FERMILAB
JAN 02 1990
LIBRARY

ctober, 1989 HLRZ Jülich 89-77

HLRZ-Jülich-89-77

Test of the Reflection Cluster Algorithm in the O(4) Spin model in 4 Dimensions and its Application in this Model at Finite Physical Temperature†

K. Jansen [1]

Abstract

The reflection cluster algorithm proposed by U. Wolff is tested in the O(4) symmetric ϕ^4 model in 4 dimensions at inifinite bare quartic coupling (O(4) spin model). It is demonstrated that it fights critical slowing down close to the phase transition and in the broken phase. The advantage of using improved estimators to measure physical observables especially in the symmetric phase of the model is shown. The numerical results show that a reduction in the statistical error by factors of 2-10 is possible. As an application we present first results of an investigation of the above model at finite physical temperature.

† Talk presented at the 1989 Symposium on Lattice Field Theory "LATTICE '89 CAPRI", Capri, Italy.

[1] *HLRZ c/o KFA Jülich, P.O.Box 1913, D-5170 Jülich, Fed. Rep. Germany*

RETURN TO
NAME
DATE

		symmetric phase $\kappa = 0.29$ $\xi = 2$	*phase transition* $\kappa = 0.304$ $\xi = \infty$	*broken phase* $\kappa = 0.31$ $\xi = 2$
χ_2	M	8.42(22)	77(-)	214(17)
	RC_{sp}	8.41(16)	76(2)	238(3)
	RC_{cl}	8.42(4)	76(1)	238(2)
τ	M	5.0(5)	30(5)	120.0(10)
	RC_{sp}	3.6(5)	7(2)	2.0(5)
	RC_{cl}	3.5(7)	6(2)	4.0(5)

Table 1: *Table of the susceptibility χ_2, the autocorrelation time τ and their errors for the Metropolis algorithm (M) and the reflection cluster algorithm in the spin representation (RC_{sp}) and in the cluster representation (RC_{cl}) in the symmetric and the broken phase and close to the phase transition. The head of the table indicates at which κ and which scalar correlation length the data have been taken. The symbol "-" means that the error plateau has not yet been reached.*

One clearly recognizes the advantage of the improved estimators in the correlation function at large distances. The signal can be easily followed up to distances 60! Compared to the Metropolis algorithm the error of the correlation function at a distance $t = 5$ is of about a factor 5 superior in favor of the cluster algorithm [3].

4 Applications

As a first application we used the reflection cluster algorithm in a simulation of the O(4) ϕ^4 theory in the symmetric phase. The results of this work have been presented by Münster at this conference and are published in ref. [7]. I want to present here some first results we have obtained in a collaboration with P. Seuferling in a study of the O(4) ϕ^4 theory at finite physical temperature [8].

The motivation is given by scenarios of the early universe. There it is assumed that at some critical temperature a phase transition from a symmetric to a spontaneously broken phase has taken place at a critical temperature T_c, where the exponential grow of the universe has set in (inflationary universe).

The model is defined in equation (1) and the finite temperature is realized by choosing one direction of the lattice much shorter than the others. A $1/N$-expansion suggests that the ratio of spatial to time extent of the lattice should be of about $5 - 6$ to have significant finite size effects for finite size scaling to be applicable. On the other hand we have chosen the time extent of the lattice to be 4 and 5 to avoid lattice effects in time direction.

In particular we want to check the -improved- perturbative relation

$$\frac{T_c}{m_{ren}} = \sqrt{\frac{6}{g_{ren}}} = \sqrt{2}\frac{v}{m_{ren}} \tag{9}$$

where the renormalized mass m_{ren} and the renormalized coupling g_{ren} are to be taken at zero temperature. If this relation holds it is possible to estimate the critical temperature by inserting

lattice	MC	$1/N$
$16^3 5$	0.3058(1)	0.3065
$20^3 5$	0.3067(1)	0.3071
$24^3 5$	0.3070(1)	0.3076

Table 2: *Table of critical values of κ from the fit to the susceptibility and from $1/N$ expansion on various lattices.*

the value of the vacuum expectation value $v \simeq 250$ GeV from the electroweak theory.

The way to obtain the desired information is as follows: Determine for a given extension of the lattice in time direction the value where the phase transition from the symmetric to the broken phase occurs, i.e. determine κ_c. At this value of κ take the values of m_{ren} and g_{ren} either from the work of Lüscher and Weisz [9] or from large scale simulations [5, 10]. In this way the relation (9) can be checked.

