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For Electron Gas : An Excitonic Phase

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Abstract:

A new state is possible for a relativistic electron gas. We argue that such a phase sets in for a value of fermi velocity close to unity.

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A dense mass of electrons is a subject of enduring novelty. Being one of the key elements of many-body physics, it has been explored since the last century; its usefulness spanning the design of communication systems all the way to the design of accelerators and beyond, and has, therefore, been an extremely well studied subject[1]. Yet, surprises have come at a pace and regularity that have defied understanding[2]. It is for this reason that every assumption of the underlying theory must be examined

While much of the theoretical work covered relatively low energy plasma, of late interest has begun to focus on the relativistic gas, which is being increasingly accessed in laboratories. In the design of non-conventional high energy accelerators, for example, the relativistic plasma, its wakes demand evermore precise understanding. In this work we focus on the possibility that in this domain the gas may undergo a phase transition into an altogether new state.

That there is a good deal of collective and coherent behaviour, both in the nonrelativistic and the relativistic domain has been investigated over the years. In the case of a low energy electron gas,with screene interactions, the effective spin couplings lead to spin-singlet coheren states lying in the middle of the spin density wave gap[3]. Again, fo a highly relativistic case, that of neutrino matter, we have indicate the possibility of a condensation[4].

The underlying procedure for exploring these aspects of many-bo systems is as follows. The polarisation function, Π , is calculated a the Green's function, G, is expanded in terms of the polarisation as follo

$$G = G^\circ + G^\circ \Pi G^\circ + \ldots$$

Since, G°, the lowest order Green's function is known, and Π calculated, the G may be obtained. It is known that the singularities of the Green's functions correspond to the physical states. An analysis of the divergences of equation (1) gives us the information we want.

In this work we are in the relativistic domain. Since the excitations we are looking for are obtained in the kinematical region of small momentum, the usual vacuum polarisation effects may be neglected in this analysis. These effects become significant for momentum of order chemical potential or above, where pair creations are important[5]. For us the important part of polarisation is the many-body effects, i.e., the particle-hole correlations.

In this framework we explore the relativistic electron gas and find that aside from the usual spectrum of excitation, there are other possibilities.

The New Modes :

In the rest frame of the system, the components of polarisation are given by[5,6]

$$\Pi_{oo} = -\frac{g^2 u^2}{4\pi^2}\left\{ X \ln\left|\frac{y+1}{y-1}\right| - 2 \right\} \tag{2}$$

$$\Pi_{11} = X^2 \Pi_{00} \; ; \Pi_{01} = \Pi_{10} = -X\,\Pi_{00} \tag{3}$$

These are valid in the region $k \ll \varepsilon_F$, $y = \frac{X}{v_F}$, $\frac{k_o}{|\vec{k}|} = x$. The coupling

constant g specifies the electron-photon interaction. There are other non-zero components of Π, which we have not written above, because these are not required for the discussions here. The quantity v_F is the fermi velocity.

The lowest order Green's functions of the photon, in the Feynman gauge is:

$$G^\circ_{\mu\nu} = -\frac{g_{\mu\nu}}{k^2} \; . \tag{4}$$

From, (1), (2) and (3) we have

$$G_{11} = G^\circ_{11} + G^\circ_{11}\,\Pi_{11}\,G_{11} + G^\circ_{11}\,\Pi_{10}\,G_{01} \tag{5}$$

with $$G_{01} = G^\circ_{oo}\,\Pi_{01}\,G_{11} \tag{6}$$

Since G°_{oo}, G°_{11}, Π_{11} and Π_{10} are all given in equations (4), (2) and (3), it is possible to solve for G_{11} as is usually done. If we solve for G_{11}, as is the common practice, because Π_{11} diverges at $y = 1$, G_{11} at $y=1$ is zero. This is the standard solution.

There is another equally plausible solution for equation (1). Since Π_{11}, Π_{10}, Π_{01} all diverge for $y=1$ on the right hand side of (1), G_{11} is divergent at $y=1$. This is a consistent solution. The modes around $y=1$ we call, for reasons that will become clear later, X1+ modes.

There are two possible solutions of the Schwinger-Dyson equations. Thus, there must exist the possibility of a new phase for the electron gas.

The Excitonic Phase :

The usual longitudenal plasma excitation spectrum characterise the normal phase of the electron gas. A few of X1+ excitation may be innocuous in this state. It is, however, possible to argue that in a certain domain, the X1+ modes lead to the possibility of a condensation.

As it is, the X1+ modes are difficult modes in the sense that long-wavelength modes with arbitrarily small energy may mix with the ground state of the system, destabilizing the state.

To proceed further we look at the imaginary part of the polarisation Π . This quantity has been calculated before and is of the form

$$\mathrm{Im}\ \Pi_{11} \sim g^2 \mu^2 x^2 y\, \theta\,(1 - |y|) \qquad (7)$$

This indicates that modes with $X > V_F$ are the stable ones. The $X<V_F$ are unstable and decay into particles and holes.

If now we approach the domain $v_F \sim 1$, the stable X1+ modes would occur for X values greater than one. Such an excitation would be acausal, because it would imply a mode travelling faster than light[7]. A way out is to assume that the X1+ modes condense.

The Order Parameter :

In calculating the many-body part of the polarisation, Π , because of the pole structure of the fermion propagator, particle-hole, particle-antiparticle and antiparticle-hole correlations give non-vanishing contributions. Since our investigation is restricted to the region $k \ll \varepsilon_F$, only particle-hole type correlations are important.

The X1+ are particle-hole modes, and the description of its condensation in terms of an order parameter was carried out in the context of neutrino physics earlier. Identical considerations apply. The electron-hole field is defined as[8]

$$e^h(r,t) = \gamma_2 \gamma_1 \gamma_3\, e^c\,(r, -t)\,, \qquad (8)$$

where e is the electron field; the superscript c means charge conjugation. The true ground state may then be characterised by a lorentz invariant order parameter[4]

$$\langle\, e^+ e^h + \mathrm{h.c.}\, \rangle \qquad (9)$$

That this object, indeed, is a lorentz scalar has been shown in an earlie work[4].

Conclusions :

A study of the Schwinger-Dyson equations for a system of electrons reve that, aside from the usual state, there is another consistent soluti to the problem. It is, therefore, possible for the usual state to transiti to the new one under thermodynamic conditions.

We argue that the new state comes about when the fermi momentum and the fermi energy are close to one another. Their ratio, the fermi velocity, when it approaches the vicinity of unity, stable tachyonic modes arise in the system. These modes must condense.

The condensation of these modes is described by an order parameter, first discussed in the context of neutrinos[4]. The particle-hole condensation is described by an elegant lorentz-invariant quantity given in (9).

The Green's function G_{11} of the photon, because of infinite particle hole correlations, is significantly altered in the kinematical region $X > 1$. They propagate in a background of stable nondecaying particle-hole modes, the excitons.

In text books we find that for systems with massless gauge interactions we have longitudinal plasma oscillations. For massive gauge interactions on the other hand, we have the usual zero-sound. Since the X1+ modes in this work are zero-sound-like, but prevail for a massless U(1) theory, we wonder, if the U(1) theory with condensed particle-hole modes are massive[9]? Do the particle-hole modes somehow disappear into the gauge bosons making them massive? We know now that symmetric and symmetry breaking solutions simultaneously exist for innumerable problems. The condensate (9), it is worth noting, is not an U(1) invariant.

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