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Astrophysics Preprint Series
IASSNS-AST-93/2
January 1993

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Submitted to *The Astrophysical Journal*

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ABSTRACT

The distance to the Large Magellanic Cloud, $D_{\rm LMC}$, plays a major role in the "age-of-the-universe" problem because $H_0 t_{\rm gc} \propto D_{\rm LMC}^{-2.9}$, where H_0 is the Hubble parameter and $t_{\rm gc}$ is the age of the globular clusters. Panagia et al. (1991) have measured $D_{\rm LMC} = 50.1 \pm 3.1\,{\rm kpc}$ from the fluorescence of the ring around SN 1987A. Using this distance together with widely accepted methods of estimating H_0 and $t_{\rm gc}$, one finds $H_0 t_{\rm gc} \gtrsim 1.2$, in contradiction to all conventional cosmological models. I re-calculate $D_{\rm LMC}$ using the supernova-ring method both with and without the assumption that the ring is circular. For a circular ring, I find $D_{\rm LMC} = 53.2 \pm 2.6\,{\rm kpc}$, 3 kpc larger than the result of Panagia et al. If the assumption of a circular ring is relaxed, then this estimate of $D_{\rm LMC}$ is transformed into an upper limit. The $2\,\sigma$ upper limit (58.4 kpc) is sufficiently high to remove the age problem for a low-Ω universe with or without a cosmological constant.

Subject Headings: distance scale, Magellanic Clouds, astrometry

1. Introduction

Panagia et al. (1991) have derived a distance to SN 1987A in the Large Magellanic Cloud (LMC) from the light curve of the ring illuminated by the supernova. They find a distance of 51.2±3.1 kpc. Correcting for the position of the supernova relative to the LMC center of mass, they find a distance to the latter of

$$D_{\rm LMC} = 50.1 \pm 3.1\,{\rm kpc} \quad \text{(Panagia et al.)}. \tag{1.1}$$

Because this distance determination has small error bars and relies on relatively straight forward physics, it has begun to take precedence over other methods of measuring $D_{\rm LMC}$. For example, RR Lyraes were formerly regarded as providing a fairly reliable measure of $D_{\rm LMC}$. In this method, one calibrates the luminosity of local (Galactic) RR Lyraes using statistical parallax, then measures the flux from RR Lyraes in the LMC. However, Walker (1992) has recently argued that because the distance to the LMC is so well determined, one should reverse this process and calibrate the RR Lyrae luminosity from RR Lyraes in LMC globular clusters. He concludes that RR Lyraes are $0\overset{m}{.}3$ brighter than the Galactic calibration would indicate. Walker cites the supernova ring measurement [eq. (1.1)] as well as the Galactic Cepheid calibration (see below) as the basis for his confidence in the LMC distance measurement.

The measurement of the distance to the LMC plays a crucial role both in the measurement of the Hubble parameter, H_0, and in the measurement of the age of the globular clusters, $t_{\rm gc}$. Most methods of determining distances to distant galaxies rely directly or indirectly on the Cepheid period-luminosity relation (see Jacoby et al. 1992 for a review). This relation is calibrated from LMC Cepheids. Hence, any change in $D_{\rm LMC}$ translates into a proportional change in the measured distance to all other galaxies, and so

$$H_0 \propto D_{\rm LMC}^{-1}. \tag{1.2}$$

Similarly, the distances to the Galactic globular clusters are measured from their

RR Lyraes or more generally from the horizontal branch (e.g. Chaboyer, Sarajedini, & Demarque 1992). If RR Lyraes are calibrated in the LMC, then the distances to the globular clusters rise and fall with $D_{\rm LMC}$. According to equation (1) of Walker (1992), $\log t_{\rm gc} = -0.41 + 0.37 M_v({\rm TO}) - 0.43Y - 0.13[{\rm Fe/H}]$, where $M_v({\rm TO})$ is the absolute magnitude of the turnoff and the remaining terms account for helium and metal abundance. Hence, a change in $D_{\rm LMC}$, implies a change in $t_{\rm gc}$ of

$$t_{\rm gc} \propto D_{\rm LMC}^{-1.9}. \tag{1.3}$$

Furthermore, both H_0 and $t_{\rm gc}$ enter directly into the so-called age-of-the-universe problem (see e.g. van den Bergh 1992). Within standard cosmology $H_0 t_0 = \alpha$, where t_0 is the age of the universe and α is a parameter that depends on the cosmological model. For $\Omega = 1$, $\alpha = 2/3$, where Ω is the density of the universe in units of the closure density. For $\Omega \ll 1$, $\alpha \sim 1$. Since the universe is presumably older than the globular clusters, we have

