Z$.
The large increase in the 3-jet rate due to the increase in phase space is the dominant source of the Q^2 dependence of the fragmentation function. It can be derived either from the evolution equations or from a direct numerical integration of the exact QCD matrix element. Since the splitting functions are derived from the integration of the matrix element, both methods are in principle equivalent. However, higher order differences may occur, since the integration of the evolution equations can be done numerically in small steps. At each step a gluon can be emitted. This corresponds to summing up all higher order terms $\alpha_s{}^n \ln^n Q^2$, which appear as leading terms in a calculation to order n. In the integration of the second order matrix element at most two gluons can be emitted. However, since the relative change in energy is only a factor 3, the exponentiated form of the leading logarithms is close to its second order expansion. Therefore, we expect no large difference between the methods and have determined the Q^2 dependence from the integration of the exact second order QCD matrix element, using the formulae from Ref.[20], which have been implemented in the LUND string model[19]. This method has the advantage that the convolution of the splitting– and fragmentation functions is done in a consistent way, i.e. the cuts to separate the nonperturbative infrared region from the perturbative part is the same for the splitting– and fragmentation functions. Furthermore, the weights $W_i(Q^2)$ in Eq. 1 from the electroweak theory have been incorporated in this model.
Two ways to extract α_s have been tried. First the x dependence at a center of mass energy of 35 GeV was parametrized by fitting the a parameter and fixing ϵ_b to 0.0075. The latter value is the one which yields an averaged B-meson energy of 0.70, as required by the semileptonic b-decays[21]. One cannot fit both parameters simultaneously from data at a single energy because of the strong correlation. After obtaining a good fit of the x dependence at the reference energy, $\Lambda^{(5)}_{\overline{MS}}$ was fitted from the inclusive spectra at 91 GeV keeping the parametrization for the x dependence found at the reference energy. The result was $\Lambda^{(5)}_{\overline{MS}} = 200 \pm 10$ MeV for x above 0.2. However, such a fit does not include the correlations between the parametrization of the x dependence and $\Lambda^{(5)}_{\overline{MS}}$. In order to take this into account a simultaneous fit of all parameters at all energies was made. We minimized the χ^2 defined as:

$$\chi^2 = \sum_j \left[\sum_i \left(\frac{(f_j D(i) - T(i))^2}{(f_j \sigma)^2} \right) + \frac{(1 - f_j)^2}{\sigma_n^2} \right],$$

where f_j is the normalization factor for experiment j with data $D(i)$ in a given x bin and an overall normalization error σ_n. The fitted function $T(i)$ is the x parametrization from the string model convoluted with the Q^2 dependence from the integration of the exact QCD matrix element and the Q^2 dependence of α_s. Fitting simultaneously all data of Fig. 3 in the range $0.18 < x < 0.8$ and $29^2 < Q^2 < 91.2^2$ GeV^2, as indicated by the dotted lines, yielded:

$$\begin{aligned} \Lambda^{(5)}_{\overline{MS}} &= 200 \pm 15 \text{ MeV} \\ a &= 0.89 \pm 0.02 \\ \epsilon_b &= 0.008 \pm 0.001 \end{aligned} \tag{8}$$

The correlations between $\Lambda^{(5)}_{\overline{MS}}$ and the fragmentation parameters are less than 40%. These values were obtained for a value of $b = 0.283$ in the LUND symmetric fragmentation function[§] and a renormalization scale $E_{cm}/10$ as argument of α_s.
The fit results are shown as the solid lines in Fig. 1b. A good agreement is observed for all x values above 0.1 with the *same* fragmentation parameters at both 35 and 91 GeV, so the difference between the energies is determined by $\Lambda^{(5)}_{\overline{MS}}$ only.
The results include all errors from the data, both the systematic and statistical errors, as well as the errors from the correlation between the fragmentation uncertainties and $\Lambda^{(5)}_{\overline{MS}}$. In addition there are theoretical uncertainties from the unknown higher order corrections, which are usually estimated by varying the renormalization scale. To get the complete error estimate, we have investigated the following:

