GANIL-P-92-02
Fermilab Library
0 1160 0006801 9

GANIL
GRAND ACCELERATEUR NATIONAL D'IONS LOURDS - CAEN
LABORATOIRE COMMUN IN2P3 (CNRS) - IRF (CEA)

10

Fluctuations, dynamical instabilities and clusterization processes*

G.F. Burgio[1,2,†], Ph. Chomaz[1,2,‡] and J. Randrup[1,3]

[1]) Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720, USA

[2]) GANIL, B.P. 5027, F-14021 Caen Cedex, France

[3]) GSI, Postfach 11 05 52, D-6100 Darmstadt 11, Germany

FERMILAB
MAR 11 1992
LIBRARY

INTERNATIONAL WINTER MEETING ON NUCLEAR PHYSICS. 30 - BORMIO, 1992

ROUTE TO
NAME
LOCATION
RETURN TO FERMI LIBRARY
GANIL P 92 02

Fluctuations, dynamical instabilities and clusterization processes*

G.F. Burgio[1,2,†], Ph. Chomaz[1,2,‡] and J. Randrup[1,3]

[1]) Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720, USA

[2]) GANIL, B.P. 5027, F-14021 Caen Cedex, France

[3]) GSI, Postfach 11 05 52, D-6100 Darmstadt 11, Germany

Abstract

We report on recent progress with regard to the numerical simulation of fluctuations in nuclear dynamics. Indeed, we study cluster formation in unstable nuclear matter within the framework of a Boltzmann-Langevin equation developed to describe large amplitude fluctuations. Through the Fourier analysis of the fluctuating nuclear density in coordinate space, we relate the onset of the clusterization to the dispersion relation of harmonic density oscillations. This detailed study on the simple two-dimensional case demonstrates the validity of the general approach. We also show how the inclusion of fluctuations implies a description in terms of ensemble of trajectories and we discuss why the presence of a a stochastic term may cure the intrinsic unpredictability of deterministic theories (such as mean-field approximation) in presence of instabilities and/or chaos.

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

† Supported in part by INFN Sezione di Catania and LNS, I-95129 Catania, Italy.

‡ On leave from Division de Physique Theorique, Institut de Physique Nucleaire, F-91406 Orsay Cedex, France.

1 Introduction

The mean-field approximation provides a useful starting point for describing many-body systems, especially when the properties of interest can be extracted from the one-body density matrix alone. In particular, in nuclear physics mean-field theories have become valuable tools for the treatment of static (and stable) situations. The utility of the mean-field approximation is less obvious for the time evolution of complex systems displaying dynamical bifurcations, which occur when a system passes through a region of instability. The complete treatment of such problems requires the consideration of the correlated evolution of a large number of degrees of freedom and is impractical. However, in many situations there exists a tractable approximate approach, in which the full (and deterministic) many-body evolution is replaced by a stochastic equation of motion for the one-body density alone, thus keeping the instantaneous information within the boundaries of the mean-field approximation, while simulating the effect of the many-body correlations by a stochastic term in the mean-field equation of motion.

This alternative avenue retains only a small part of the dynamical information [1] whose reduction naturally invites the concepts of from non-equilibrium statistical mechanics, such as irreversibility and entropy. In general, the equation of evolution for the retained variables will be non-linear and therefore the dynamics may exhibit chaotic features such as instabilities and bifurcations.

One way to restore the predictive power of the theory is to realize that the lack of full information about the state of the system, both initially and in the course of time, will act as a stochastic term in the effective equation of motion for the retained variables, because they characterize an entire ensemble of microscopically different states of the system. In effect, this ensemble acts as the heat bath in Brownian motion and the equation of motion will be of the Langevin type. In this case one is forced to consider not only one single trajectory, but an ensemble of trajectories which differ because samll fluctuations in the evolution may be propagated very differently by the deterministic but non-linear part of the effective equation of motion.

