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The Even-Odd Anomalous Tunneling Effect

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P. Kamiński[1,2], S. Drożdż[2], M. Ploszajczak[1], E. Caurier[3]

1. GANIL, BP 5027, F-14021 Caen Cedex, France

2. Institute of Nuclear Physics, Radzikowskiego 152 , PL-31-342 Kraków, Poland

3. Groupe de Physique Nucléaire Theorique, Centre de Recherches Nucléaires et Université Louis Pasteur de Strasbourg, BP 20, F-67037 Strasbourg Cedex, France

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The Even-Odd Anomalous Tunneling Effect

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Abstract

The analysis of the interacting fermion models with the SU(2) symmetry indicates the different behaviour of the splittings of opposite parity lowest energy eigenstates for even and odd number of fermions. The imaginary time-dependent mean field approach reproduces the exact results only for the even number of particles. For the odd particle numbers, an introduction of the universal logarithmic term to the action integral is needed. The even-odd effect appears in a wide class of models with the SU(2) symmetry and is connected with the presence of p^4-terms in the classical large - N limit.

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The notion of tunneling refers to nonperturbative quantum phenomena which are energetically forbidden in the classical limit of the theory. This effect plays a very important role in many branches of physics. In the context of nuclear physics it manifests itself in spontaneous fission and fusion reactions, exotic radioactivity and band mixing in deformed nuclei.

In many-body systems, which we usually deal with, our understanding of tunneling is based on approximations. Because of the success of the mean-field concept in physics of atoms, atomic nuclei, metallic clusters, *etc.* , the time-dependent Hartree-Fock (TDHF) approximation provides the most natural approach. The description of tunneling within TDHF is based on the concept of evolution in the imaginary time. Using this formalism one obtains the barrier penetration formula similar to the semiclassical WKB prescription [1].

The question is: do the available approximations reflect the whole complexity of the problem and possible peculiarities?

To answer such question one needs an exactly soluble model. As such we use simple two-level models with the SU(2) symmetry. Generally, the SU(2) models consist of two $N-$times degenerate shells which can be occupied by N fermions. The two-body interaction can move a pair of particles between the levels in different shells. The Hamiltonian, written in terms of fermion creation and annihilation operators, reads [2] :

$$H = \sum_{k=1}^{2} \varepsilon_k \left(\sum_{n=1}^{N} a_{nk}^{\dagger} a_{nk} \right) - \frac{1}{2} \sum_{k,l=1}^{2} V_{kl} \left(\sum_{n=1}^{N} a_{nk}^{\dagger} a_{nl} \right)^2 , \tag{1}$$

where $V_{kl} = V(1-\delta_{kl})$ and $V \geq 0$. The SU(K) models ($K \geq 2$) , in spite of their simplicity, contain many features of the realistic shell-model Hamiltonian and has been widely used to test various many-body theories. One can define the collective operators $G_{kl} = \sum_{n=1}^{N} a_{nk}^{\dagger} a_{nl}$ and the quasi-spin operators $K_0 = \frac{1}{2}(G_{22} - G_{11})$, $K_+ = G_{21}$, $K_- = G_{12}$ which respect the commutation properties of angular momentum operators. In terms of these operators, the SU(2) Hamiltonian (1) takes the form:

$$H = \varepsilon K_0 - \frac{V}{2} (K_+^2 + K_-^2) , \tag{2}$$

where $\varepsilon \equiv \varepsilon_2 - \varepsilon_1$. Using the finite basis of collective, symmetric states labelled by the number

of particles in the upper unperturbed shell one can find the exact eigenenergies and eigenstates by diagonalization. The diagonalization splits into two independent sectors because the two-particle interaction does not mix states of the odd and even occupation numbers in the upper shell. Consequently, there are two distinct classes of states with different parity.

Using the Hermitean linear and bilinear combinations of the quasi-spin operators, like K_0, K_0^2, K_+K_-, K_-K_+, $K_+^2 + K_-^2$ one can construct another models with the SU(2) symmetry which do not mix the even and odd parity states. So the general Hamiltonian of the SU(2) model can be written in the following form:

$$H = \varepsilon K_0 + BK_0^2 + CK_+K_- + DK_-K_+ + E(K_+^2 + K_-^2) \ , \tag{3}$$

where B, C, D, E are arbitrary parameters. Without loss of generality, this Hamiltonian can be rewritten :

$$H = \varepsilon K_0 - \frac{V_1}{2}(aK_+K_- + bK_-K_+) - \frac{V_2}{2}(K_+^2 + K_-^2) \ , \tag{4}$$

using the relation defining the Casimir operator for the SU(2) group :

$$K_0^2 + \frac{1}{2}(K_+K_- + K_-K_+) = \frac{N}{2}\left(\frac{N}{2} + 1\right) \ . \tag{5}$$