$$H_0 t_{\rm gc} < \alpha. \tag{1.4}$$

If one takes $H_0 \sim 80\,{\rm km\,s^{-1}Mpc^{-1}}$ (e.g. Jacoby et al. 1992) and $t_{\rm gc} \gtrsim 15$ Gyr (e.g. Walker 1992), both obtained under the assumption $D_{\rm LMC} \sim 50\,{\rm kpc}$, then $H_0 t_{\rm gc} \gtrsim 1.2$, in contradiction to equation (1.4). However, equations (1.2) and (1.3) imply

$$H_0 t_{\rm gc} \propto D_{\rm LMC}^{-2.9}. \tag{1.5}$$

If it turned out that the LMC were really at 55 kpc rather than 50 kpc, the age problem would be removed for $\Omega \ll 1$. If $D_{\rm LMC} \sim 60\,{\rm kpc}$, then the age problem would be removed for $\Omega = 1$ as well.

Therefore, it is important to determine just how secure the value $D_{\rm LMC} = 50\,{\rm kpc}$ really is. As mentioned above, Walker cites two methods of determining $D_{\rm LMC}$. One is by Cepheids calibrated in nearby open clusters. This method

is fundamentally rooted in the distance to the Pleiades, the nearest young open cluster. Using a distance to the Pleiades of 130 pc (as measured by comparing A and F stars in the Pleiades with similar nearby stars which have trigonometric parallaxes) and assuming that the Pleiades are not in some way anomalous, Walker finds $D_{\rm LMC} \sim 51$ kpc in agreement with equation (1.1). However, Gatewood et al. (1990) have recently measured the distance to Pleiades by trigonometric parallax and find a distance of 150 ± 18 pc. Thus it remains possible that the distance to the Pleiades has been underestimated. Moreover, until there are direct distance measurements to several open clusters, one cannot be certain that the Pleiades are not anomalous.

The second method cited by Walker is the supernova-ring method, which is also open to some question. It rests on a number of model-dependent assumptions. In this paper, I analyze the assumptions underlying the supernova-ring method to determine the strength of their observational and theoretical support. I show that the only assumption that lacks strong support is that the ring is intrinsically circular. I re-analyze the distance determination both with and without this assumption. When the ring is assumed to be circular, I find a distance to the LMC of

$$D_{\rm LMC} = 53.2 \pm 2.6 \text{ kpc} \qquad \text{(circular ring)}. \tag{1.6}$$

The difference between this value and that given by Panagia et al. [eq. (1.1)] is due to two approximately equal effects. First, I have made a more careful analysis of how the observations of the ring should be combined to obtain a best estimate of the distance to SN 1987A. Second, I have adopted the value used by Jacoby et al. (1992) for the relative distances of SN 1987A and the center of mass of the LMC. I show that when the assumption of a circular ring is relaxed, equation (1.6) is transformed from a best estimate into an upper limit. That is, if the ring is eccentric, the distance estimate can only be reduced.

2. The Panagia et al. Measurement

The measurement of the distance to SN 1987A by Panagia et al. (1991) rests on three assumptions:

1) The observed ring of illuminated gas is indeed a thin planar structure, rather than a density caustic in a three-dimensional (e.g. ellipsoidal) structure.

2) The caustics in the ionized emission curves seen at ~ 83 and ~ 413 days identify the extreme light travel times for the paths going from supernova to ring to observer.

3) The ring is actually circular and appears elliptical because it is seen in projection.

Briefly, the argument given by Panagia et al. is as follows. For a circular ring, the light travel times to the far and near sides of the apparent minor axis (less the light travel time directly from the supernova) are

$$t_+ = \frac{d}{2c}(1 + \sin i), \tag{2.1}$$

and

$$t_- = \frac{d}{2c}(1 - \sin i), \tag{2.2}$$

where d is the physical diameter of the ring and i is the angle of inclination of the plane of the ring to the line of sight. The first term in these equations is the travel time from the center to the circumference of the ring. The second term is the travel time from the plane of the ring to the plane of the sky at the ring circumference. From these equation (2.1) and (2.2), one finds

$$\sin i = \frac{t_+ - t_-}{t_+ + t_-}. \tag{2.3}$$

Alternatively, one may estimate the angle of inclination from the apparent axis

ratio of the ring

$$\cos i = \frac{\theta_-}{\theta_+}, \tag{2.4}$$

where $\theta_\pm$ are the major and minor angular diameters of the apparent ellipse.