In this paper we present a first illustration of this approach for a catastrophic evolution within the framework of nuclear physics, namely the clusterization of matter at subsaturation density. The employed Boltzmann-Langevin equation has been recently proposed and investigated in order to extend the mean-field description to such phenomena as heavy-ion induced nuclear multifragmentation processes. [2, 3, 4, 5, 6, 7]

Our starting point is the socalled Boltzmann-Uehling-Uhlenbeck equation, which provides a self-consistent semi-classical evolution of the one-nucleon phase-space density $f(r,p,t)$ taking into account the average effect of Pauli-blocked two-body collisions between individual nucleons in the system. (This approach was first developped by Nordheim for the description of electrons in a solid.[8]) The BUU theory has been extensively applied to nuclear collisions and has proven fairly successful for describing one-body observables.[7] However, the model can-

not provide a description for large fluctuation phenomena, such as multifragmentation, because it determines only a single ("average") dynamical history. By contrast, a Langevin-type evolution is very well suited for such phenomena.

In ref. [2] a formalism was developped for treating stochastic one-body dynamics within the framework of transport theory, starting from the Fokker-Planck equation for the distribution of one-body densities in phase-space, $\phi[f]$. In subsequent works [4], we have developed a specific implementation of the Langevin equation associated with the *BUU* approach; it has a well-defined prescription for calculating the fluctuating part of the collision integral and applies to arbitrary fluctuations in the one-body dynamics. In those studies, we neglected the effective one-body field in order to facilitate the analysis. However, its presence is of course essential. In fact, at low densities nuclear matter is unstable against oscillations, so the attractive part of the effective field will amplify any stochastic density enhancements and thus drive the system towards fragmentation. We have now included the effective field and show our first results regarding cluster formation in nuclear dynamics. [5]

2 Summary of the model

In refs. [4] we developed a new method for incorporating fluctuations within the *BUU* description. This extended Boltzmann-Langevin equation governs the time evolution of the one-particle phase-space density due to the effects of the effective one-body field (Vlasov dynamics) *and* the two-body collisions, the latter processes being described in terms of the collision integral which is stochastical in nature. Therefore, the evolution of the one-particle density can be considered as a "generalized Langevin process", analogous to to the motion of a Brownian particle in a heat bath (except that the Brownian coordinate is not merely the momentum of a particle but the whole single-particle density itself $f(r,p)$).

The effects of the two-body collisions are twofold: (i) to produce dissipation by degrading macroscopic motion into statistical motion of the individual particles, and (ii) to induce correlated fluctuations that can subsequently be propagated by the evolution in the non-linear effective field. Therefore it is reasonable to split the collision term $I[f]$ into two parts, one which takes into account the average effect of the two-body collisions, $\bar{I}[f]$, and the other one which includes the fluctuating part, $\delta I[f]$. The stochastic *BUU* equation then reads

$$\frac{\partial f}{\partial t} + \{H[f], f\} = I[f] = \bar{I}[f] + \delta I[f] \ , \tag{1}$$

where $H[f]$ is the self-consistent effective one-body Hamiltonian. The collision term acts as a random "kick" on the one-body distribution function and its fluctuating part produces sudden branchings of the dynamical histories resulting in a bundle of trajectories. By contrast, the effective mean field leads to a very smooth evolution. It is therefore convenient to consider the two evolutions separately.

In order to solve the above equation of motion, we represent $f(r,p)$ on a lattice of grid points in phase space. The size of each lattice cell is given by

$\Delta s = \Delta r \Delta p / h^D$, and the value f_k at the lattice point $s_k = (r_k, p_k)$ represents the average value of f over the cell k. Since each physical particle represents a phase-space volume h^D, we first divide the phase space into cells K of the order of the unity.

The Vlasov part of the propagation is made by means of a standard matrix-inversion technique. The collision integral $I[f]$ expresses all possible two-body collisions consistent with locality and energy-momentum conservation. During a given small time interval Δt, the expected number of such elementary transitions $\bar{\nu}$ leading from within the phase-space subcells 1 and 2 into the subcells $1'$ and $2'$ is given by

$$\bar{\nu}_{1,2;1'2'} = \Delta t \Delta s^4 f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \delta(r_1 - r_2) \delta(r_1 - r_{1'}) \delta(r_2 - r_{2'}) \omega(p_1, p_2; p_{1'}, p_{2'}) \ , \quad (2)$$

where $\omega(p_1, p_2; p_{1'}, p_{2'})$ is the elementary transition rate and the Pauli blocking factors $\bar{f} \equiv 1 - f$ express the availability of the final one-particle states.