The quantum systems with the SU(2) symmetry have a well-defined classical limit. It can be constructed using the SU(2) coherent state representation of Slater determinants [3, 4] :

$$|\ \psi(z)\rangle \equiv |\ z\rangle = \exp(z^* G_{21})\ |\ 0\rangle \ , \tag{6}$$

where $|\ 0\rangle$ is a state with all particles in the lower shell and z is the complex amplitude of particle-hole excitation. As the total number of particles tends to infinity $N \to \infty$, various coherent states become orthogonal, the quantum interference effects disappear and, consequently, the quantum dynamics becomes equivalent to the classical dynamics in the space of SU(2) coherent states [5] . In particular, the energy per particle of the quantum SU(2) ground state in the $N \to \infty$ limit corresponds *exactly* to the energy of the classical Hartree-Fock (HF) state.

Performing the suitable change of variables :

$$z, z^* \ \to \ \beta = \frac{z}{\sqrt{1 + z^* z}} \ , \ \beta^* = \frac{z^*}{\sqrt{1 + z^* z}} \tag{7}$$

and

$$\beta, \beta^* \quad \longrightarrow \quad q = \frac{1}{\sqrt{2}}(\beta + \beta^*) \ , \ p = \frac{1}{\sqrt{2}}(\beta - \beta^*) \tag{8}$$

one obtains the classical Hamiltonian $\mathcal{H}_{cl} = \langle \psi \mid H \mid \psi \rangle / N\varepsilon$. The equations of motion for q and p take the form of classical Hamilton equations and are equivalent to the time-dependent Hartree-Fock (TDHF) equations for the evolution of coherent states.

For the SU(2) quantum Hamiltonian (4) one obtains:

$$\begin{aligned} \mathcal{H}_{cl} = & -\frac{1}{2}\left(q^2 + p^2 - 1\right) - \frac{V_1}{2\varepsilon}\left[\frac{a}{2}\left(q^2 + p^2\right) + b\left(1 - \frac{1}{2}q^2 - \frac{1}{2}p^2\right)\right] \\ & -\frac{N-1}{2\varepsilon}\left(1 - \frac{1}{2}q^2 - \frac{1}{2}p^2\right)\left[V_1 \frac{a+b}{2}\left(q^2 + p^2\right) + V_2\left(q^2 - p^2\right)\right] \ . \end{aligned} \tag{9}$$

The static part of $\mathcal{H}_{cl}$:

$$V(q) = \mathcal{H}_{cl}(q, p = 0) \tag{10}$$

can be interpreted as an analog of the potential energy. Its form depends on the values of the parameters. For large and positive values of V_1, V_2 , the potential $V(q)$ displays two symmetric minima on both sides of the $q = 0$ axis, separated by a barrier whose height depends on V_1 and V_2. The minima correspond to the HF ground state energy.

For excitation energies such that the total energy is lower than $V(0)$, the whole q-interval splits into the two classically accessible regions and the classically forbidden region under the barrier. In TDHF the evolution under the barrier is possible after the so-called Wick transformation $t \longrightarrow i\tau$, or equivalently, $p \longrightarrow i\kappa$ [6]. However, in general, this is not simply equivalent to the classical motion in an inverted potential as it is customary for the Hamiltonians with the p^2 dependence only [7]. Under this transformation the terms proportional to p^2 in (9) change their sign, but those involving p^4 do not. Formally, one can interpret the motion as taking place in an inverted non-local or momentum-dependent effective potential:

$$-\tilde{V}(q, p) = -\left(V(q) - cp^4\right) = -\left(V(q) - c\kappa^4\right) \equiv -\tilde{V}(q, \kappa) \ . \tag{11}$$

The resulting tunneling probability leads to the energy splitting ΔE between the two originally degenerate states. The magnitude of this splitting is given by [1]:

$$\Delta E_{sc} = \frac{\omega}{\pi} \exp(-NW) \ . \tag{12}$$

where ω is the frequency of the motion in either of the wells and $W = \int \kappa dq$ is the imaginary-time analog of the classical action integral. The splitting ΔE between the lowest states of different parity can be calculated both quasiclassically, using formula (12) , and exactly, by diagonalization of the quantum Hamiltonian.

