A New Set of Transverse Phase Space Variable in High Energy Multiparticle Production

Liu Lianshou Wu Yuanfang

Institute of particle physics, Hua-Zhong Normal University, Wuhan 430070, China

ABSTRACT

The defects of commonly used phase space variables for the study of anomalous scaling in multiparticle production are discussed, and a new set of transverse variable \((w_1, w_2)\) appropriate for this kind of study are introduced. The statistical characteristic quantities of these variables and the corresponding cummulate variables are given.
Since the pioneer work of A. Bialas and R. Peschansky\textsuperscript{[1]} the possible existence of anomalous scaling of factorial moments $F_q$ in high energy multiparticle production has attracted much attention\textsuperscript{[2]}. This property depends on which phase space variables are used. In this letter the problem of currently used variables will be pointed out and a set of new variables, which has some advantage in the study of anomalous scaling, will be defined.

The natural variables in 3-D phase space is of course the three components $(p_1, p_2, p_3)$ of momentum vector $\vec{p}$. However, in high energy collisions there is a special direction, i.e. the direction of motion of the incident particles (taken as the third direction for convenience), which is called "longitudinal direction". All the physical quantities in multiparticle final states should be longitudinal Lorentz invariant, i.e. invariant (or at most change for an additive constant) with respect to the Lorentz transform along the longitudinal direction. The set of variables $(p_1, p_2, p_3)$ obviously do not fulfil this requirement. The longitudinal variable that meets the requirement of longitudinal Lorentz invariance is rapidity $y$, while the transverse variables can be any function of $(p_1, p_2)$ independent of $p_3(= p_3)$. Since the dynamical fluctuations in transverse plane are physically interesting\textsuperscript{[3]}, the appropriate choice of transverse variables is worth while careful discussion. This is the aim of the present letter.

Accounting for the cylindrical symmetry of the system, the "polar coordinate" $(\ln p_\perp, \phi)$ in the transverse plane of phase space are commonly used. However, for the study of anomalous scaling this choice causes some problem, because a ring-form region ($a \leq \sqrt{p_1^2 + p_2^2} \leq b$, cf. A in Fig.1) in the transverse phase space, when mapping into the plane of these variables ($A \rightarrow B$ in Fig.1), is cut by the arbitrarily chosen $p_1$ axis (the heavy line segment in Fig.1A), and the mapping of this axis is one-to-two instead of one-to-one, cf. Fig.1B. This will affect the scaling property of the one-dimensional projection of factorial moment $F_q$. For example, when $F_q$ is projected on the $p_1$ (or $p_2$) axis, the particles in two quadrants are projected into a single bin $\delta p_1$ ($\delta p_2$), in contrary both with the projection on the $(\ln) p_\perp$ axis, where the particles in all four quadrants are projected into a bin $\delta \ln p_\perp$, and with the projection on the $\phi$ axis, where only the particles in one quadrant are projected into a bin $\delta \phi$. This difference in particle aggregation will obviously influence the anomalous scaling property of factorial moments.

![Fig. 1 Mapping of transverse phase space into $(\ln p_\perp, \phi)$ plane. The ring-form region in transverse phase space is cut by the heavy line segment, which in turn is mapped into two segments.](image)

On the other hand, if the two components $(p_1, p_2)$ of transverse momentum are used directly, another problem arises, i.e. the region

$$a \leq |p_i| \leq b, \quad i = 1, 2$$

(1)

in $(p_1, p_2)$ space is divided into four disconnected parts due to the existence of transverse momentum cut $p_\perp \geq a$ in real experiment, cf. Fig.2A. This is an obstacle for the study of anomalous scaling.

In view of the above we try to define a new set of variable. Using the lower transverse momentum bound $a$ as a scale to measure $p_i$ ($i = 1, 2$) we define

$$w_i = \text{sgn}(p_i) \ln \left( \frac{|p_i|}{a} \right) = \text{sgn}(p_i)(\ln |p_i| - \ln a), \quad i = 1, 2,$$

(2)
and inversely
\[ p_i = a \text{ sgn}(w_i) e^{\|w_i\|}, \quad i = 1, 2, \]  
where sgn\( (p_i) = p_i / |p_i| \) is the sign of \( p_i \). The disconnected region Eq.(1) after mapping into \((w_1, w_2)\) plane becomes a simply-connected region
\[ -\ln \left( \frac{b}{a} \right) \leq w_i \leq \ln \left( \frac{b}{a} \right), \quad i = 1, 2, \]  
cf. Fig.2B.

