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January 2001

Universidade Católica de Petrópolis
ICEN-Instituto de Ciências Exatas e Naturais
Rua Barão do Amazonas, 124 - Centro
Petrópolis - RJ - CEP - 25.685-070

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ANTI-SYMMETRIC TENSOR MATTER FIELDS AND NONLINEAR SIGMA-MODELS

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Abstract

The equivalence between 2-form gauge potentials and torsion-free non-linear σ-models motivates us to study the possibility of coupling tensorial matter to Yang-Mills fields, by gauging the isometries of a σ-model. We show that this coupling is actually possible; however, matter appears no longer as elementary degrees of freedom, but rather as composite fields, expressed in terms of the scalars that define the σ-model. A possible phenomenological application is presented here that describes the interactions among vector mesons in terms of the geometrical quantities of the target manifold. Also, spin-2 meson resonances may naturally be accommodated whenever the σ-models target manifold is a non-symmetric homogeneous space with torsion.

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1 Introduction

It is well-known that 2-form gauge fields, $B_{\mu\nu}$, that undergo the gauge transformations $\delta B = d\zeta$, are equivalent to massless scalar intermediate particles [1, 2, 3, 4]. Starting off from the Abelian theory, we can iterate consistency conditions to build up the non-Abelian version where self-interaction terms will be present.

In a first-order formulation of this tensor gauge theory, it is easy to show that there is an equivalence between these non-Abelian tensor fields and a non-linear σ-model without torsion [5]. On the other hand, a few years ago, Avdeev an Chizlov [6] have proposed an interesting Abelian model where a 2-form may describe charged matter that interacts via Maxwell fields. Following this idea, the works of ref.[7] adopted this viewpoint and a number of results were obtained in connection with this gauge theory for tensor matter. Next, pursuing the investigation of 2-forms as non-Abelian fields, a no-go theorem was proposed [8] that prevents the construction of a gauge theory whereby spin-1 gauge bosons interact with 2-form non-Abelian potentials by means of the Yang-Mills prescription.

In this letter, it is our aim to adopt the viewpoint of considering anti-symmetric tensor fields to describe matter degrees of freedom , rather than gauge mediating particles. With this purpose in mind, we establish a connection between the σ-model geometry and composite matter fields in the space-time. These are then identified with meson resonances.

We actually propose that these matter fields are introduced in connection with the torsion on the target manifold. Depending on the isometries of the

latter, we are able to construct a phenomenological approach in which the interactions between multiplets of vector mesons are understood in terms of the geometrical properties of the torsion generated by the tensor matter fields. Vector and spin-2 resonances are associated to composite objects written down in terms of the connection, torsion and curvature of the σ-model target space and their multiplet structure is determined by the group-theoretical properties of the homogeneous target space associated to the σ-model.

2 ON THE GEOMETRY OF σ-MODELS

The action of a non-linear σ-model is given by [9]

$$S = \int d^4x \frac{1}{2}\partial_\mu \varphi^m \partial^\mu \varphi^m, \tag{1}$$

where the set $(\varphi^1, \varphi^2, ..., \varphi^N)$ defines the coordinates of the target space, endowed with a metric $g_{mn}(\varphi)$. These fields fulfill, for example, a non-linear constraint:

$$\varphi_1^2 + \varphi_2^2 + ... + \varphi_N^2 = 1. \tag{2}$$

This particular constraint induces the mapping $\mathcal{M}^{1,3} \to S^{N-1}$. The global symmetry group (actually, the isometry) is $SO(N)$, and it is straightforward to obtain the metric of this manifold:

$$g_{mn}(\varphi) = \delta_{mn} + \frac{\varphi_m \varphi_n}{1 - \varphi_m \varphi_m}. \tag{3}$$

Consequently, we might rewrite our original action under the form:

$$S = \int d^4x \frac{1}{2} g_{mn}(\varphi) \partial_\mu \varphi^m \partial^\mu \varphi^n. \tag{4}$$

The coset space in this case is $S^{N-1} = \frac{SO(N)}{SO(N-1)}$, and the Maurer-Cartan equations define quantities like curvature and metric connection with torsion on this manifold:

$$de + e \wedge e = 0, \tag{5}$$

$$de + w \wedge e = T, \tag{6}$$

$$dw + w \wedge w = R, \tag{7}$$

where the forms e, w, T and R stand respectively for the vielbein and the connection 1-forms and the torsion and the curvature 2-forms.