In Fig.1 a,b we present the results for the Lipkin model (2): $V_1 = 0$, $V_2 = V$, for even ($N = 30$) and odd ($N = 31$) particle numbers. The energy splitting is plotted as a function of a dimensionless coupling constant $\chi = V(N-1)/\varepsilon$. The energy scale of the problem, *i.e.* the barrier height and the HF energy is proportional to χ for large χ. Hence, if one wants to compare the properties of the solutions for different energy scales, then the trivial χ-dependence of the results should be cancelled out. Such a scale-independent splitting can be defined as: $\delta \equiv \Delta E/\chi$.

One can observe the striking difference between the behaviour of the exact solutions expressed in terms of δ (stars in Fig. 1). For even N it tends asymptotically to a finite limiting value for large coupling constant χ, whereas for odd N it tends to zero. The imaginary-time-dependent Hartree-Fock (ITDHF) result (the full line) does not distinguish even and odd N. It follows closely the quantum solution only for the even particle numbers. In the quasiclassical formula (12) N enters as a factor multiplying the action "under barrier". The action integral in the large χ limit tends to a constant value $W_{max} = \int_{q_{+max}}^{q_{-max}} \kappa_{max} dq$, where $q_{\pm max} = \pm 1$, $\kappa_{max}(q) = \lim_{\chi \to \infty} \kappa(q)$ and the preexponential factor $\omega = \sqrt{2(\chi^2 - 1)}$ grows as χ, so δ tends to a constant.

Appearance of such different behaviour for the even and odd particle numbers can be partially related to a discrete symmetry of the quantum Lipkin model. The inversion of the occupation numbers in the two shells , i.e. the exchange of particles and holes in both shells, leads to the change of the parity of the wave-function for odd N, whereas for even N it does not. The classical system is symmetric with respect to the particle - hole inversion. Consistently, ITDHF describes better the solutions for even N.

The presence of the quantum discrete symmetry has no correspondence in the classical

limit. The quantum splitting δ_Q tends to zero for odd N, when the energy scale of the problem ($\sim \chi$) tends to infinity. Assuming that this convergence is strong, i.e. non-logarithmic, one obtains immediately:

$$\delta_Q \simeq \delta_{sc} * \mathcal{O}\left(\frac{1}{\chi}\right) , \tag{13}$$

the modification of the classical scale-independent splitting $\delta_{sc} \equiv \Delta E_{sc}/\chi$. This modification leads to the introduction of logarithmic correction to the classical action integral.

The observed even-odd effect appears not only in the Lipkin model. This effect is present for many other SU(2) systems. The calculations performed for different models with positive values of V_1 and V_2 give qualitatively the same result. The ITDHF curve follows again the exact results for even N, whereas for odd N the exact splitting has different asymptotics identically as in the Lipkin model.

As it was noticed before, the ITDHF description of the energy splitting for odd N requires introduction of some additional terms to the classical action. In the exponent (12), one needs for odd particle numbers a N - independent extra term $-ln\chi$, compensating the growth of the preexponential factor ω and leading to the proper asymptotic value of the splitting of the ground states of different parities. This term is universal for all odd N, i.e. it does not have the character of a quantum correction, and moreover, it is generic for all $SU(2)$ models displaying the even-odd effect. Its form and the role it plays suggests connection with quantum fluctuations around the classical path because such processes are known [7] to determine the preexponential factor. Such correction is not sufficient for the whole region of χ. For small χ the calculated splitting is too large because the semiclassical prescription used (12) is not valid.

There are only three models with the SU(2) symmetry which do not display the different behaviour for the odd and even N:

$$H' = \varepsilon K_0 - \frac{V}{2}(K_+ + K_-)^2 , \tag{14}$$

$$H'_{1,2} = \varepsilon K_0 - \frac{V}{2}\left(K_+^2 + K_-^2 + 2K_\pm K_\mp\right) . \tag{15}$$

For these particular models the classical Hamiltonians do not contain p^4-terms. Instead, some

p^2-terms are multiplied by the nontrivial, q-dependent "mass parameter". For these cases the ITDHF calculations match the exact results very well for both even and odd N. For the first of the models (14) the splitting tends to zero (Fig. 2 a,b), whereas for the second and third model (15) it tends to some small constant value (Fig. 2 c,d).