- <u>The renormalization scale uncertainty.</u>
  The argument of α_s was varied between $E_{cm}/10$ and E_{cm}. $\Lambda^{(5)}_{\overline{MS}}$ should be independent of the choice of this scale, if all higher order corrections would be known. However, in fixed order a lower scale implies a larger value of α_s. For the 3-jet cross section the change in the Born cross section can be compensated by a different coefficient in the higher order correction. However, the 4-jet cross section is only known up to the Born term in second order QCD, so a lower scale for the argument of α_s implies a higher 4-jet rate. This change in the 4-jet rate can be compensated by a change in the fragmentation parameters, which parametrize the contributions of the non-perturbative soft gluons causing the hadronization. The χ^2 of the fit does not change significantly by changing the scale, but $\Lambda^{(5)}_{\overline{MS}}$ varied 200 to 280 MeV, if the scale was changed from $M_Z/10$ to M_Z. This error, originating from the unknown higher order corrections, was found to be the dominant error, as in all other α_s determinations. The scale dependence is still relatively small, since we study the difference between the

[§]Although a and b are strongly correlated, one cannot leave b at an arbitrary value and just fit a or vice-versa. A good parametrization is only obtained, if b is choosen in the range around 0.3.

spectra at different energies, so higher order contributions and fragmentation effects partly cancel. Similar observations have been for other "difference" variables, like the Asymmetry in the Energy-Energy Correlations (AEEC) or the difference in jetmasses[7].

- Differences between experiments.
  In contrast to the deep inelastic lepton scattering experiments, which measure the Q^2 dependence in a single experiment, we have to combine the data from experiments at different accelerators. In this case one has to worry about possible systematic effects from differences between the experiments. Since we have several experiments at various energies, one can check for such effects by comparing the results of different combinations. Recently, the ALEPH collaboration has published the inclusive spectrum at 91 GeV for a fraction of their event sample[12], which we have used in the various combinations. As shown in Fig. 4, we find the spread in $\Lambda^{(5)}_{\overline{MS}}$ to be about 20 MeV, so we attribute an error of $\pm$ 10 MeV to this systematic uncertainty. The reason for this surprising small error is simple: all experiments use large 4π solenoidal detectors, which implies well understood corrections, both for detector effects and initial state radiation. Furthermore, it is the change in the *shape* of the x spectra which is important, not the absolute normalization, and all experiments measure the shape well over the whole x range.

- x-dependence.
  At small x values the contribution from soft gluons dominates, in which case one has to worry about higher order corrections and infrared divergencies. Furthermore the gluon fragmentation contributes mainly below x=0.4.

  To estimate the uncertainty from the small x range, we fitted between x_{min} and x_{max} and varied x_{min} between 0.18 and 0.5. For lower values of x_{min} the χ^2 of the string model parametrization becomes worse (see Table1), as expected from the cuts introduced to remove the infrared singularities in the gluon emission. Nevertheless, including the lower x bins does not change the value of $\Lambda^{(5)}_{\overline{MS}}$. It only increases the χ^2. x_{max} was kept at 0.8, since the experimental correction factor becomes large for $x > 0.8$. Within the statistical error no variation in $\Lambda^{(5)}_{\overline{MS}}$ was seen for the varied range of x_{min}, so we did not attribute an additional error for the uncertainty from the small x range.

- Fragmentation effects.
  As mentioned before, the fragmentation effects largely cancel in the difference between the spectra at different energies. However, since the primary quark composition changes with energy, one has to worry about the difference in fragmentation between light and heavy quarks. The latter have a much harder

fragmentation function, although after decays the spectra look rather similar again.

It turns out to be impossible to mimick the characteristic change in shape expected from the QCD scaling violations by the difference in quark compositions. This is expressed by the small correlations between the QCD scale $\Lambda^{(5)}_{\overline{MS}}$ and the fragmentation parameters, as quoted above.

Actually, fitting the x spectra at 35 and 91 GeV simultaneously turns out to be a good way to determine the fragmentation of both light and heavy quarks, since the different quark composition at the different energies combined with the somewhat softer x spectrum of the heavy quarks after decay yields only a moderate correlation of 0.6 between the fragmentation parameters a and ϵ_b. The quoted value of the latter parameter gives an average energy of the B-mesons of 0.70 ± 0.01, which is in good agreement with the value obtained from the lepton spectra in semileptonic B decays[21]. Note that the determination of ϵ_b from the inclusive hadron spectra includes all decays and is therefore practically independent from the value determined from the lepton spectra.