Panagia et al. model the light curves to determine $t_\pm$ and find

$$t_+ = 413 \pm 24 \text{ days}, \qquad t_- = 83 \pm 6 \text{ days}. \tag{2.5}$$

From these and equation (2.3), they make one measurement of the inclination, $i = 42° \pm 5°$. They then use the ellipse diameters measured by Jakobsen et al. (1991),

$$\theta_+ = 1\farcs66 \pm 0\farcs03, \qquad \theta_- = 1\farcs21 \pm 0\farcs03, \tag{2.6}$$

and equation (2.4) to make another measurement of the inclination, $i = 43° \pm 3°$. (I estimate smaller error bars on both inclination measurements. See § 3). They combine the two measurements to form an average value $\langle i \rangle = 42\fdg8 \pm 2\fdg6$, substitute into equation (2.1) to find d, and compute the distance to SN 1987A, $d/\theta_+ = 51.2 \pm 3.1$ kpc.

Before proceeding to an examination of the assumptions that underlie this calculation, I note that equations (2.1) and (2.2) play symmetric roles in the distance derivation and either might have been used in the penultimate step to derive the physical ring diameter, d. If Panagia et al. had used equation (2.2), they would have found $d/\theta_+ = 54$ kpc, almost $1\,\sigma$ higher than the value derived using equation (2.1). I return to this point in the next section.

I now turn to the assumptions. The ring appears planar, but as Dwek & Felton (1992) have emphasized, one should be cautious. Planetary nebulae are ellipsoidal shells and often appear as ellipsoidal rings in projection. Crotts & Heathcote (1991) have measured the redshift of the ring emission and find expansion along the minor axis, but essentially no expansion along the major axis. This is consistent with a ring seen with an inclination vector that is approximately aligned with the

apparent minor axis, but not with an ellipsoid. Dwek & Felton (who appear to have originally preferred an ellipsoid model) now agree that a planar structure is favored. They adduce a second argument that the ring structure is indeed planar, namely that there is a delay of ~ 80 days between the supernova and the beginning of the fluorescent emission. This is characteristic of an open topology such as a ring tilted to the line of sight, but not a closed topology such as an ellipsoid. To these, let me add a third argument. The hourglass appearance of the "Napoleon's Hat" nebula near SN 1987A (Podsiadlowski, Fabian, & Stevens 1991) shows that gas has indeed been blown out along the directions orthogonal to the ring. Only if an ellipsoid had formed after the Napoleon's Hat nebula, could it have maintained its structure. Finally, Panagia et al. argue that there appears to be very little light coming from the interior of the ring. This argument could be made into a quantitative test as follows: While the *density contrast* of the ring produced by an ellipsoid in projection can be arbitrarily high, the *integrated light* should be roughly equally distributed between the inner 3/4 of the area and the outer 1/4. One could fit the supernova and the ring to the point spread function (PSF) and the PSF convolved with a ring, subtract them off, and see whether the inner 3/4 of the ring really accounts for 1/2 of light. However, at this point, the other arguments seem so compelling that this additional test appears unnecessary.

Another non-planar geometry should also be considered. The ring may well be a "belt" around the center of the hourglass in the Napoleon's Hat nebula. In this case, there might be a near-cylinder of gas extending out of the ring plane. However, since the ring is inclined at $\sim 45°$, such a cylinder would appear almost exactly like a ring with finite thickness in the plane, and the effect on the timing arguments would likewise be almost exactly the same. Thus, to the extent that this geometry is allowed, it has no special consequences for the problem.

Once the geometry is established as planar, the meaning of the caustics seen in the light curve (see Fig. 2 of Panagia et al.) is clear: A burst of light incident on an arbitrary, smooth, convex, *reflection* nebula will always produce caustics in the light curve at the extreme times of reflection. The supernova ring does not reflect

but rather fluoresces, and therefore the (theoretical) reflection light curve must be convolved with a transfer function which characterizes the fluorescence. However, only a pathological transfer function could create, destroy, or move the caustics in the underlying reflection light curve. The one possibility that really must be considered is that the fluorescence has an extremely slow start up and peaks very quickly well after the burst of illumination. While it may be possible in principle to construct physical models which would have this slow start-up character, one would then expect the transfer function to have a broad peak, at least of order the start-up delay and probably longer. However, the data show an extremely rapid drop immediately following the second caustic, which excludes such a broad peak in the transfer function. (Note that to the small extent this effect is allowed by the light-curve data, it would tend to cause one to *overestimate* the distance to the supernova.)

