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Earth as an Orbiting Neutrino Observatory

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Abstract

Earth's eccentric orbit about the sun offers us a moving laboratory from which we can find or exclude hypothetical neutrino oscillations over a wide range of truly astronomical wavelengths. The proposed Borexino solar neutrino experiment will record the arrival times of thousands of almost monochromatic neutrinos arising from electron capture by ^{7}Be nuclei in the solar core. We develop a simple and efficient algorithm with which to exhibit in advance its reach in neutrino masses and mixing angles and determine these parameters if a signal is present in the data. Oscillations can be observed for neutrino mass differences satisfying $10^{-8}\,\mathrm{eV}^2 > \delta m^2 > 10^{-11}\,\mathrm{eV}^2$, an interesting domain that is otherwise difficult to access.

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Imagine searching for neutrino oscillations with a mobile detector whose baseline variation is 5×10^6 kilometers and an intense monochromatic source. Such an experiment was proposed [1]. Borexino at Gran Sasso intends to measure the arrival times of ~ 50 solar neutrinos daily. Most of them arise from the electron capture reaction $e + {}^7\mathrm{Be} \to \nu_e + {}^7\mathrm{Li}$ and are nearly monochromatic with energy $E = 0.862$ MeV.

The distance d from sun's center to the laboratory varies. Neutrino masses and mixing may lead to observable time variations of the Borexino event rate, a point stressed elsewhere [1][2]. When data become available, they can be compared to various hypotheses. In the meantime, it is prudent to know what region of neutrino–oscillation parameter space may be explored. For expository clarity, assume that N events, all due to ^{7}Be neutrinos, are collected in the course of a year with both the detector's acceptance and the sun's neutrino luminosity remaining constant.[1]

Oscillations of astronomical length may pertain to the solar neutrino problem, whose solution (if, indeed, a solution is called for) could involve unknown properties of the sun or of neutrinos. We find a three–decade range of detectable oscillation lengths to which the Borexino experiment will be sensitive. It lies above those needed for the MSW effect [3] but includes those relevant to just–so neutrino oscillations [4]. Quite aside from this issue, few can doubt the importance of a search for long–wavelength neutrino oscillations of which we have neither *a priori* knowledge nor ready alternative access.

The largest change in d results from earth's elliptical orbit, whose semimajor axis (the astronomical unit) is $a \simeq 1.5 \times 10^{11}$ m and whose eccentricity is $\epsilon \simeq 0.0167$. The distance to the sun is expressed in terms ψ, the eccentric anomaly:

$$d = a(1 - \epsilon \cos \psi). \tag{1}$$

Its annual excursion defines the baseline variation of the experiment,

$$L_1 \equiv 2\epsilon a \simeq 5.0 \times 10^9 \text{ m}.$$

The angle ψ is related to time through Kepler's equation:

$$\frac{2\pi t}{\tau} = \psi - \epsilon \sin \psi,$$

[1] In fact, a few of the detected neutrinos are those of the solar neutrino continuum and the acceptance is unlikely to be uniform for a year. These complications do not vitiate our arguments.

with τ a sidereal year and t measured from perihelion. We incur a negligible error in our study of neutrino oscillations by ignoring the effect of eccentricity on the relation between t and ψ. Accordingly, we put

$$\psi = 2\pi t/\tau \quad \text{and define} \quad z \equiv \cos\psi.$$

Were detection independent of neutrino flavor (or were there no neutrino oscillations), the event rate would be affected only by the inverse–square law:

$$\left(\frac{dN}{d\psi}\right)_0 = \frac{\hat{N}}{2\pi}\,\frac{(1-\epsilon^2)^{3/2}}{(1-\epsilon z)^2}. \tag{2}$$

$\hat{N}$ is the number of neutrinos detected yearly under this circumstance. In the presence of oscillations, $\hat{N}$ is to be compared with predictions of solar models. Since it is not known with certainty, we phrase our tests for neutrino oscillations in terms of N, the number of events descried in a year.