- Experimental uncertainties.
  The experimental systematic uncertainties are twofold:

  - An overall normalization error of typically 2%.
  - A point-to-point systematic uncertainty, which may vary from bin to bin due to the fact that the Monte Carlo simulation has larger uncertainties at very small and large x values. These effects are more difficult to estimate and are typically of the order of 2 %.

  We varied both errors by a factor two compared with the values quoted by the experiments or if not quoted, assumed the typical values. The value of $\Lambda^{(5)}_{\overline{MS}}$ turned out not to be sensitive to the overal normalization factor, since the scaling violations are determined by the change in shape, not by the normalization. The point-to-point errors σ_{ptp} change the error on $\Lambda^{(5)}_{\overline{MS}}$ and the quoted error of 15 MeV corresponds to $\sigma_{ptp} = 0.02$. This point-to-point error was added in quadrature to the statistical error and leads to a χ^2 of 74 for 71 data points (using a common normalization error of 2% for each experiment). Increasing all systematic errors by a factor two reduces the χ^2 to 43 and increases the error on $\Lambda^{(5)}_{\overline{MS}}$ from 15 MeV to 20 MeV, but leaves the central value of $\Lambda^{(5)}_{\overline{MS}}$ practically unchanged. It should be noted that the fitted normalization factors were all consistent with one within the normalization errors.

- Cut-off dependence. As mentioned above, a cut on the invariant masses of 9.1 GeV between quarks and gluons has been applied to avoid the infrared

divergent region of soft and collinear gluons. The scaling violations are not very sensitive to this cut, since they just require a different parametrization of the nonperturbative part for a different cut. The only important thing is that one gets a good parametrization of the x dependence at the reference energy. Of course, if the cut becomes too high, one cannot describe the data anymore over the whole x and Q^2 range with the same parameters. However, this effects mostly the small x range, since the cut eliminates soft and collinear gluons. From a Monte Carlo study we know that the gluons contribute mainly below x=0.4. Increasing $y_{min} = Q_{ij}^2/s$ by a factor two (0.02 instead of 0.01 at 91 GeV) results in an increase of $\Lambda_{\overline{MS}}^{(5)}$ of 60 MeV. Therefore we attribute an error of ± 30 MeV, although part of this contribution is presumably already absorbed in the scale error, since in both cases the higher order contributions are varied.

We see that the observed range of $\Lambda_{\overline{MS}}^{(5)}$ varies between 200 and 280 MeV. Taking the average as the central value and combining the various experimental errors in quadrature (15 and 10 MeV from the fit and the comparison between experiments, respectively) and adding the non-gaussian errors from the cut-off and the renormalization scale (30 and 40 MeV, respectively) linearly, results in

$$\alpha_s(M_Z) = 0.119 \pm 0.006$$

This corresponds to $\Lambda_{\overline{MS}}^{(5)} = 240^{+90}_{-70}$ if we assume symmetric errors in α_s.
These results in the timelike region are in good agreement with the results on scaling violation in the spacelike region ($\alpha_s = 0.113 \pm 0.005$)[6] and with other α_s determinations at LEP from jet rates and shape variables ($\alpha_s = 0.122 \pm 0.007$)[7].

Acknowledgments

We are greatly indebted to our technical staffs, collaborators and funding agencies for their support in building the DELPHI detector and to the members of the LEP Division for the speedy commissioning and superb performance of the LEP machine. Furthermore, we would like to thank G. Altarelli, G. Schüler, T. Sjöstrand, N. Skachkov and B. Webber for useful discussions.

References

[1] BCDMS Collab., A.C.Benvenuti et al., Phys.Lett. B223 (1989) 490.

[2] EMC Collab., I.I.Aubert et al., Nucl.Phys. B293 (1987) 740.

[3] CHARM Collab., F.Bergsma et al., Phys.Lett. B123 (1983) 269; ibid. B153 (1985) 111.

[4] TASSO Collab., M.Althoff et al., Zeit.Phys. C22 (1984) 307.
TASSO Collab., W.Braunschweig et al., Zeit.Phys. C47 (1990) 187.