I conclude that two of the three assumptions used by Panagia et al. are well founded in the data. The third assumption, that the ring is intrinsically circular, is less secure. Panagia et al. give two arguments for circularity. First, they say that "it is *physically* very hard to produce a high-eccentricity structure *centered* on its source." Second they point to the agreement in the inclinations as calculated from equations (2.3) and (2.4) as being consistent with the hypothesis of a circle. Neither of these arguments is compelling.

The fact that the inclinations as calculated from equations (2.3) and (2.4) are consistent with a circular ring does not make the ring circular. I construct explicit counter-examples below. The fact that we cannot think of a mechanism to produce an ellipse does not mean that an ellipse is excluded; nature is more clever than we are. There are several reasons for believing that the ring may have a low or moderate eccentricity. First, the gas in the ring is clumpy which may result from inhomogeneities in the medium into which it is expanding. Such inhomogeneities might deform an initially circular ring to be elliptical. Second, the model of the Napoleon's Hat nebula constructed by Podsiadlowski et al. (1991) shows the axis of the hourglass inclined at 19° to the apparent axis of the ring. This could be

because the ring actually lies in the plane perpendicular to the hourglass but has intrinsic ellipticity that makes it appear to lie in an inclined plane when the ring is treated as circular. Alternatively, the ring may not lie in the hourglass plane, but then the very presence of two planes shows that some physical process has broken the symmetry about the ring axis. This same process may have produced some ellipticity in the ring. Thus, there is no reason to exclude *a priori* the possibility that the ring has some intrinsic ellipticity. I therefore investigate to what extent the distance determination may depend on the ellipticity of the ring. Before doing so, I re-examine the circular case.

3. Calculation For A Circular Ring

Here I recalculate the distance to SN 1987A under the assumption that the ring is circular. In doing so, I introduce most aspects of the formalism that will be required for the non-circular case. The value that I derive for the distance is 1.5 kpc larger than that derived by Panagia et al.

First, I define four new multiplicative combinations of the measured quantities,

$$t_\times \equiv \sqrt{t_+ t_-} = 185 \pm 9\,\text{days}, \tag{3.1}$$

$$\theta_\times \equiv \sqrt{\theta_+ \theta_-} = 1\overset{''}{.}42 \pm 0\overset{''}{.}02, \tag{3.2}$$

$$\eta_t \equiv \frac{t_-}{t_+} = 0.201 \pm 0.019, \tag{3.3}$$

and

$$\eta_\theta \equiv \frac{\theta_-}{\theta_+} = 0.729 \pm 0.022 \tag{3.4}$$

The correlation coefficients of $t_\times$ with η_t and $\theta_\times$ with η_θ are 0.15 and 0.3 respectively. The four other pairs of quantities are independent.

From equations (2.1) and (2.2), one finds that $t_\times = (d/2c)\cos i$. Equation (2.4) implies that $\theta_\times = (d/D_{\rm SN})\sqrt{\cos i}$, where $D_{\rm SN}$ is the distance to the supernova. Hence,

$$D_{\rm SN} = D_\times G(i), \tag{3.5}$$

where

$$D_\times \equiv c\frac{t_\times}{\theta_\times} = 22.6 \pm 1.1\,{\rm kpc}, \tag{3.6}$$

and

$$G(i) = 2\sqrt{\sec i}. \tag{3.7}$$

Under the assumption that the ring is circular, the inclination can be measured by two independent methods. First, from η_θ using equation (2.4),

$$i = \cos^{-1}\eta_\theta = 43.^\circ 2 \pm 1.^\circ 8, \tag{3.8}$$

and second from η_t, using a transformation of equation (2.3),

$$i = \frac{\pi}{2} - 2\tan^{-1}\sqrt{\eta_t} = 41.^\circ 7 \pm 2.^\circ 0. \tag{3.9}$$

In both cases, I have determined the error bars by using the chain rule. Since the methods are independent, they can be combined to yield $i = 42.^\circ 5 \pm 1.^\circ 4$, or

$$G(i) = 2.329 \pm 0.025 \tag{3.10}$$

The quantities $D_\times$ and $G(i)$ are virtually independent (correlation coefficient $\sim$ 0.01), so the errors in equation (3.5) can be combined in the standard way to yield

$$D_{\rm SN} = 52.7 \pm 2.6\,{\rm kpc}. \tag{3.11}$$

Note that this result is 1.5 kpc larger than that found by Panagia et al. based on the same data. The reason for the difference is that by using equation (2.1) alone

rather than averaging over equations (2.1) and (2.2), Panagia et al. in effect gave higher statistical weight to some data than others. By transforming to the new variables, $t_\times$, $\theta_\times$, η_t, and η_θ, I have been able to carry out the calculation with all measured quantities weighted according to their observational errors.