In our numerical study we found that the cluster algorithm is about a factor 3-4 superior to the Metropolis algorithm and therefore reduces the computer time substantially. We measured the susceptibility and fitted the data for the suceptibility according to [11]

$$\chi_2 = A_1 (L_S^{-\frac{2}{\nu}} + A_{2,3} |\kappa_c^\infty - \kappa_c^{L_S}|^2)^{-\frac{\gamma}{2}}. \tag{10}$$

Here ν and γ are the critical exponents associated to the correlation length and the susceptibility respectively. The formula (10) contains two limits:

$$\begin{aligned} \chi_2 &= A_1 L_S^{\frac{\gamma}{\nu}} && ; |\kappa - \kappa_c^{L_S}| \ll 1 \\ \chi_2 &= A |\kappa - \kappa_c^{L_S}|^{-\gamma} && ; |\kappa - \kappa_c^{L_S}| \gg 1 \end{aligned}$$

In figure 2 we show the susceptibility together with the fit (10). From that fit we determine $\kappa_c^{L_S}$. In table 2 we compare the critical values of κ obtained in this way together with the ones we have obtained from the $1/N$ expansion in first order.

The table shows a reasonable agreement between Monte Carlo data and the $1/N$ expansion. By using the finite size scaling formula

$$\kappa_c^\infty = \kappa_c^{L_S} + \frac{const}{L_S^{1/\nu}} \tag{11}$$

one finds, imposing the critical exponent ν in $d = 3$, $\nu = 0.68$, a critical value of κ on the infinite lattice $\kappa_c^\infty = 0.3084(4)$. Inserting the values of m_{ren} and g_{ren} from ref. [9] one finds

$$0.606 = \frac{T_c}{m_{ren}} \simeq \sqrt{\frac{6}{g_{ren}}} = 0.612 \tag{12}$$

indicating that perturbation theory may hold.

5 Conclusion

We have demonstrated that the reflection cluster algorithm proposed by U. Wolff works well in the $O(4)\phi^4$ theory in d=4 dimensions. In particular in the symmetric phase of that model the

use of the improved estimators leads to a substantial reduction of the statistical error. Close to the phase transition and in the broken phase the autocorrelation time is found to stay small, which leads to an additional error reduction. In total the statistical error can be reduced by factors of $2-10$ with the cluster algorithm. This makes it possible to perform precision tests of that model. As a first example we presented results of the O(4) ϕ^4 model at finite physical temperature that indicate that perturbation theory is applicable for the ratio $\frac{T_c}{m_{ren}}$ leading to a -preliminary- value of $T_c = \sqrt{2}v \simeq 350GeV$.

The work presented here have been obtained in collaborations with C. Frick and P. Seuferling. I want to thank them for giving me the oppurtunity to present the results at this conference.

References

[1] R.H. Swendsen and J.-S. Wang, Phys. Rev. Lett. 58 (1987) 86.

[2] U. Wolff, Phys. Rev. Lett. 60 (1988) 1461; Phys. Rev. Lett. 62 (1989) 361; DESY-Report 88-176 (1988); DESY-Report 89-21 (1989).

[3] C. Frick, K. Jansen and P. Seuferling, HLRZ Jülich preprint, HLRZ 89-38.

[4] R. Dashen and H. Neuberger, Phys. Rev. Lett. 50 (1983) 1897.

[5] A. Hasenfratz, K. Jansen, C.B. Lang, T. Neuhaus and Y. Yoneyama, Phys. Lett. B199 (1987) 531; A. Hasenfratz, K. Jansen, J. Jersák, C.B. Lang, T. Neuhaus and H. Yoneyama, Nucl. Phys. B317 (1989) 81.

[6] M. Sweeny, Phys. Rev. B27 (1983) 4445.

[7] C. Frick, K. Jansen, J. Jersák, I. Montvay, G. Münster and P. Seuferling, DESY Report 89-90 (1989).

[8] K. Jansen and P. Seuferling, in preparation.

[9] M. Lüscher and P. Weisz, Nucl. Phys. B300 [FS22] (1988) 798; Phys. Lett. B212 (1988) 472; Nucl. Phys. B318 (1989) 705.