[5] M. Virchaux and A. Milsztajn, Phys. Lett. B274 (1992) 22

[6] A.D.Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B266(1991) 73

[7] The LEP Coll. (ALEPH, DELPHI, L3, OPAL), . Phys. Lett. B276 (1992) 247
ALEPH Coll., D. Decamp et al., Phys. Lett. B255 (1991) 623; ibid. B257 (1991) 479
DELPHI Coll., P.Abreu et al., Phys. Lett. B247 (1990) 167; ibid. B252 (1990) 159; Z. Phys. C 54 (1992) 55
L3 Coll., B. Adeva et al., Phys. Lett. B248 (1990) 464; ibid. B271 (1991) 461;
OPAL Coll., M.Z. Akrawy et al., Phys. Lett. B235 (1990) 389; ibid. B252 (1990) 159; Z. Phys. C 49 (1991) 375; P.D. Acton et al., CERN preprint PPE/91-214 (1991)

[8] DELPHI Collab., P.Aarnio et al., Phys. Lett. **B240** (1990) 271.

[9] Y.K. Li at al., Phys. Rev. D41(1990)2675

[10] O. Podobrin, Ph.D. thesis, Univ. of Hamburg, 1992

[11] MARK-II Coll., A. Petersen et al., Phys. Rev. D37 (1988) 1

[12] ALEPH Coll., CERN preprint PPE/92–62 (1992)

[13] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298
V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 78
G. Altarelli, Phys. Rep. 81 (1982) 1

[14] A.Devoto, D.W.Duke, I.F.Owens, R.G.Roberts, Phys. Rev. D27 (1983) 508.

[15] G.Curci, W.Furmanski, R.Petronzio, Nucl. Phys. B175 (1980) 27; W.Furmanski, R.Petronzio, Phys. Lett. B97 (1982) 437.

[16] E.G.Floratos, C.Kounnas, R.Lacaze, Nucl.Phys. B192 (1981) 417.

[17] T.Munehisa, M.Okada, K.Kudoh, K.Kitani, Prog.Theor.Phys. 67 (1982) 609.

[18] The effect has been first observed at PETRA by the JADE Collab., W. Bartel et al., Phys. Lett. B101 (1981) 129; ibid. B134 (1984) 275. Lateron it has been confirmed by many other groups.

[19] T. Sjöstrand, Comp. Phys. Comm. 39 (1986) 347;
T. Sjöstrand and M. Bengtsson, Comp. Phys. Comm. 43 (1987) 367
We used version JETSET 7.3.

[20] R.K. Ellis, D.A. Ross, E.A. Terrano, Phys. Rev. Lett. **45**(1980) 1225;
Nucl. Phys. **B178**(1981) 421

[21] P. Roudeau, Proc. Joint Int. Lepton-Photon Symp. and Europhys. Conf. on High Energy Phys., Vol. 2, p. 303, Geneva, (1991)

[22] B. Anderson, G. Gustavson, B. Söderberg, Z. Phys. C20 (1983) 317

[23] C. Peterson, D. Schlatter, I. Schmitt, P. Zerwas, Phys. Rev. D27 (1984) 105

Bin	χ^2	Data	σ_{stat}	σ_{syst}	Generator
0.00 – 0.01	4.22	400.83	0.806	8.017	414.03
0.01 – 0.02	15.64	409.26	0.699	8.185	438.21
0.02 – 0.03	15.04	264.65	0.566	5.293	283.01
0.03 – 0.04	6.49	185.05	0.478	3.701	193.01
0.04 – 0.05	1.55	137.37	0.415	2.748	139.71
0.05 – 0.06	0.19	105.30	0.365	2.106	105.39
0.06 – 0.07	0.23	83.57	0.327	1.671	82.09
0.07 – 0.08	1.26	68.35	0.298	1.367	66.24
0.08 – 0.09	6.35	56.88	0.274	1.138	53.50
0.09 – 0.10	3.94	47.20	0.250	0.944	44.90
0.10 – 0.12	4.77	37.08	0.157	0.742	35.15
0.12 – 0.14	2.19	27.61	0.136	0.552	26.56
0.14 – 0.16	0.98	20.90	0.119	0.418	20.30
0.16 – 0.18	3.84	16.59	0.108	0.332	15.78
0.18 – 0.20	0.75	12.92	0.095	0.258	12.58
0.20 – 0.22	0.51	10.37	0.086	0.207	10.13
0.22 – 0.24	1.16	8.36	0.077	0.167	8.09
0.24 – 0.26	0.65	6.72	0.069	0.134	6.79
0.26 – 0.28	0.62	5.67	0.064	0.113	5.52
0.28 – 0.30	0.23	4.61	0.057	0.092	4.63
0.30 – 0.32	0.75	3.85	0.053	0.077	3.90
0.32 – 0.34	0.62	3.19	0.047	0.064	3.23
0.34 – 0.36	0.14	2.70	0.044	0.054	2.66
0.36 – 0.40	5.43	2.09	0.027	0.042	2.19
0.40 – 0.44	0.13	1.50	0.024	0.030	1.47
0.44 – 0.48	0.23	1.08	0.019	0.022	1.00
0.48 – 0.52	0.00	0.75	0.016	0.015	0.76
0.52 – 0.56	0.07	0.56	0.014	0.011	0.56
0.56 – 0.60	1.18	0.40	0.011	0.008	0.38
0.60 – 0.66	1.17	0.27	0.007	0.005	0.26
0.66 – 0.72	0.62	0.16	0.006	0.003	0.16
0.72 – 0.78	4.16	0.10	0.004	0.002	0.09
0.78 – 0.84	7.35	0.05	0.003	0.001	0.05
0.84 – 0.90	7.80	0.02	0.002	0.0005	0.02
0.90 – 1.00	5.06	0.006	0.0006	0.0001	0.005