The models, for which one can observe the even-odd effect have always p^4-terms in the classical limit. The second common feature of these models is the characteristic behaviour of the admixture of the higher static HF (SHF) state $\mid q, 2\rangle$ (constrained HF equations solved at q with $p = 0$) in the ITDHF single-particle state $\mid z, 1\rangle$ under the barrier. These states and their overlaps for the Lipkin model are discussed more in details in ref. [8]. Both of the two SHF single-particle states are equally present in the ITDHF single particle state at $q = 0$ in the limit of large coupling constants (Fig. 3 a), *i.e.* the overlap of the single-particle SHF excited state and ITDHF state tends to $\frac{1}{2}$. Such form of the overlaps indicates the presence of the single particle level-crossing at $q = 0$. The overlap has completely different character for the modified SU(2) models which do not possess p^4-term in the classical limit. It is small during the whole evolution under the barrier (Fig. 3 b,c).

To sum up, the analysis of the different models with the SU(2) symmetry indicates that the even-odd effect is observed for all the models possessing in the classical limit the p^4-terms. Then, the evolution under the barrier is equivalent to the evolution in the inverted effective potential $-\tilde{V}(q,p) \equiv -\tilde{V}(q,\kappa)$ which is momentum-dependent or equivalently non-local.

Although the nuclear interactions at low energies can be fitted using the static potentials depending only on the coordinates, there are also many indications, mainly from the optical model analysis of the scattering data, of the presence of the velocity dependent terms. Potentials of this type were used in the analysis of the nucleon-nucleon scattering [9]. Independently, one can argue that the self-saturating system of independent fermions *must* be described by the velocity (momentum) dependent potential [10]. The effective two-particle interaction between nucleons [11] in unoccupied shells, which contain contributions from the virtual excitations of rest of nucleons, have much higher symmetry than the forces between

nucleons in free space. This permits the existence of the potentials containing different terms with the higher momentum powers. So far they were not considered in the investigations of the structure of heavy nuclei, in particular in the connection to the tunneling phenomenon. The anomalies which may accompany the presence of such terms in the nuclear potential can manifest themselves in the nonapplicability of the classical limit of the shell model (SHF) in the description of tunneling for certain configurations of valence shells and for some nucleon numbers.

The requirement of introduction of an additional universal term to the classical action for odd N in the broad class of the SU(2) models for which the level-crossing occurs, may suggest a close relation of the above problem to the anomalies. Construction of the effective theory leads to the appearance of some additional terms in the lagrangian [12], which usually have a topological origin and are closely related to Berry's phase [13]. The anomalies are associated with the discrete symmetries, nonlocalities and degeneracies, but it is not obvious that all those related problems can be explained in terms of anomalies and Berry's phase, especially for the imaginary-time evolution. A deeper understanding of the even-odd effect in tunneling for momentum dependent potentials and its connection to the anomalies remains a challenging problem for further investigations.

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Figure captions

Fig. 1

The rescaled energy splitting $\delta \equiv \Delta E/\chi$ between the two lowest states of different parity in the Lipkin SU(2) model for different number of particles : (a) N=30; (b) N=31. The stars and solid lines denote exact solutions and ITDHF results respectively.

Fig. 2

The rescaled energy splitting $\delta \equiv \Delta E/\chi$ between the two lowest states of different parity in the modified SU(2) models for different number of particles: (a) N=10 (eq. (14), H'); (b) N=11, (eq. (14), H'); (c) N=10 (eq. (15), H_2'); (d) N=11, (eq. (15), H_2'). The stars and solid lines denote exact solutions and ITDHF results respectively.

Fig. 3

The admixture of the higher lying SHF state to the ITDHF state under the barrier is plotted as a function of q for different values of χ for: (a) the Lipkin SU(2) model, (b) the modified SU(2) model (eq. (14), H'), (c) the modified SU(2) model (eq. (15), H_2'). Solid lines correspond to $\chi = 3$, dashed lines to $\chi = 10$ and dotted to $\chi = 10000$.