Vacuum neutrino oscillations lead to additional time dependence and to an overall reduction of the event rate. Let δm^2 be the relevant neutrino mass splitting and θ the mixing angle. The oscillation length appropriate to ^{7}Be solar neutrinos with $E = 0.862$ MeV is:

$$L = \frac{4\pi E}{\delta m^2} \simeq \frac{2.14\,\mathrm{eV}^2}{\delta m^2} \text{ meters.}$$

Borexino detects neutrino–electron elastic scattering, to which all neutrino flavors contribute. The cross section is smaller for ν_μ or ν_τ than it is for ν_e. We define the parameter r according to:

$$1 - r \equiv \frac{\sigma(\nu_{\mu,\tau} + e \to \nu_{\mu,\tau} + e)}{\sigma(\nu_e + e \to \nu_e + e)} = \frac{1 - 4\sin^2\theta_w + 8\sin^4\theta_w}{1 + 4\sin^2\theta_w + 8\sin^4\theta_w},$$

where θ_w is the weak mixing angle. We find $r \simeq 0.79$.

If all the neutrinos were created with the same energy at sun's center, the Borexino event rate (including contributions from all neutrino flavors) would be:

$$\left(\frac{dN}{d\psi}\right)_1 = \left(1 - \frac{r}{2}\sin^2 2\theta\right)\left[1 + \lambda\cos\frac{2\pi d}{L}\right]\left(\frac{dN}{d\psi}\right)_0, \tag{3}$$

where

$$\lambda = \frac{r\sin^2 2\theta}{2 - r\sin^2 2\theta}$$

measures the strength of the observed oscillations. But, the neutrinos are neither exactly monochromatic nor produced at a point.

Neutrino pathlengths vary because their production is distributed in the sun's core, thereby suppressing oscillations. For ^{7}Be neutrinos, the source density is $\propto \exp -(z/s)^2$, where z is projected along the earth–sun direction and $s \simeq R_\odot/17 \simeq 4.1 \times 10^7$ m [5]. Since $R_\oplus \ll s \ll L_1$, we neglect the small daily and monthly perturbations of d in estimating the suppression. Even more important are the thermal motions that broaden the neutrino line — a Doppler shift due to nuclear motion and a contribution of the electron's kinetic energy. The mean solar temperature in the region that ^{7}Be neutrinos are produced is $kT \simeq 1.15$ KeV and the nuclear mass M is ~ 6.5 GeV/c^2.

The size and temperature of the source reduce oscillations[2] by a multiplicative factor $S(L)$ given approximately by:

$$S(L) = \frac{e^{-(L_0/L)^2}}{\sqrt{1 + (L_0'/L)^2}}, \tag{4}$$

where:

$$L_0 = \pi\sqrt{s^2 + 2a^2\, kT/M} \simeq 3.1 \times 10^8 \text{ m}, \quad \text{and} \quad L_0' = 2\pi\, a\, kT/E \simeq 1.25 \times 10^9 \text{ m}.$$

Thus, there is a lower limit to L (or upper limit to δm^2) for which oscillations can be seen [2]. At $L = L_0$ (or $\delta m^2 \simeq 10^{-8}$ eV^2), they are suppressed by $S(L) = 0.09$ and are will not be detectable by Borexino even if $\sin^2 2\theta = 1$. The daily excursion of d due to earth's rotation is $< 1.3 \times 10^7$ m or $0.04\, L_0$; its monthly excursion due to moon's attraction is 9.3×10^6 m or $0.03\, L_0$. The above considerations show that these motions make no measurable modulations.

An upper limit to detectable values of L (or lower limit to δm^2) is not sharply defined. Less than one oscillation is completed if $L > L_1$. We find that its effect is detectable with a sensitivity that falls erratically with L.