Table 1: The inclusive hadron momentum spectrum as measured by DELPHI

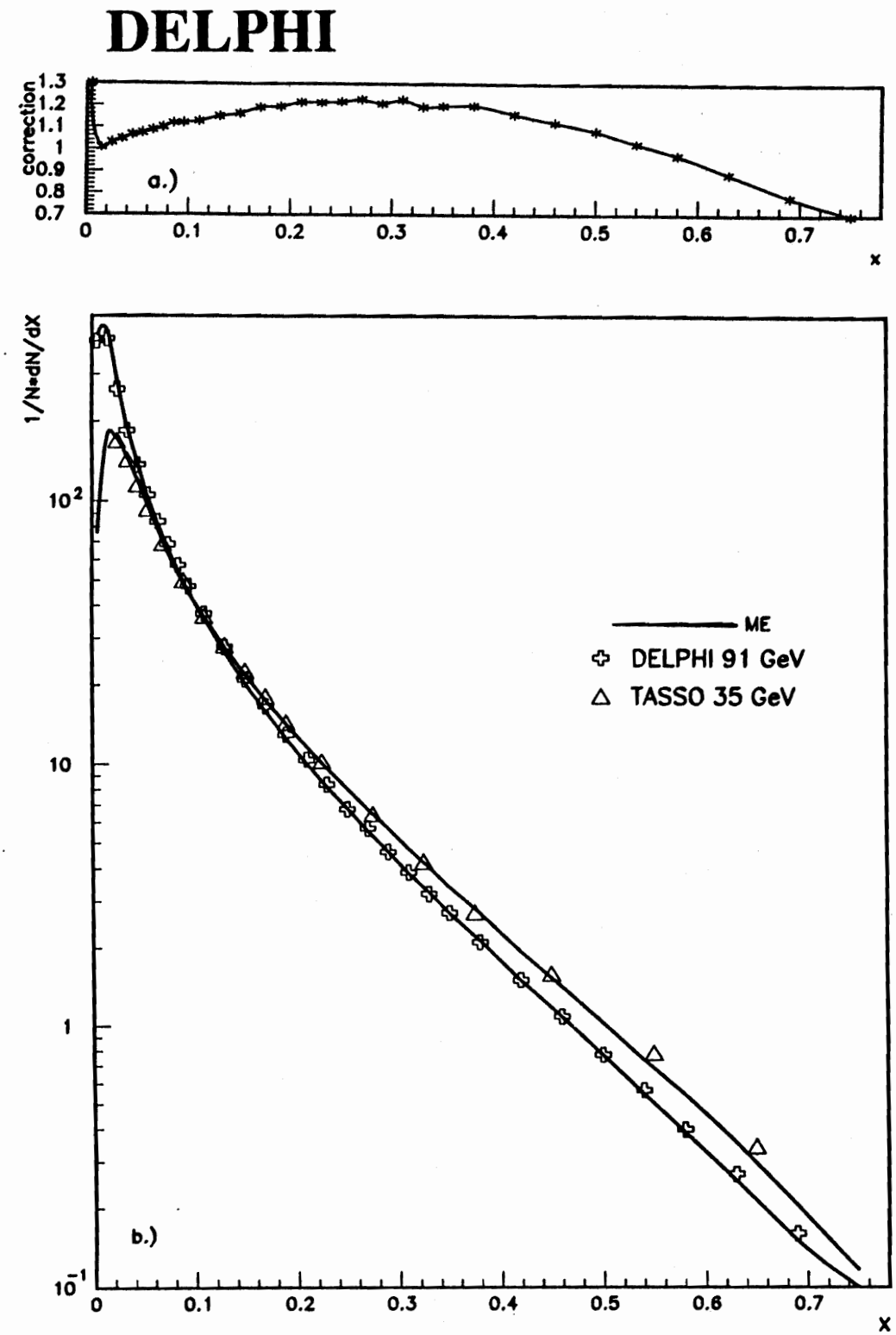

Figure 1: The correction factor (a) and the corrected inclusive momentum distribution $\frac{1}{\sigma}\frac{d\sigma}{dx}$, where $x = p_{hadron}/E_{beam}$ from TASSO data at 35 GeV and DELPHI data at 91 GeV. The solid curves are results of the fits to the second order QCD matrix element Monte Carlo.