The actual number of elementary transitions ν is a stochastic variable, having a Poisson distribution characterized by the above mean value $\bar{\nu}$. The average part of the collision integral, $\bar{I}[f]$, expresses the average changes corresponding to putting $\nu = \bar{\nu}$, while the fluctuating part accounts for the stochastic remainder, $\delta\nu = \nu - \bar{\nu}$. This latter quantity has the variance $\sigma_\nu^2 = \bar{\nu}$ and the fluctuating part $\delta I[f]$ is thus fully determined by the mean transition rates $\bar{\nu}$. In our particular numerical implementation, we simulate the fluctuations $\delta\nu$ of each physically distinct transition by a normal (*i.e.* Gaussian) noise term with the variance $\sigma_\nu^2 = \bar{\nu}$, which has been found to be a numerically accurate procedure. It should also be noted that this manner of introducing the fluctuations at the elementary level preserves the correlations induced by the fact that a given collision involves four phase-space locations.

However, in order to achieve sufficient accuracy in the propagation of the collisionless (Vlasov) part of the evolution, a finer lattice is required and we therefore further divide each physical cell into sufficiently small subcells k.

In order to ensure that the fluctuations associated with the transitions between subcells are properly correlated over the scale of a physical cell, we proceed as follows. From the quantities $\bar{\nu}_{ijkl}$ given above in eq. (2) we may compute the expected number of transitions between the physical cells I, J, K, L,

$$\bar{N}_{IJKL} = \sum_{ijkl} \bar{\nu}_{ijkl} \ , \quad (3)$$

where, for example, i refers to the subcells of I. The associated fluctuation δN has the variance $\bar{N}$ and so can be simulated by a corresponding normal noise. The quantity δN is then shared between the subprocesses in proportion to the mean changes,

$$\delta\nu_{ijkl} = (\bar{\nu}_{ijkl} / \delta N_{IJKL}) \bar{N}_{IJKL} \ . \quad (4)$$

This method introduces the appropriate correlation of the noise over volumes h^D, thus ensuring that the relation between mean and variance is preserved on the physical scale, and it has the numerical advantage that both the collision term

and the mean field propagation are computed with the same small cells, thus avoiding degradation in accuracy.

3 Application to a Model System

We have studied a gas of Fermions situated on a two-dimensional torus. For the effective one-body field we employ a simplified Skyrme interaction,

$$U(x) = A\frac{\bar{\rho}(x)}{\rho_0} + B(\frac{\bar{\rho}(x)}{\rho_0})^2, \tag{5}$$

with $A = -100.3$ MeV and $B = 48$ MeV. Moreover, ρ_0 is the saturation density and $\bar{\rho}(x)$ is the average of the density $\rho(x,y)$ with respect to the transverse direction y. Since the effective field U then depends on x only, the same holds for those modes that can be excited, and this simplifies the analysis considerably. In determining the parameters of U, we have sought to mimic standard three-dimensional matter. Thus we have required a Fermi momentum of $P_F = 260$ MeV/c, and a binding energy of 16 MeV per nucleon, and that a density doubling lead to approximately zero binding (corresponding to a compressibility modulus of $K \approx 300$ MeV for a calculation in three dimensions). In the present two-dimensional model system, the saturation density is $\rho_0 = 0.55$ fm^{-2}.