[10] J. Kuti, L. Lin and Y. Shen, Phys. Rev. Lett. 61 (1988) 678.

[11] P. Seuferling, DESY report 89-052 (1989).

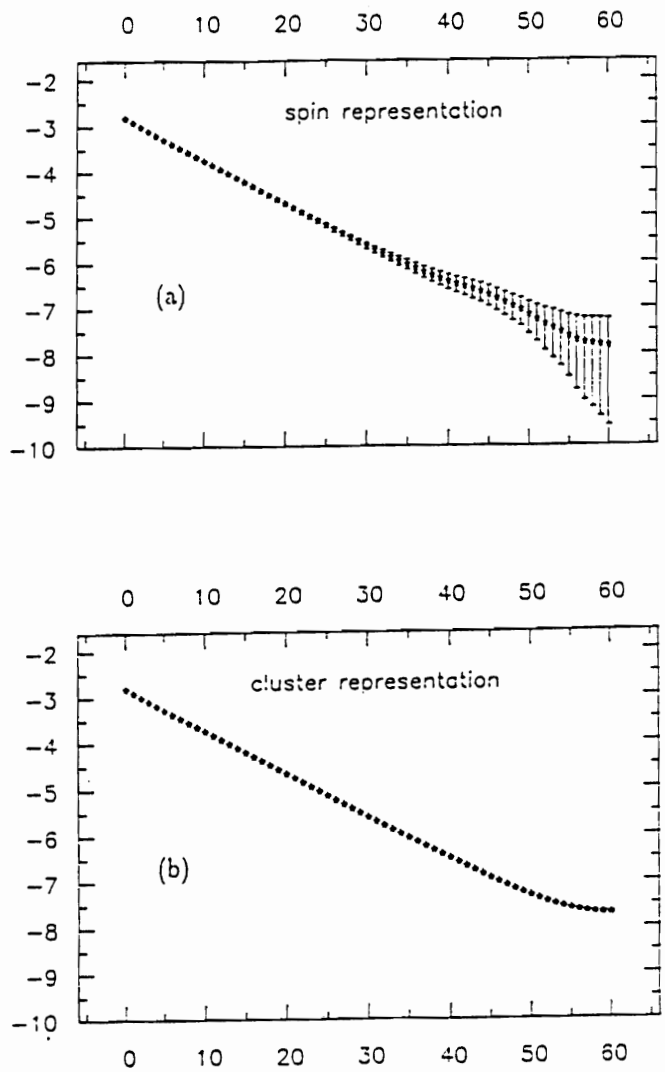

Figure 1: The O(4) invariant correlation function on an $8^3 120$ lattice measured in the spin representation (a) and in the cluster representations (b). Both correlation functions have been obtained with the cluster algorithm.

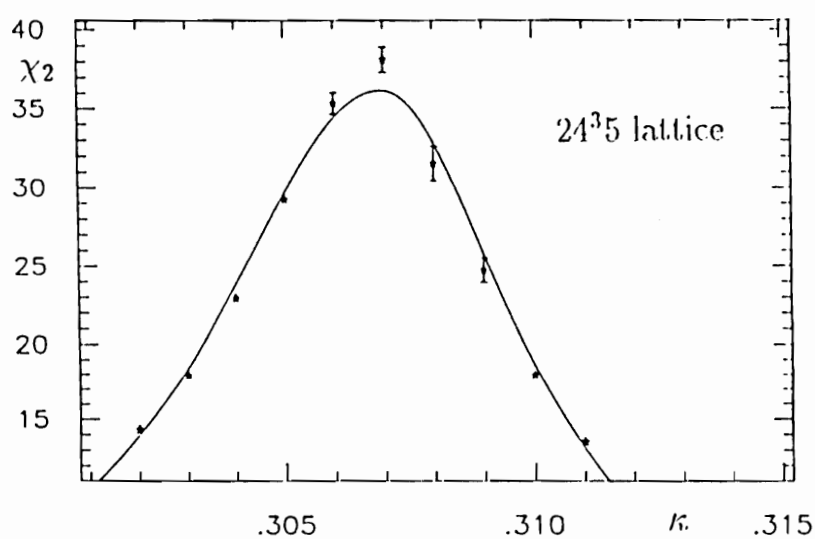

Figure 2: The susceptibility at finite temperature on an $24^3 5$ lattice together with the fit according to (10).