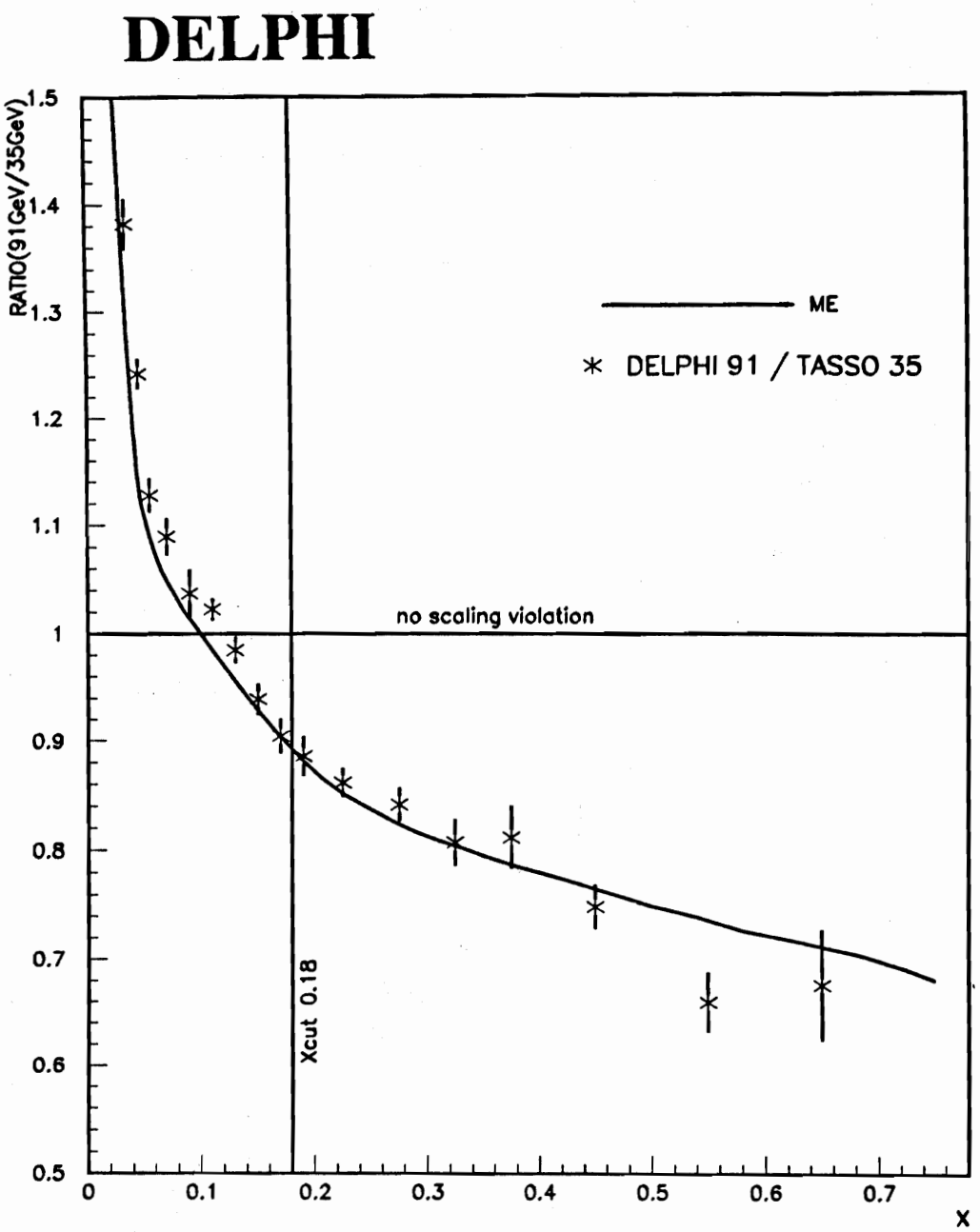

Figure 2: The ratio of the curves in Fig. 1b.

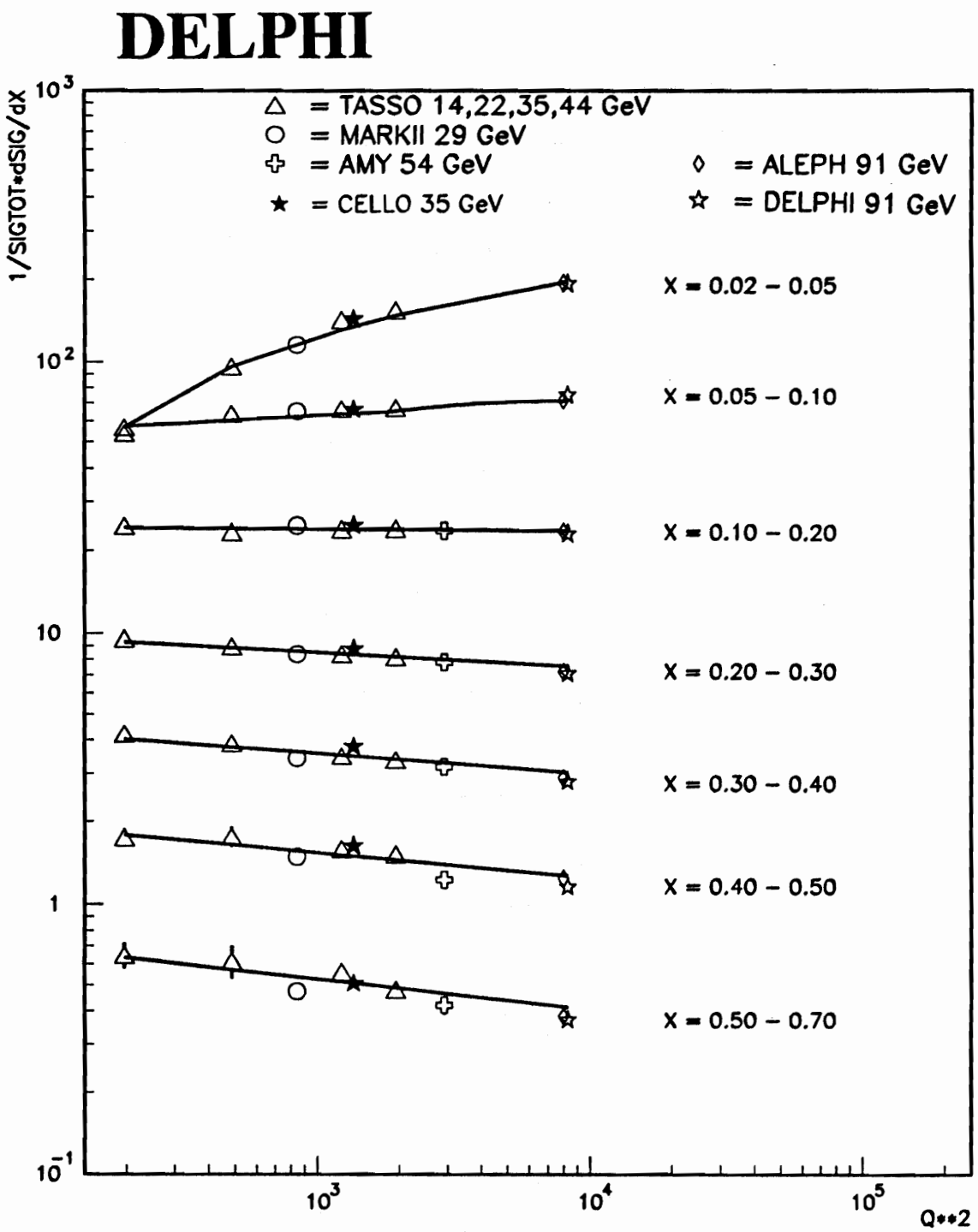

Figure 3: The Q^2 dependence of the inclusive momentum cross section for various x bins.