We have then solved the stochastic BUU equation on a lattice of 21 cells in the x direction and 25×25 cells in momentum space. The physical cells had a spatial resolution of 1 fm so that the torus extends 21 fm in the x direction. The value $\Delta x = 1$ fm was chosen to correspond approximately to the spatial extension of a nucleon. Since there is no dependence of y, the length in the y direction was chosen to be large ($L_y = 5000$ fm). The momentum resolution was 65 MeV/c, so that the p-space was extending up to $p_x = p_y = \pm 682.5$ MeV/c. The subcells were identical to the macrocells, except for the x-direction for which the accuracy of the gradients requires a minimum of $\delta x = \frac{1}{3}$ fm. The Vlasov propagation was made with time increments of $\delta t = 0.5$ fm/c, while the collisions were considered at time intervals of $\Delta t = 5$ fm/c. The interaction range in the collision integral was taken as 1.2 fm.

We have initialized the system as a uniform gas at half the saturation density, with Fermi-Dirac occupancies corresponding to the small temperature 3 MeV; this is in the mechanically unstable regime of the phase diagram. We analyze an ensemble of 100 trajectories; for each trajectory, the calculation was stopped at 90 fm/c.

3.1 Fragment formation

First, let us focus our attention on the evolution of the spatial density of the system. In Fig. 1 we show the time evolution of the density vs. the position for one trajectory of the ensemble. Initially the system has a uniform density, but soon the fluctuations are breaking this initial symmetry. Subsequently, the

fluctuations are rapidly amplified by the action of the effective one-body field, thus leading towards fragment formation. We note that the density distribution of each particular system keeps changing with time, with the clusters exhibiting fusion and fission. In this manner, each particular trajectory explores the various accessible configurations, while the distribution of trajectories quickly approaches the appropriate statistical limit.

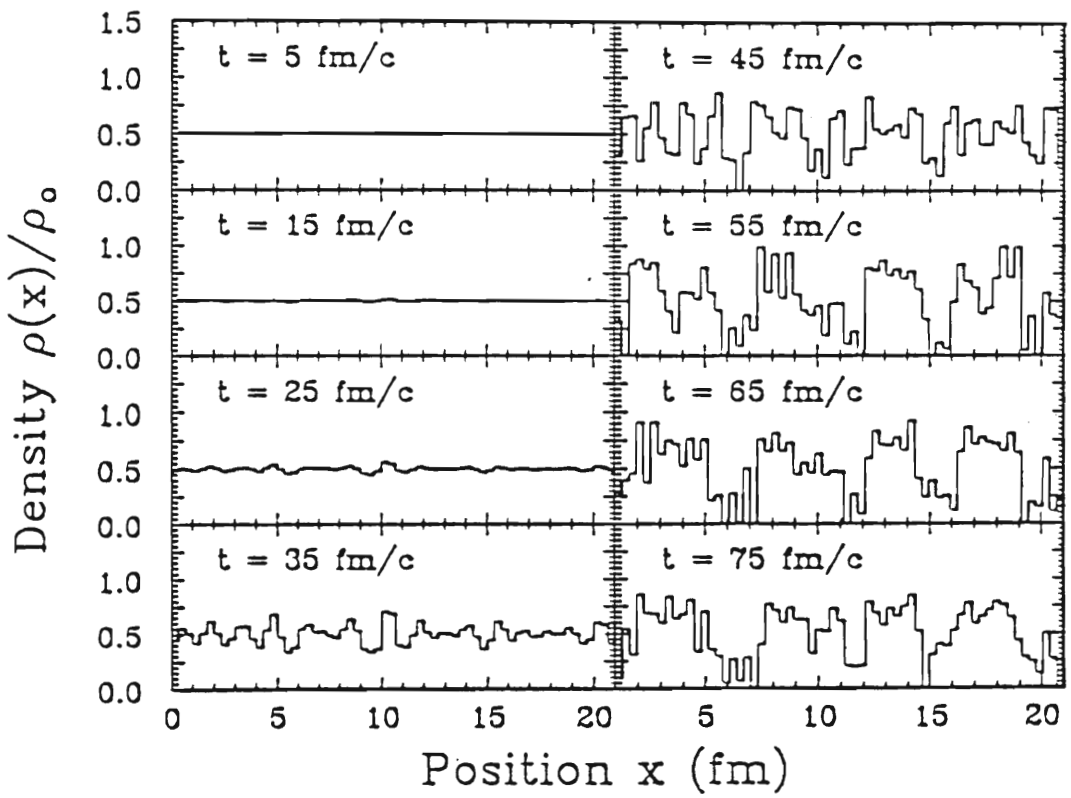

Fig. 1. **Density profiles:** The density profile $\bar{\rho}(x)$ associated with one particular trajectory picked from among the ensemble of different evolutions, shown as a function of the position x at eight different times t.