The observed event rate, (3) with the suppression factor taken into account, may be written:

$$\frac{dN}{d\psi} = \left(1 - \frac{r}{2}\sin^2 2\theta\right)\left[1 + \hat{\lambda}\cos\left(p - p\epsilon z\right)\right]\left(\frac{dN}{d\psi}\right)_0, \tag{5}$$

[2] The phase of the oscillation is also changed [2], but this effect is irrelevant to our considerations.

where

$$\hat{\lambda} = S(L)\,\lambda \quad \text{and} \quad p = 2\pi a/L.$$

Integration of (5) over the year yields the relation between N and $\hat{N}$:

$$N = \zeta \hat{N}, \quad \text{where} \quad \zeta = \left(1 - \frac{r}{2}\sin^2 2\theta\right)\,(1 + \hat{\lambda}\,J_0(\epsilon p)\,\cos p). \tag{6}$$

J_0 is a Bessel function of the first kind.

Were Borexino data in hand, we might fit them to (5) with $\hat{\lambda}$ and p as free parameters. Absent the data, we show how the sensitivity of the proposed experiment may be determined in advance without performing a plethora of Monte–Carlo simulations. dN/dz (which is periodic in the interval $1 \geq z \geq -1$) is the product of a sinusoidal oscillation in z with the slowly–varying function $(dN/d\psi)_0$ and the Jacobian $d\psi/dz$. We search for neutrino oscillations by performing a weighted Fourier transform of a hypothetical data set of N events. To this end, z_j is computed for each event and the following Fourier sums are evaluated:

$$Q_m(N) = \frac{1}{N}\sum_{j=1}^{N} G(z_j)\,\cos \pi m z_j \quad \text{for} \quad m = 0,\,1,\,\ldots$$

$$R_m(N) = \frac{1}{N}\sum_{j=1}^{N} G(z_j)\,\sin \pi m z_j \quad \text{for} \quad m = 1,\,2,\,\ldots$$

The positive weight function G cancels the Jacobian and undoes the small direct effect of earth's annual motion:

$$G(z) \equiv \frac{\pi\,(1-\epsilon z)^2\,\sqrt{1-z^2}}{2\,(1-\epsilon^2)^{3/2}}. \tag{7}$$

The sums' expected values with no oscillations ($\theta = 0$ and $N = \hat{N}$) are given by their continuum limits:

$$\langle Q_m \rangle = \frac{1}{N}\int_0^{2\pi} G(z)\,\cos \pi m z\,\left(\frac{dN}{d\psi}\right)_0 d\psi,$$

and similarly for $\langle R_m \rangle$. With the event rate given by (2), we find $\langle Q_0 \rangle = 1$, and $\langle Q_m \rangle = \langle R_m \rangle = 0$ for $m > 0$.

Our choice for G ensures that any non–zero sum with $m > 0$ signals neutrino oscillations, as does a departure of Q_0 from unity. To assess the the statistical significance of a potential signal, we need the rms fluctuations of the sums from their expected values:

$$\Delta Q_m = \sqrt{(\langle Q_m^2 \rangle - \langle Q_m \rangle^2)/N}, \quad \text{where} \quad \langle Q_m^2 \rangle = \frac{1}{N}\int_0^{2\pi} [G(z)\,\cos \pi m z]^2\,\left(\frac{dN}{d\psi}\right)_0 d\psi,$$

and similarly for R_m. We obtain:

$$\begin{aligned}
\Delta Q_0 &= \sqrt{(\pi^2-8)/8N}, \\
\Delta Q_m &= \frac{\pi}{4\sqrt{N}}\sqrt{1+J_1(2m\pi)/m\pi}, \\
\Delta R_m &= \frac{\pi}{4\sqrt{N}}\sqrt{1-J_1(2m\pi)/m\pi}.
\end{aligned}$$