To find the distance to the LMC, Panagia et al. assumed that SN 1987A lies 1.1 kpc further from us than does the center of mass of the LMC. However, Jacoby et al. (1992) point out that the eastern side of the LMC disk is known to be closer, so that if (as seems plausible) 30 Dor lies in the plane of the LMC, then SN 1987A lies 0.5 kpc *closer* than the LMC center of mass. I adopt this correction and find,

$$D_{\rm LMC} = 53.2 \pm 2.6\,{\rm kpc}. \tag{3.12}$$

4. Calculation For An Elliptical Ring

The formalism developed in the previous section for a circular ring can be generalized to the elliptical case. The resulting equations can be solved analytically in limit of small eccentricity, e, that is for $e^2 \ll 1$. To first order in e^2, the distance measurement is unchanged from the circular case. To second order, a finite eccentricity moves the LMC closer by a fractional amount $\sim 0.4(e^2)^2$. This systematic effect becomes significant (relative to the statistical errors) when $e^2 \gtrsim 0.3$, that is, for axis ratios $b/a \lesssim 0.85$. As far as I know, no measurements exclude such axis ratios. A numerical solution confirms these analytic results.

Suppose that the ring has major and minor semi-axes, a and b, and that the unit vector normal to the plane in which it lies is inclined to the line of sight at an angle i. Let ϕ be the position angle of the minor axis relative to the line in the plane which is maximally inclined to the line of sight. The geometry of the ellipse is then characterized by its distance $D_{\rm SN}$, its physical scale, a, and three

dimensionless parameters, i, ϕ, and e where,

$$e^2 \equiv 1 - \frac{b^2}{a^2}. \tag{4.1}$$

Projected on the sky, the ellipse will appear as a smaller ellipse with projected major and minor semi-axes a' and b'. The product of these axes is proportional to the area of the projected ellipse, that is

$$a'b' = ab\cos i. \tag{4.2}$$

After some algebra, one finds that the ratio of the axes is

$$\frac{b'}{a'} = f(i,\phi,e) - \sqrt{f^2(i,\phi,e) - 1}, \tag{4.3}$$

where

$$f(i,\phi,e) \equiv \frac{1}{4}\left(\frac{a}{b} + \frac{b}{a}\right)(\sec i + \cos i) + \frac{1}{4}\left(\frac{a}{b} - \frac{b}{a}\right)(\sec i - \cos i)\cos(2\phi). \tag{4.4}$$

Let γ be the position angle of an arbitrary point on the ellipse. The distance from the supernova to that point is then given by

$$r^2 = a^2\sin^2(\gamma - \phi) + b^2\cos^2(\gamma - \phi). \tag{4.5}$$

The light travel time from the supernova to an arbitrary point to the observer (less the time of travel directly from the supernova) is

$$t = \frac{r}{c}(1 + \sin i\cos\gamma). \tag{4.6}$$

This equation may be rewritten as

$$t = \frac{\sqrt{ab}}{c}g(i,\phi,e,\gamma), \tag{4.7}$$

where

$$g(i,\phi,e,\gamma) \equiv \sqrt{1+\xi}\,(1+\sin i\cos\gamma)\sqrt{1-\frac{e^2}{2-e^2}\cos(2\gamma-2\phi)}, \tag{4.8}$$

and

$$\xi \equiv \frac{(a-b)^2}{2ab} \sim \frac{e^4}{8}. \tag{4.9}$$

The caustics in the light curve occur at extreme times, which are found by differentiating equation (4.7), and setting $dt/d\gamma = 0$. Thus, the caustics lie at the γ which solve the equation,

$$\sin\gamma = \frac{e^2}{2-e^2}[\csc i\sin(2\gamma-2\phi)+\sin(3\gamma-2\phi)]. \tag{4.10}$$

This equation has at least two solutions. The cases that have more than two solutions correspond to light curves with more than two caustics. Since the actual light curve has only two caustics, I will ignore the more complicated cases.* I label the coordinates of the two solutions $\gamma_\pm$, and define $g_\pm$ by

$$g_\pm(i,\phi,e) \equiv g(i,\phi,e,\gamma_\pm). \tag{4.11}$$

The measurable quantities defined in the previous section may now be written in terms of the parameters of the ellipse:

$$t_\times = \frac{\sqrt{ab}}{c}\sqrt{g_+g_-}, \tag{4.12}$$

$$\theta_\times = 2\frac{\sqrt{ab}}{D_{\rm SN}}\sqrt{\cos i}, \tag{4.13}$$

* Numerically I find that the regions of parameter space with more than two caustics are adjacent to the regions that are permitted with low probability. They have lower inclinations than the permitted regions.