For the convenience of further application, let us turn to the study of the statistical properties of \( w_i \)'s, i.e. their probability distribution, variance and covariance and the degree of independency between them. The probability distributions of \( p_\perp \) and \( \phi \) are known from experiment and can be expressed approximately as \( P(p_\perp) = 3 e^{-3p_\perp} \); \( P(\phi) = 1/2\pi \). Assuming these as independent distributions for granted we have for the combined distribution of \( w_1 \) and \( w_2 \):
\[ P(w_1, w_2) = P(p_\perp)P(\phi) \frac{D(p_\perp, p_\perp) D(p_1, p_2)}{D(w_1, w_2)} = A e^{3\sqrt{\sigma^2\|w_1\| + \sigma^2\|w_2\|}} \frac{e^{\|w_1\| + \|w_2\|}}{\sqrt{\sigma^2\|w_1\| + \sigma^2\|w_2\|}}, \]  
where \( A \) is a normalization constant making \( P(w_1, w_2) \) to be normalized in the region Eq.(4). This distribution has four peaks in the four quadrants with a dip at the center and descends steeply at the boundaries.

In general, the correlation (or independency) of two random variables \( x_1 \) and \( x_2 \) are characterized by their covariance \( \text{cov} \) and correlation coefficient \( \rho \) defined as \( \text{cov}(x_1, x_2) = \langle (x_1 - \langle x_1 \rangle)(x_2 - \langle x_2 \rangle) \rangle; \rho(x_1, x_2) = \frac{\text{cov}(x_1, x_2)}{\sigma(x_1)\sigma(x_2)} \) having the property \( \text{cov}(x_1, x_2) = \rho(x_1, x_2) = 0 \) for \( P(x_1, x_2) = P_1(x_1)P_2(x_2) \). In the present case, however, due to the symmetry of \( P(w_1, w_2) \), \( \text{cov}(w_1, w_2) = 0 \), and cannot be used to characterize the correlation strength between \( w_1 \) and \( w_2 \). Therefore, for a quantitative characterization of the correlation strength we use the "quadratic covariance"
\[ \text{cov}^2(x_1, x_2) = \langle (x_1 - \langle x_1 \rangle)^2(x_2 - \langle x_2 \rangle)^2 \rangle - \sigma_1^2\sigma_2^2, \]  
and the corresponding "quadratic correlation coefficient"
\[ \rho^2(x_1, x_2) = \frac{\text{cov}^2(x_1, x_2)}{\sigma^2(x_1)\sigma^2(x_2)}, \]  
which vanish for independent \( x_1 \) and \( x_2 \), \( \text{cov}^2(x_1, x_2) = \rho^2(x_1, x_2) = 0 \), for \( P(x_1, x_2) = P_1(x_1)P_2(x_2) \).

In our case with the distribution Eq.(5), taking \( a = e^{-9}, b = e^{3} \), the variance, quadratic covariance and quadratic correlation strength of \( w_1 \) and \( w_2 \) are respectively \( \sigma(w_1) = \sigma(w_2) = 6.825; \text{cov}^2(w_1, w_2) = 166; \rho^2(w_1, w_2) = 7.66 \times 10^{-2} \). The fact that \( \rho^2 < 1 \) shows that the correlation is weak, and so one can ignore the correlation between them approximately and use them as independent variables.

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In the study of anomalous scaling in multiparticle production the "cummulate variables"[1] are usually introduced to eliminate the influence of fluctuating distribution. When the correlation between \( w_1 \) and \( w_2 \) is neglected the cummulate variables are defined as

\[
X_i(w_i) = \int_{w_i}^{w_i'} P(w_i)dw_i, \quad i = 1, 2, \tag{8}
\]

where \( P_i(w_i) = \int_{-\ln[(b/a)]}^{\ln[(b/a)]} P(w_1, w_2)dw_2, \quad (i, j = 1, 2; \ i \neq j) \) is the marginal distribution, which has two strong peaks at about \( w_i = \pm 7.5 \). Taking the correlation into account the corresponding definition becomes

\[
X_1(w_1) = \int_{w_1}^{w_2} P_1(w_1)dw_1, \quad X_2(w_1, w_2) = \frac{1}{P_i(w_i)} \int_{w_2}^{w_2'} P(w_1, w_2)dw_2. \tag{9}
\]

Note that in the later case \( X_2 \) is a function of \( w_2 \) for fixed value of \( w_1 \), so that it depends on both \( w_2 \) and \( w_1 \).

The cummulate variables \( X_2 \) as function of \( w_2 \) for five different values of \( w_1 \) are shown in Fig.3. In the same figure is also shown the \( X_2(w_2) \) under the assumption of independency between \( w_2 \) and \( w_1 \) (curve 0). It can be seen that when \( w_2 \) takes its most probable value (\( \sim \pm 7.5 \), cf. Fig.2), the cummulate variables \( X_2 \) with and without correlation between \( w_2 \) and \( w_1 \) almost coincide. This is a further evidence showing that it is a good approximaton to neglect the correlation between \( w_1, w_2 \) and use Eq.(8) as the definition of the corresponding cummulate variables.

\[\text{Fig. 3 The cummulate variable } X_2 \text{ as function of } w_2 \text{ for different values of } w_1\]

In this letter we have discussed the various choices of phase space variables in multiparticle production. A new set of transverse variable \( (w_1, w_2) \) is introduced, which is more appropriate for anomalous scaling study.

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References