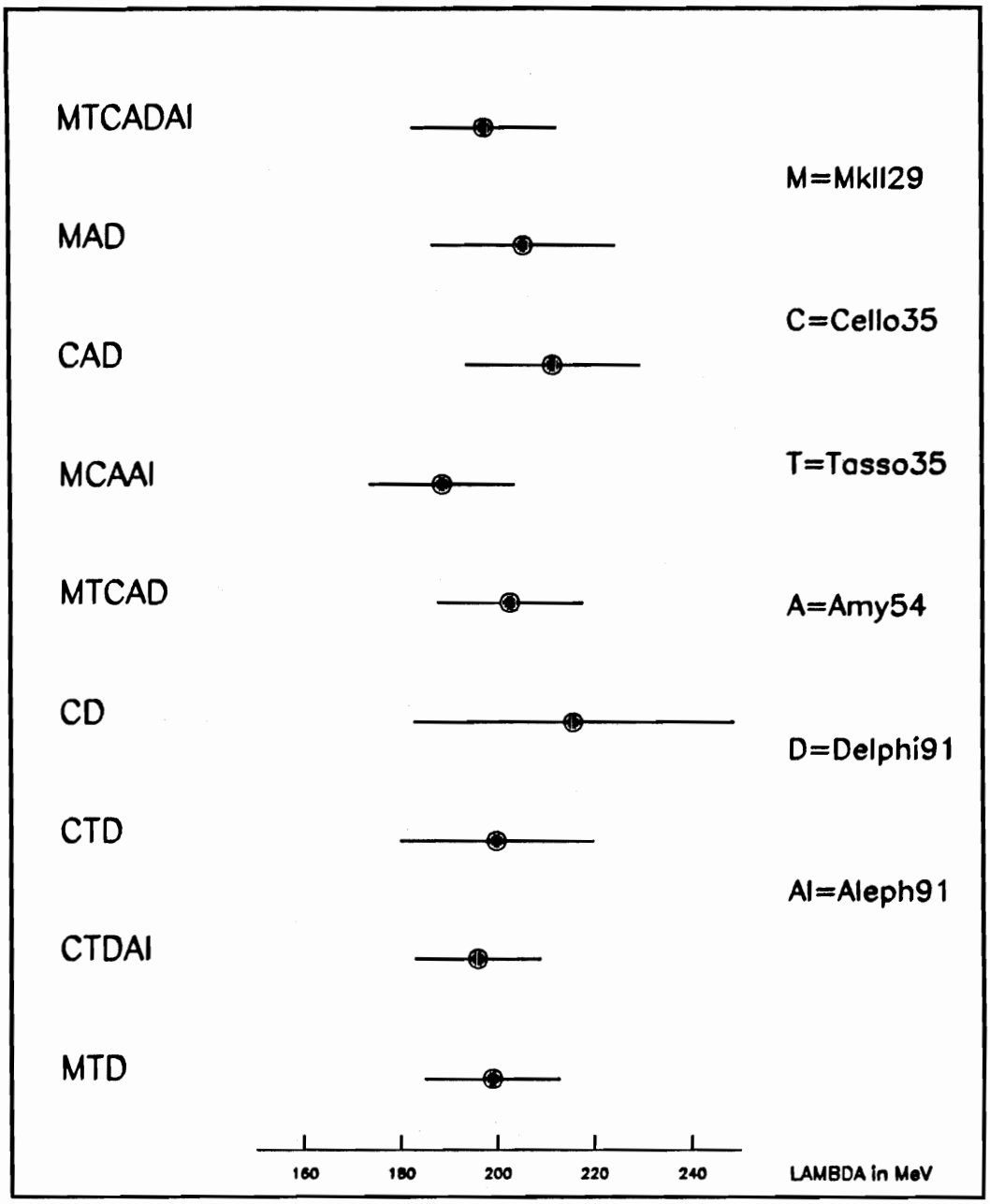

Figure 4: The $\Lambda^{(5)}_{\overline{MS}}$ values obtained from fitting various combinations of experiments.