$$\eta_t = \frac{g_-}{g_+}, \tag{4.14}$$

and

$$\eta_\theta = f - \sqrt{f^2 - 1}, \tag{4.15}$$

where f and $g_\pm$ are given by equation (4.4) and (4.11). Hence, one may generalize equation (3.5), and write

$$D_{\rm SN} = D_\times G(i, \phi, e), \tag{4.16}$$

where

$$G(i, \phi, e) \equiv 2\sqrt{\frac{\cos i}{g_+ g_-}}. \tag{4.17}$$

The problem can now be solved by using the observed values of η_t and η_θ together with equations (4.14) and (4.15) to constrain the ellipse parameters i, ϕ, and e. For the allowed parameters, one may evaluate G and so the distance $D_{\rm SN}$ using equations (4.16) and (4.17).

Of course, it is impossible to evaluate the three ellipse parameters with only two equations. Even with perfect data there would be one degree of degeneracy in the allowed range of these parameters. However, the primary interest is not in these parameters *per se*, but only in the distance that they imply. In order to explore the nature of this degeneracy and its implications for $D_{\rm SN}$, I begin with a perturbative solution to the equations, expanding in the parameter $\epsilon = e^2$. For simplicity, I assume that the data are perfect and that the measured values of η_t and η_θ imply the same inclination i_0 when a circular ring is assumed [see eqs. (3.8) and (3.9)].

4.1. Zeroth Order

From equation (4.10), $\gamma_{+,0} = 0$ and $\gamma_{-,0} = \pi$. The position angle ϕ is indeterminate, that is all values are equally acceptable. For purposes of continuity with the first order equation, however, I choose $\phi_0 = \pi/4$, $\cos(2\phi_0) = 0$. From equation (4.8), $g_{+,0}g_{-,0} = \cos^2 i_0$.

4.2. First Order

From equation (4.10),

$$\gamma_{+,1} = -\frac{\epsilon}{2}(\csc i_0 + 1), \qquad \gamma_{-,1} = \pi + \frac{\epsilon}{2}(\csc i_0 - 1). \tag{4.18}$$

Note that for an elliptical ring, the caustics do not come from opposite sides of the ring, that is $\gamma_{-,1} - \gamma_{+,1} \neq \pi$. From equation (4.8), I find

$$\frac{g_{-,1}}{g_{+,1}} = \frac{1 - \sin i_1}{1 + \sin i_1}, \tag{4.19}$$

which implies that to first order there is no change in the inclination,

$$i_1 = i_0 \ . \tag{4.20}$$

Equations (4.4) and (4.15) require that

$$\cos(2\phi_1) = -\frac{\epsilon}{4}(2\csc^2 i_0 - 1), \tag{4.21}$$

and I choose $\phi_1 \sim \pi/4$. With this result, equation (4.8) implies that

$$g_{+,1}g_{-,1} = \cos^2 i_0. \tag{4.22}$$

Inserting equation (4.22) into equation (4.17) and comparing with equation (3.5), we see that to first order in e^2, the distance determination is independent of eccentricity.

4.3. Second Order

Substituting the first order parameters into equation (4.8) yields

$$g_{+,2}g_{-,2} = \left(1 + \frac{1 + \csc^2 i_0}{4}\epsilon^2\right)\cos^2 i_0. \tag{4.23}$$

This implies that to second order the distance is reduced by a fraction $0.40\,e^4$:

$$D_{\rm SN} = 2D_{\rm x}\sqrt{\sec i}(1 - 0.40\,e^4). \tag{4.24}$$

4.4. Geometrical Interpretation

There are two observational constraints on the geometry of the ellipse: the apparent axis ratio and the timing ratio. Suppose that these are consistent with a circular ring at inclination i. Any ellipse that lay at this inclination and had its minor axis aligned with the inclination would have a larger apparent axis ratio than the observed one. If the major axis of the ellipse were aligned with the inclination, the apparent axis ratio would be smaller. Hence, there will always be some intermediate position angle where the ellipse has the same apparent axis ratio as a circle with the same inclination. The above perturbative analysis tells us that this occurs when the position angle is $\pi/4 + \mathcal{O}(e^2)$, that is, halfway between the axes. For an ellipse in this position, the timing ratio is very similar to the circular case, exactly the same to first order in e^2. The reason is that while the positions with extreme delay times no longer lie along a diameter, the change in the light-travel times from the diametric case is second order in the angular displacement and hence $\mathcal{O}(e^4)$. For an ellipse with position angle $\phi = \pi/4$, the relevant linear dimension for light travel is $\sqrt{a'b'\sec i}$. Since the physical scale of the ring is judged from the de-projected area $\pi D_{\rm SN}^2\theta_+\theta_-\sec i$, one infers essentially the same distance for an elliptical ring as one would for a circular ring. As the eccentricity becomes large, however, two effects come into play. First, the light path deviates