3.2 Instabilities

To get a deeper insight into the mechanism of fragmentation and the role of the instabilities, it is important to study the onset of the phenomenon. When the fluctuations are still small their evolution is described by linear response theory, and it is instructive to study the modes propagated by the Vlasov equation. If the system is stable, the dispersion relation yields real values for the energies of the normal modes, which can be represented by plane waves propagating in opposite directions. If the system is unstable, as the present one, some eigenenergies are complex and the imaginary part is inversely related to the time τ characterizing the development of the fluctuations.

We have therefore derived and solved the dispersion relation associated with the Vlasov equation linearized around a uniform distribution at zero temperature. The energy is imaginary when $\epsilon_0/\rho_0 + \partial U/\partial\rho < 0$, where ϵ_0 is the Fermi energy at the saturation density ρ_0. This condition is exactly the instability condition of the Vlasov solution, $\partial^2 E/\partial\rho^2 < 0$, where E is the energy density. In this regime the imaginary energy reads

$$iE = \frac{1}{\tau} = \pm k \left(\frac{\epsilon_0}{\rho_0} + \frac{\partial U}{\partial\rho}\right) \sqrt{\frac{\rho}{m(-\epsilon_0/2\rho_0 - \partial U/\partial\rho)}}\,, \tag{6}$$

where k is the wave number of the unstable mode of the system. In the above relations the partial derivative of the mean field U is calculated at the actual density ρ. The strict proportionality of the energy E to the wave number k is due to the absense of a finite range term in the effective potential U. For simplicity, in the present exposition, we effectively mimic such a term by the finite size of the spatial grid (which is $\frac{1}{3}$ fm).

The above result does not correspond to the actual instabilities in the *BUU* dynamics, because it ignores both the two-body collisions and the finite temperature. Therefore, we have computed the actual dispersion relation by considering the *BUU* evolution (without fluctuations) of a uniform density at 3 MeV of temperature perturbed with a small (1%) harmonic variation in the x direction. For each initial perturbation characterized by wave number k, we have performed a *BUU* calculation and analyzed how the perturbation evolves in time, through a Fourier transform in the x direction.

We have found that the linear response expression remains valid up to a time of about 20 fm/c. After this time frequency doubling appears in the Fourier spectrum due to the non-linearity of the *BUU* equation. Around 30 fm/c the dynamics begins to appear chaotic and looses the memory of the initial perturbation. This happens for all initial wave numbers, except for those that are commensurable with the total number of cells. For these the initial state retains some periodicity, which is preserved by the *BUU* equation; the fact that these initial symmetries indeed remain throughout the evolution constitutes an important check on the numerical stability of the code.

Within the linear regime, for each k, we have analyzed how the peak of the Fourier transform evolves in time and we have observed an exponential evolution with a characteristic time τ (in fact we observe a mixing between an exponentially growing and an exponentially decaying solutions with the same time τ). In Fig. 2 we show the evolution of $1/\tau$ vs. the wave number k (solid curve), while the dashed line is the prediction of the dispersion relation (6). The curve is symmetric around $k = 0.75$ fm^{-1}, because of our discretization of the gradients where a double step is used. Therefore, within the truncation due to the lattice, we observe that the actual dispersion relation is very close to the expression (6) derived for the Vlasov equation at zero temperature.