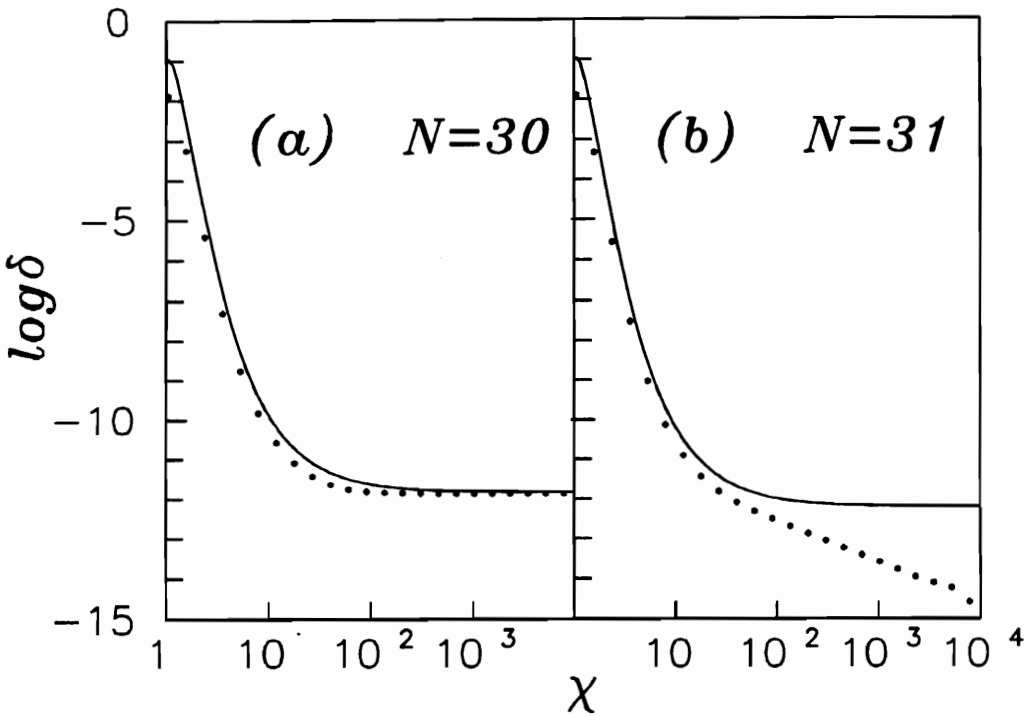

Fig. 1

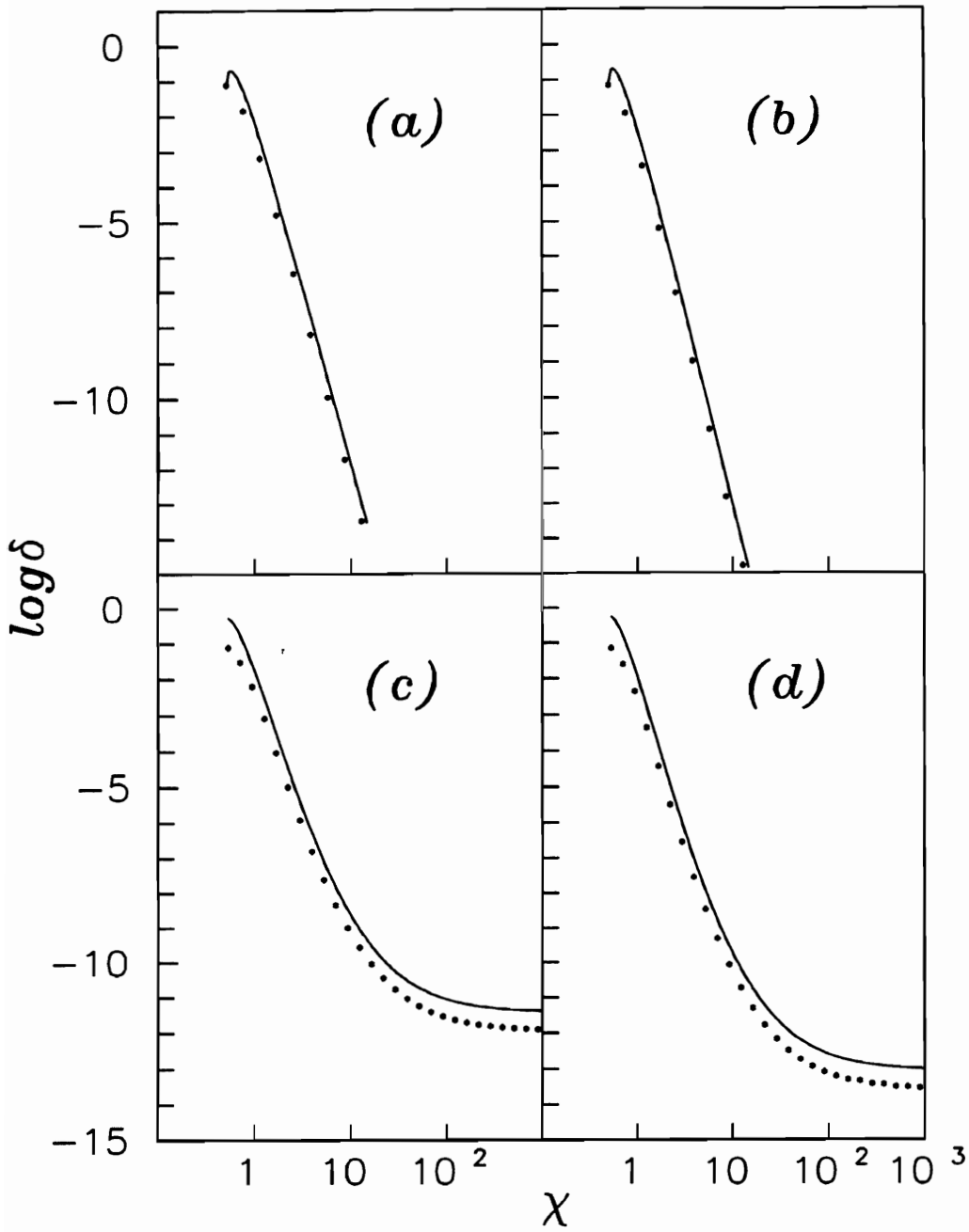