considerably from a straight line across the ellipse. The observed time delays are therefore longer than would be the case if the two extreme trajectories lay along a single straight line. One therefore overestimates the distance to the supernova if one assumes a circular ring (for which the light paths do lie along a single straight line). Second, the position angle that reproduces the observed projected geometry lies slightly closer to the major axis than to the minor axis. Hence the projected diameter which most closely approximates the extreme light trajectories is slightly larger than $2\sqrt{a'b'}$. The light travel time is therefore longer than it would be for a circular ring having the same apparent size. This also causes one to overestimate the distance.

4.5. Numerical Solution

It is straight forward to solve equation (4.10) numerically. One may then find the predicted values for η_t and η_θ for any set of ellipse parameters i, ϕ, and e. The likelihood of the data given the model can be evaluated by assuming gaussian measurement errors. In principle, one should multiply this probability by the prior probability of the set of ellipse parameters, and sum over the whole of parameter space. The prior probability of the orientation parameters i and ϕ is well determined: We have no prior knowledge of how the ellipse is oriented, so the prior distribution is uniform in $\sin i$ and in ϕ. By contrast, there is no generally agreed upon prior probability for the eccentricity. For example, Panagia et al. believe that it is "physically very hard" to produce high eccentricity. I gave several arguments why the ring might be eccentric. I avoid this controversy by summing over the orientation angles, but reporting the differential distribution in eccentricity.

Figure 1 shows likelihood contours for G/G_0 versus e^2, where $G_0 = 2.33$ is the best estimate of G for the circular case ($e^2 = 0$). The contours form a sharply peaked ridge. For a given eccentricity, the measurement uncertainty of G is small compared to the uncertainty of $D_\times$, and it can therefore be ignored. However, the systematic uncertainty due to the possible eccentricity of the ring may be important. To find the most likely value of G, one must estimate the prior probabilities