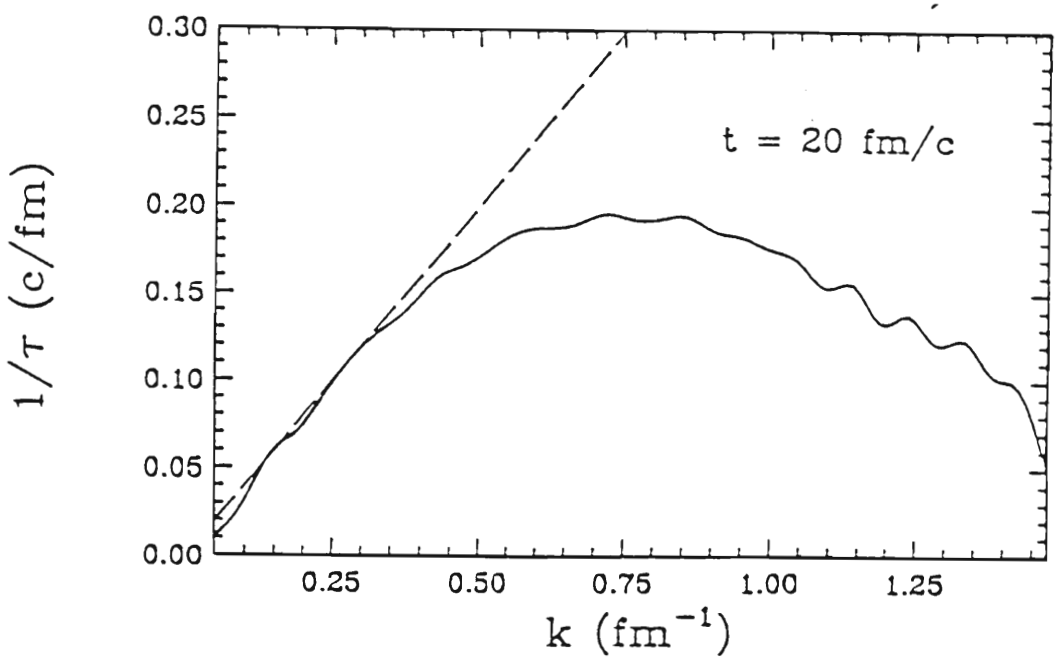

Fig. 2. **Dispersion relation:** The inverse of the characteristic time τ_k associated with an unstable harmonic mode of uniform, two-dimensional nuclear matter at half the saturation density, as a function of the wave number k of the mode. The solid curve represents the actual dispersion relation as obtained in our numerical implementation of the *BUU* model, while the dashed curve is the analytic expression (6) pertaining to the collisionless (Vlasov) limit.

3.3 The onset of multifragmentation

This normal mode analysis will allow us to study the interplay between instabilities and fluctuations. Indeed, considering the Fourier transform of the fluctuations of the density, $F(k,t) = \int dx e^{i2\pi kx} \delta\rho(x,t)$, we find

$$|F(k,t)|^2 = \int\int dx dx' \; e^{i2\pi k(x-x')} \; \delta\rho(x,t) \; \delta\rho(x',t) \; , \tag{7}$$

Taking the expectation value of this quantity of the ensemble, then yields the fluctuation of modes with wave number k,

$$\sigma_k(t) = \prec |F(k,t)|^2 \succ = \int\int dx dx' \; e^{i2\pi k(x-x')} \; \sigma(x,x',t) \; . \tag{8}$$

Within the linear response regime and for a given instability mode k, the random two-body collisions act as a continuous source of fluctuation C_k. Therefore the variance σ_k evolves according to the equation

$$\dot{\sigma}_k = C_k + \frac{2}{\tau_k}\sigma_k \; , \tag{9}$$

where the second term is due to the unstable character of the normal mode k, which has a growth time τ_k. The solutions of this equation are given by

$$\sigma_k(t) = \frac{C_k \; \tau_k}{2} \; (e^{2t/\tau_k} - 1) + \sigma_k(0) e^{2t/\tau_k} \; , \tag{10}$$