Expected values of Q_m and R_m in the presence of neutrino oscillations are indicated by square brackets:

$$[Q_m] = \frac{1}{N}\int_0^{2\pi} G(z)\,\cos\pi m z\,\frac{dN}{d\psi}\,d\psi.$$

Making use of (2), (5) and (7), we obtain:

$$[Q_m] = \frac{\hat{N}}{2N}\left(1-\frac{r}{2}\sin^2 2\theta\right)\int_{-1}^{+1}\cos m\pi z\,\left(1+\hat{\lambda}\,\cos(p-\epsilon p z)\right)\,dz,$$

and the same for $[R_m]$ with $\cos m\pi z$ replaced by $\sin m\pi z$. We use (6) to express the results in terms of $\hat{\lambda}$ and p:

$$\begin{aligned}
[Q_0] &= \left(1+\hat{\lambda}\,\cos p\,\frac{\sin\epsilon p}{\epsilon p}\right)\,(1+\hat{\lambda}\,\cos p\,J_0(\epsilon p))^{-1}, \\
[Q_m] &= \hat{\lambda}\,(-1)^m\,\cos p\,\frac{\epsilon p\,\sin\epsilon p}{(\epsilon p)^2-(m\pi)^2}\,(1+\hat{\lambda}\,J_0(\epsilon p)\,\cos p)^{-1}, \qquad (8) \\
[R_m] &= \hat{\lambda}\,(-1)^m\,\sin p\,\frac{m\pi\,\sin\epsilon p}{(\epsilon p)^2-(m\pi)^2}\,(1+\hat{\lambda}\,J_0(\epsilon p)\,\cos p)^{-1}.
\end{aligned}$$

We have shown that there is a lower limit $\sim 10^8$ m to detectable oscillation lengths. Thus, only a finite number of sums need be examined. Retrospectively, we find that only those with $m \le 15$ (corresponding to $L \le L_1/15$) are required.

Suppose the data were in hand, the sums Q_m and R_m evaluated. The relevant signal–to–noise ratios,

$$\frac{Q_0-\langle Q_0\rangle}{\Delta Q_0},\quad \frac{Q_m}{\Delta Q_m},\quad \frac{R_m}{\Delta R_m},\qquad \text{for}\quad 1\le m\le 15, \qquad (9)$$

are defined as the 31 random variables S_j where m is the integral part of $(j+1)/2$ and $0 \le j \le 30$. Detectable neutrino oscillations cause one or more of the S_j to depart significantly from zero. The value(s) of j for which this happens offers a rough but immediate determination of the oscillation length, $L \simeq 2L_1/j$.

The S_j correspond to different Fourier components of the data. In the absence of neutrino oscillations, each of them has a normal probability distribution about zero with unit variance. Let $P(s)$ be the probability that $|S_j| > s$ for any fixed j and $Q(s)$ be the probability that $|S_j| > s$ for at least one j. We find $Q(s) = 1 - (1 - P(s))^{31}$ and fix s to be that value for which $Q(s) \leq 0.0027$, corresponding to a three standard deviation effect. It is straightforward to show that $s \simeq 3.9$. The appearance of at least one S_j exceeding 3.9 is taken as a positive signal for vacuum neutrino oscillations.

To proceed further, we first compute the expected values of S_j in the presence of oscillations as a function of δm^2 and $\sin^2\theta$. This is done by replacing Q and R in (9) by their square–bracketed values as given by (8). The results are denoted by $[S_j]$. Oscillations are detectable if and only if δm^2 and $\sin^2\theta$ are such that $[S_j] \geq 3.9$ for at least one value of j.

To be explicit, we assume that $N = 18,000$ events are collected in the course of one or several runs extending between aphelion to perihelion, each with uniform acceptance. In fig. 1 we exhibit the region of δm^2 and $\sin^2 2\theta$ to which Borexino is sensitive. A large domain of possible neutrino squared mass differences, extending between 10^{-11} eV^2 and 10^{-8} eV^2 can be explored.