The connection can be expressed as below:

$$\omega_m^{ab} = \frac{1}{2}\Omega_m^{ab}(e) + \frac{1}{2} e_m^c K_c^{ab}(T), \tag{8}$$

where Ω and K are, respectively, the non-holonomicity coefficients and the contortion tensor [10]. This is the content of the Cartan-Maurer equation given by (6).

We now quote how Yang-Mills fields and their corresponding field strengths may be identified with the pull-back of the $SO(N)$-connection naturally defined by the structure group of the target space:

$$V_\mu^{ab} = \omega_m^{ab} \partial_\mu \varphi^m \to F_{\mu\nu}^{ab} = R_{mn}^{ab} \partial_\mu \varphi^m \partial_\nu \varphi^n, \tag{9}$$

where $R_{mn}^{ab}(\varphi)$ is the Riemann curvature tensor of the target manifold and may be found by the second Maurer-Cartan equation:

$$R_{mn}^{ab} = \partial_m \omega_n^{ab} - \partial_n \omega_m^{ab} + \omega_m^{ac} \omega_{nc}^b - \omega_n^{ac} \omega_{mc}^b. \tag{10}$$

Actually, it may readily be proven that V_μ^{ab} and $F_{\mu\nu}^{ab}$ in Eq.(9) transform respectively like an $SO(N)$-gauge field and its associated field strength.

In order to introduce 2-form fields as matter, rather than gauge, degrees of freedom, we need torsion in the target manifold, mainly because it is a tensor and its pull-back defines a rank-2 skew-symmetric tensor field over space-time:

$$W^a_{\mu\nu} = T^a_{mn}(\varphi)\partial_\mu\varphi^m\partial_\nu\varphi^n. \tag{11}$$

T^a_{mn} is real, so, $W^a_{\mu\nu}$ are real anti-symmetric tensor matter fields, which describe spin-1 particles belonging to the $(1,0)\oplus(0,1)$-representation of $SO(1,3)$.

So, Eqs.(9) and (11) are the key elements for our claim: V^{ab}_μ acts as an Yang-Mills field with associated field strength $F^{ab}_{\mu\nu}$, and $W^a_{\mu\nu}$ may transforms as massive spin-1 matter fields. Therefore, none of these is taken as elementary; incidentally, we already know of interesting examples of models where gauge potentials appear as composite fields in connection with non-linear σ-models [11]. Our model has the character of an effective theory describing some low-energy physics. A good application of this effective model might be in the interactions among mesons: not only scalar mesons, as in the standard non-linear σ-model approach, but also vector and possibly spin-2 mesons, as we shall see later on, may be accomodated in the framework of σ-models.

3 MANIFOLDS G/H WITH TORSION

To present a concrete application, it is crucial to find manifolds of the form G/H, such that the embedding of H into G allows for non-trivial torsion.

The splitting of the generators, which specify the embedding of H into G,

is given as follows [10, 12]:

$$\begin{aligned}[Q_i, Q_j] &= f_{ijk}Q_k,\\ [Q_i, Q_a] &= f^b_{ia}Q_b,\\ [Q_a, Q_b] &= f^i_{ab}Q_i + f^c_{ab}Q_c,\end{aligned} \tag{12}$$

which yield a connection 1-form and a torsion 2-form given as below:

$$\begin{aligned}T^a &= k f^a_{bc} e^b \wedge e^c,\\ \omega^a_b &= -f^a_{bi}e^i - (1+k)f^a_{bc}e^c.\end{aligned} \tag{13}$$

The generators Q_i lie in the subalgebra H of G, whereas the Q_a's stand for the generators that are in G but not in H. The non-symmetry of the embedding is the group-theoretic counterpart of the torsion on the manifold G/H. Examples of homogeneous spaces with torsion are displayed below that may naturally account for spin-1 and spin-2 meson resonances.