where $\sigma_k(0)$ is the initial fluctuation, which is zero in the present case. The characteristic amplitude of the fluctuations is determined by C_k, and their time scale by τ_k. We note that for $t < \tau_k/2$, eq. (10) reduces to $\sigma_k(t) \simeq C_k t$, *i.e.* the fluctuations are dominating the dynamics, while for $t > \tau_k/2$ both fluctuations and instabilities are present, leading asymptotically to a pure exponential increase, $\sigma_k(t) = \frac{1}{2} C_k \tau_k \exp(2t/\tau_k)$. Eq. (9) governs the evolution of the modulus squared of the Fourier transform of the density $\bar{\rho}(x)$, and the form of its solution, Eq. (10), provides a good understanding of the growth of the fluctuations. This quantity is displayed in Fig. 3. At the early stage of the evolution ($t < \tau_k/2$) one mainly observes the Fourier component of the noise and indeed we observe that the spectrum at $t = 5$ fm/c is characteristic of a system where the fluctuations must be correlated over a domain of at least 1 fm in size. In particular, the spectrum does not extend beyond $k = 1$ fm^{-1} , and is symmetric around $k = 0.5$ fm^{-1}, corresponding to a spectrum on a lattice with a spacing of 1 fm. As the time goes on, one observes the interplay of the stochastic collisions and the exponentially increasing propagation due to the unstable effective field.

The continual action of both agents shifts the peak to slightly higher frequencies (from 0.5 to 0.6 fm^{-1}). However, it should be noted that the response above 1 fm^{-1} remains nearly zero throughout this initial stage and that the system is never fully dominated by the instabilities. This demonstrates that the system is keeping some memory of the physical processes that have induced the fluctuations.

After 30 fm/c the system enters into a non-linear regime, with frequency doubling leading soon to a chaotic transition stage, before reaching an equilibrium characterized by a statistical population of fragment configurations.

3.4 Fragment mass distribution

We finally discuss briefly the distributions of cluster configurations. In our model we can readily define a pseudofragment by adding the matter in contiguous cells having a density above a specified value ρ_s. The resulting mass distribution changes continually, because the density distribution keeps fluctuating as it explores the entire ensemble of possible configurations. This equilibrium behavior is independent of the specific early evolution, as we have explicitely verified by making an alternate calculation starting with an uncorrelated random noise on the one-body density. In an actual nuclear collision, the expansion of the system will effectively truncate the evolution after a finite time, and the resulting fragment mass distribution will in general depend on the specific character of the fluctuations employed.