Fig. 2

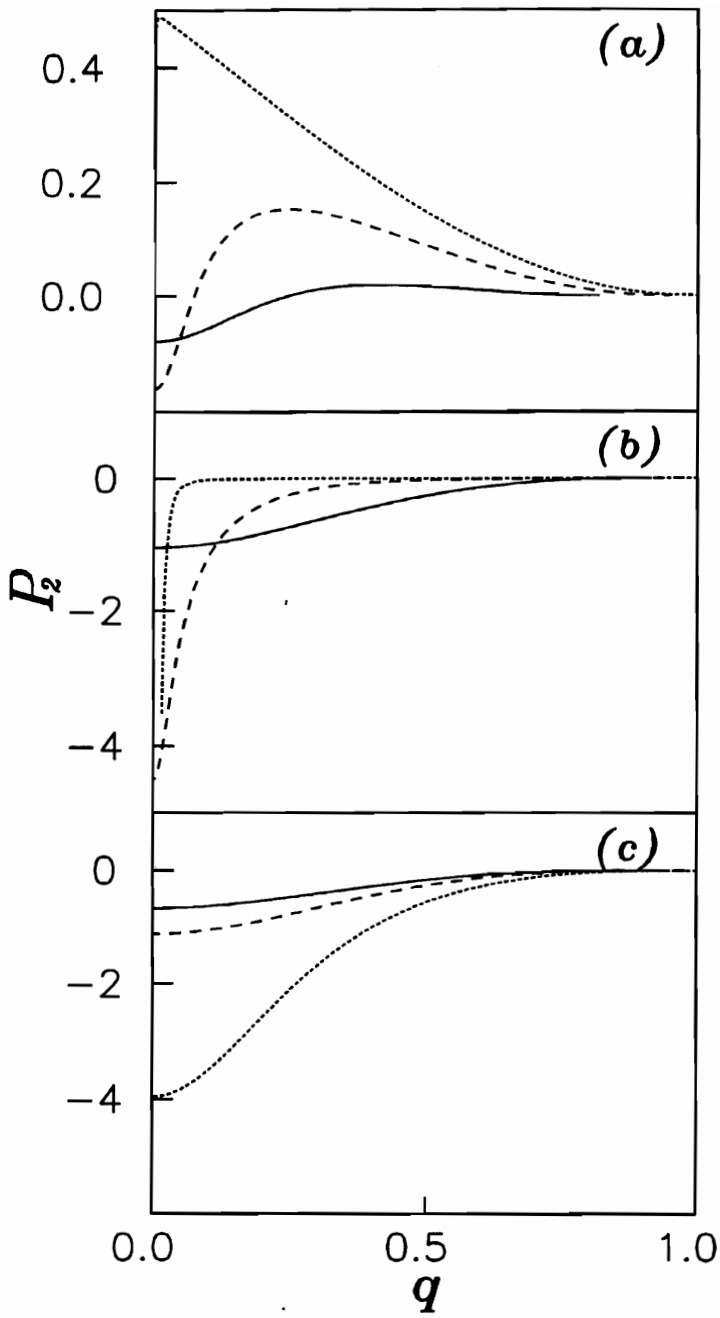

Fig. 3