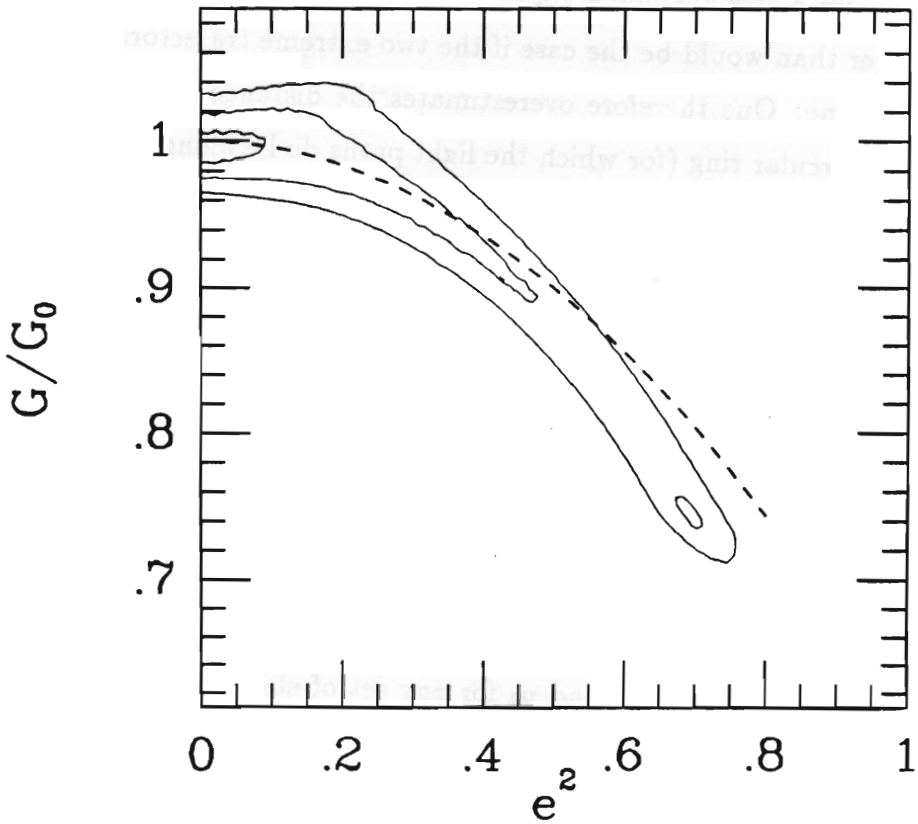

Figure 1. Likelihood contours as a function of e^2 and G/G_0, where e is the eccentricity, $G(i,\phi,e)$ is a parameter which enters the determination of $D_{\rm LMC}$, and $G_0 = 2.33$ is the value of G for a circular ring ($e = 0$). Likelihoods are averaged uniformly over the angular variables $\sin i$ and ϕ. The contours $m = 1$, 2, and 3 are shown as *solid* curves, where $\exp(-m^2/2) = L/L_{\rm max}$ and L is the likelihood. The second order perturbation result, $G/G_0 = 1 - 0.40\,e^4$ is shown as a *dashed* curve. Note that for fixed e^2, the uncertainty in G is small compared to the uncertainty of $D_\times$, the other quantity that enters $D_{\rm LMC}$.

of e^2 and sum over this variable. If one believes that the probability of $e^2 \gtrsim 0.2$ is extremely low, then $G = G_0$. However, even if one relaxes all prior constraints on e^2, one still obtains a hard upper limit,

$$G < G_0 = 2.33\ . \tag{4.25}$$

Substituting this equation into equation (4.16), I find an upper limit on $D_{\rm SN}$, and

therefore an upper limit on $D_{\rm LMC}$ of,

$$D_{\rm LMC} < 53.2 \pm 2.6 \text{ kpc}, \tag{4.26}$$

5. The Age Of The Universe

As I discussed in the introduction, the determination of the distance to the LMC is a crucial step in deciding whether there is indeed an age-of-the-universe problem because the parameter $H_0 t_{\rm gc} \propto D_{\rm LMC}^{-2.9}$. Recall that for typical values currently discussed for H_0 and $t_{\rm gc}$ (derived under the assumption that $D_{\rm LMC} = 50\,\text{kpc}$), we have $H_0 t_{\rm gc} \sim 1.2$. Table 1 lists values of $\alpha = H_0 t_0$ for a few sets of cosmological parameters. In this table, Ω is the density of the universe and λ is the cosmological constant, both expressed in units of the closure density.

TABLE 1

Age Parameter For Model Universes

Ω_0	λ_0	$\alpha \equiv H_0 t_0$
0.0	0.0	1.00
0.1	0.0	0.90
0.2	0.0	0.85
0.3	0.0	0.81
1.0	0.0	0.67
0.1	0.9	1.28
0.2	0.8	1.08
0.3	0.7	0.96

Equation (4.26) allows at the $2\,\sigma$ level a 17% increase in $D_{\rm LMC}$ relative to the presently accepted value. According to Table 1 and equation (1.5), this is sufficient to accommodate most models of the universe, but not a closed universe ($\Omega = 1$).

I conclude that the "age-of-the-universe" problem is a genuine problem only if $\Omega = 1$. For other models of the universe, the uncertainties in $D_{\rm LMC}$ alone are sufficient to account for the apparent discrepancy.

Even though there is at present no clear age problem, such a problem might re-emerge in the future. First, continued measurements of the distance to the Pleiades and other nearby open clusters may constrain $D_{\rm LMC}$ to a value lower than its present $2\,\sigma$ upper limit. Second, recall that the Galactic calibration of RR Lyraes yields $D_{\rm LMC} \sim 44\,{\rm kpc}$, which is $\sim 17\%$ closer than the supernova ring method. As it stands, this result is troubling and needs to be explained. However, the result would be substantially more troubling if $D_{\rm LMC}$ were near its $2\,\sigma$ upper limit. Third, Lee (1992) has recently argued that the Galactic bulge is $\sim 1.3\,{\rm Gyr}$ older than the globular clusters and has suggested that the central parts of larger galaxies may be even older. If Lee's results are confirmed, the age of the universe would be even larger than presently estimated. Finally, observational evidence might develop that $\Omega = 1$. These are all important questions which should be addressed. Nevertheless, based on the present best estimate (and $2\,\sigma$ upper limit) for $D_{\rm LMC}$, there is no age problem. I reach this conclusion even without considering additional uncertainties in H_0 (Jacoby et al. 1992) and $t_{\rm gc}$ (Walker 1992).

Acknowledgements: I would like to thank Dan Maoz, Richard McCray, Hans-Walter Rix, Herbert Rood, Michael Strauss, and David Weinberg for stimulating discussions. Work supported by the National Science Foundation (contract PHY92-45317).

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