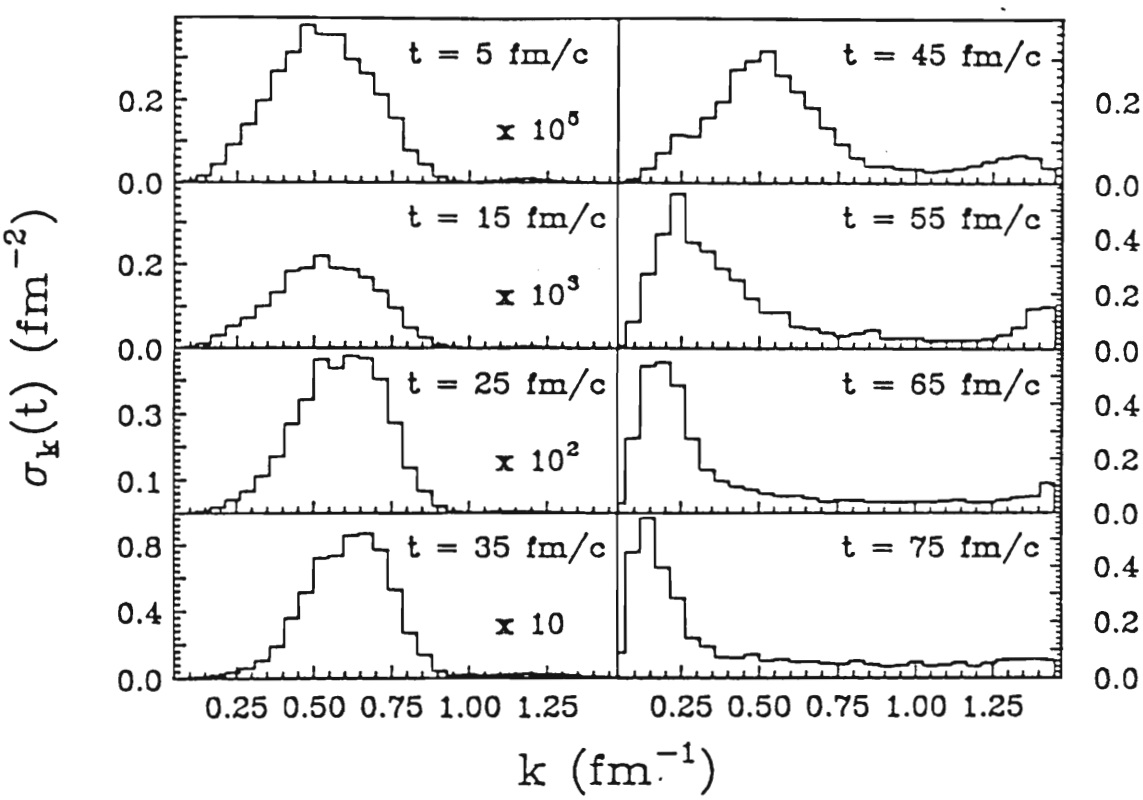

Fig. 3. Variance spectrum: The variance σ_k associated with harmonic modes of given wave number k, shown at a number of times t. The quantity σ_k is given by the square of the Fourier transform of the density $\bar{\rho}(x)$, averaged over the ensemble of histories followed, see eq. 10. The vertical scales have been magnified in each panel, so as to obtain curves of comparable appearance.

4 Conclusions

Using a recently developed nuclear Boltzmann-Langevin model, we have made a first dynamical simulation of a chatastrophic evolution leading to the multi-fragmentation of an initially uniform system. The essential new feature of the model is that it permits the spontaneous breaking of symmetries, which is essential to produce catastrophic phenomena, such as clusterization. The presence of a stochastic term solves the problem of the lack of predictability of the deterministic theory in presence of instabilities and forces us to adopt a description in terms of ensembles. Such a description is closely related to statistical mechanics and is therefore able to accomodate a high degree of dynamical branching.

In the presented results, we have been able to discern two dynamical regimes: an early linear regime characterized by a competition between the stochastic creation of fluctuations and their exponential evolution due to the instability of the effective field, and a later complex one where the system behaves chaotically and seeks to condense into fragments. Since the detailed evolution towards equilibrium has been found to be sensitive to the specific treatment of the fluctuations, it appears that the inclusion of fluctuations is important for the quantitative description of fragment production in nuclear collisions, and one may hope that multi-fragment observables may provide an informative basis for direct confrontation between theory and experiment.

References

[1] R. Balian, Y. Alhassid, and H. Reinhart, Phys. Rep. **131**, 1 (1986).

[2] J. Randrup and B. Remaud, Nucl. Phys. **A514**, 339 (1990).

[3] S. Ayik and C. Gregoire, Phys. Lett. **B212**, 269 (1988); Nucl. Phys. **A513**, 187 (1990);

E. Suraud, S. Ayik, M. Belkacem, and J. Stryjewski, Nucl. Phys. **A** (submitted).

[4] Ph. Chomaz, G.F. Burgio, and J. Randrup, Phys. Lett. **B254**, 340 (1991); G.F. Burgio, Ph. Chomaz, and J. Randrup, Nucl. Phys. **A529**, 157 (1991).

[5] G.F. Burgio, Ph. Chomaz, and J. Randrup, GSI-91-62, Preprint Dezember 1991, to be published in Phys. Rev. Lett.

[6] W. Bauer, G.F. Bertsch, and S. Das Gupta, Phys. Rev. Lett. **58**, 863 (1987)

[7] G.F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 190 (1988).

[8] L.W. Nordheim, Proc. Royal. Soc. (London) **A119**, 689 (1928).