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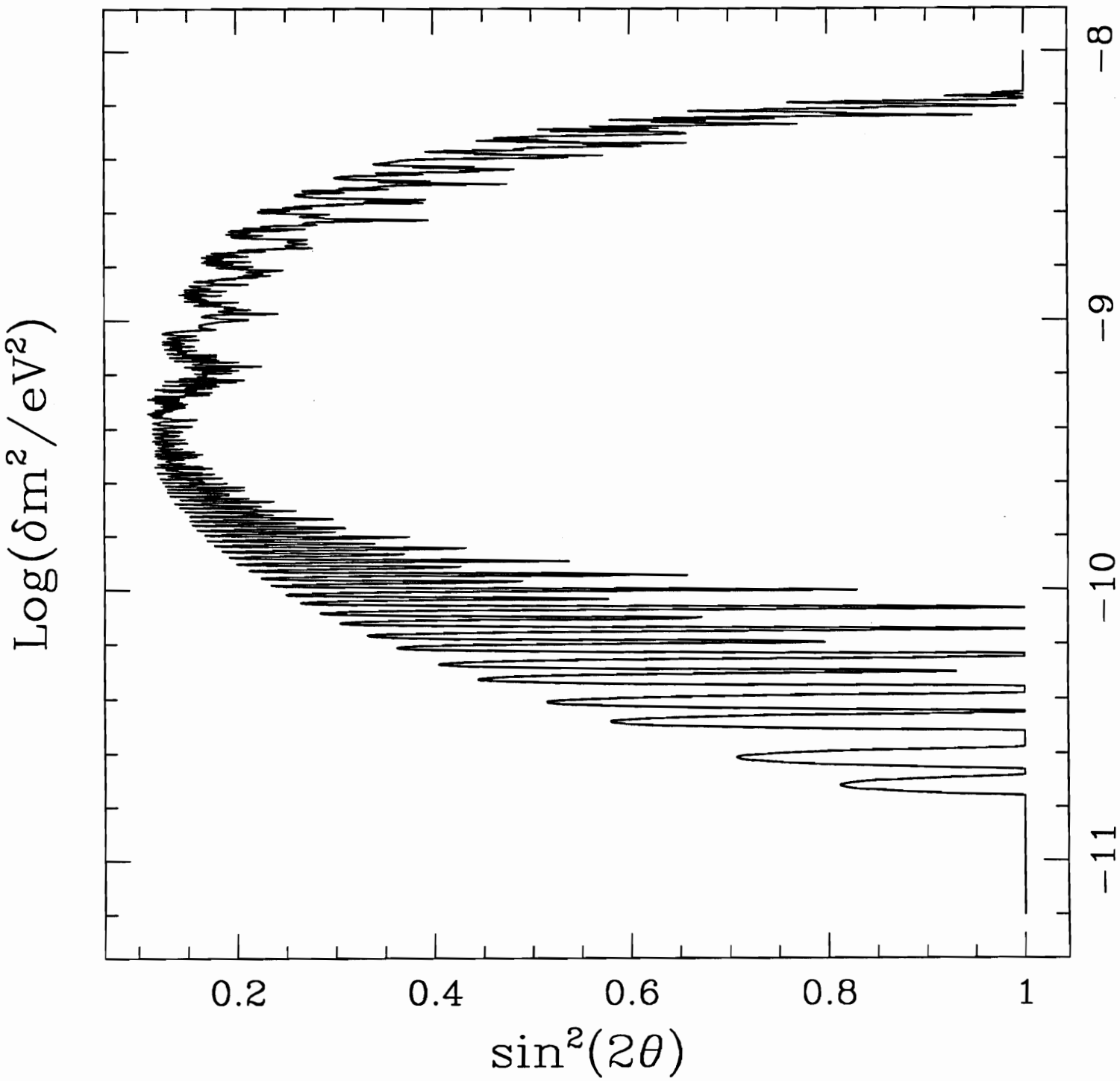

Fig.(1) The region to the right of the wiggly curve is the domain of $(\sin^2 2\theta, \delta m^2)$, with m in eV, in which Borexino may detect vacuum neutrino oscillations at the 3 standard deviation level with 18,000 events.