4 CONTACT WITH PHENOMENOLOGY

In order to accommodate multiplets of vector mesons, we needed to search for coset spaces with a non-symmetric embedding, since we need torsion to build up the 2-form $W^a_{\mu\nu}$. The superscript a is a local frame index for the G/H and so it turns out that the degeneracy of the vector meson multiplets corresponds to the dimension of G/H. With the help of the Particle Data Group, we identify the following multiplets:

1. $\frac{G_2}{SU(3)}$:

   6-dimensional manifold $\longrightarrow$ 6-plet: $a_1(1270GeV)$ and b_1 $(1235GeV)$;

2. $\frac{SO(4)}{SU(2)} = \frac{SU(2)\otimes SU(2)}{SU(2)}$:

   3-dimensional manifold $\longrightarrow$ 3-plet: $\rho\ (770GeV)$;

3. $\frac{SU(3)}{U(1)}$ and $\frac{Sp(4)}{SU(2)} = \frac{SO(5)}{SO(3)}$:

   7-dimensional manifolds $\longrightarrow$ 7-plet: $a_1(1270GeV)$, $b_1(1235GeV)$, $f_1(1285GeV)$.

For the time being, we are only concerned with a merely kinematical classification. For this reason, we are not able to choose between $SU(3)/U(1)$ and $Sp_4/SU(2)$. However, introducing dynamics and interactions among different multiplets, decay ratios, computed in terms of the structure constants of G and H may impose appropriated constraints so that a particular space may be singled out.

5 ON SPIN-2 RESONANCES

With the geometrical objects we have at our disposal, we wish now to propose a possible description for the almost-degenerate spin-2 resonances of the meson spectrum. Symmetric rank-2 tensor fields associated to massive spin-2 particles may be identified with the following composite fields:

$$S_{\mu\nu} = T^a_k T^k_{j,\varphi} \partial_\mu \varphi^i \partial_\nu \varphi^j = S_{\nu\mu} \quad (singlet) \tag{14}$$

or

$$S^a_{\mu\nu} = R^{kdv}_{(\varphi} T^{kb}_{j)} \partial_\mu \varphi^i \partial_\nu \varphi^j = S^a_{\nu\mu}, \tag{15}$$

for non-trivial multiplets. A close inspection of the meson summary table indicates the appearance of singlets, doublets, 3-plets and 6-plets of spin-2 resonances. So, the non-symmetric spaces $SO(4)/SU(2)$, $G_2/SU(3)$ and $S_p(4)/(SU(2)\otimes U(1)_{nonmax}$ may well set the geometry for the 3-plet and 6-plet of spin-2 mesons. Again, torsion seems to be the key element for the construction of composite fields describing resonances of a non-trivial spin in the case with spin-2.

6 CONCLUSIONS

This work shows that non-linear σ-models defined on a coset space, G/H, with a non-symmetric embedding of H in to G allow for the introduction of non-Abelian 2-form fields that behave like massive spin-1 matter particles. These anti-symmetric fields are introduced by thanks to the torsion of σ-model target manifold. As a consequence, it is shown that multiplets of spin-2 mesons can also be introduced in such a way that they interact with scalar mesons in the framework of an effective (therefore non-renormalisable) theory described by the σ-model.

In this context, we claim that not only scalar mesons can be effectively be described by non-linear σ-models, but also the vector and tensorial ones. For this, we must introduce torsion in the manifold.

In this work, we restricted ourselves to a classification procedure. Next, we shall be concerned about the interactions among the various multiplets classified above; calculation of decay rates must be carried out and we shall thereby set dynamical constraints on the coupling constants that govern the

interactions of the multiplets and on the isometry group themselves. These results shall be soon reported elsewhere [13].

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