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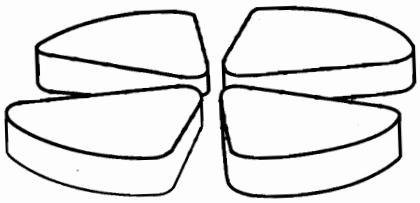

Dynamical symmetries of the shell model

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Dynamical Symmetries of the Shell Model

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The use of spectrum generating algebras in the nuclear shell model is reviewed. Older ideas due to Wigner, Racah and Elliott are succinctly summarised to put more recent advances in a proper perspective. The latter include the SO(8) model with $T = 0$ and $T = 1$ pairing, the fermion dynamical symmetry model, and pseudo-SU(3) and pseudo-SU(4) symmetries arising from the combination of the isospin and pseudo-spin degrees of freedom.

1. A SYMMETRY TRIANGLE FOR THE SHELL MODEL

The structure of the atomic nucleus is determined, in first instance, by the nuclear mean field, the average potential felt by one nucleon through the interactions exerted by all others. This average potential is responsible for the shell structure of the nucleus. For a description that goes beyond this most basic level, the *residual* interaction between the nucleons needs to be considered and what usually matters most for nuclear structure at low energies is the residual interaction between the nucleons in the valence shell. This interaction depends, in general, in a complex fashion on the numbers of valence protons and neutrons and the kinds of valence orbits available and its inclusion usually leads to the requirement of a numerical treatment. Nevertheless, many of the basic features of the structure of nuclei can be derived from a few essential characteristics of the nuclear mean field and the residual interaction.

One important element is the attractive, short-range nature of the residual interaction; in the limit of a pure delta force it gives rise to many-body nuclear wave functions that are classified by LS (or Russel–Saunders) coupling. This classification, however, is badly broken by the spin–orbit term in the nuclear mean field. The conflicting tendency between the spin-independent short-range character of the residual interaction, which favours LS coupling, and the spin–orbit term in the average potential, which leads to a jj-coupled classification, is the first crucial element in the structural determination of the nucleus. Indeed, this conflict was recognised and studied in the early days of the nuclear shell model [1]; the generally accepted conclusion was that, while the LS classification is appropriate for very light nuclei, it is gradually replaced by jj coupling which is relevant for the vast majority of nuclei [2].

The second important feature that determines the structure of the nucleus concerns the numbers of protons and neutrons in the valence shell. The residual interaction between identical nucleons has, because of its short-range nature, a pairing character which favours the formation of pairs of nucleons in time-reversed orbits. This is no longer true when the

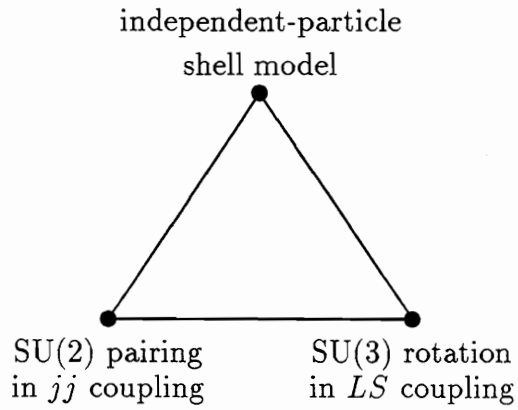

Figure 1. Schematic representation of the shell-model parameter space with three analytically solvable vertices.

valence shell contains both protons and neutrons, in which case the interaction acquires an important quadrupole component. Hence, nuclei display a wide variety of spectra, from pairing-type towards rotational-like, the determining factor in the evolution from one to the other essentially being the product $n_\nu n_\pi$ of proton and neutron numbers in the valence shell [3].

On the basis of such qualitative considerations one thus surmises that the essential characteristics of nuclei are determined by (i) the competition between residual interaction and shell structure (single-particle versus collective), (ii) the strength of the short-range interaction versus the spin–orbit term in the mean field (LS versus jj coupling) and (iii) the competition between pairing and quadrupole interaction [SU(2) versus SU(3)].

One of the gratifying aspects of symmetry techniques as applied to the nuclear shell model, is that these essential features can be represented algebraically. This is illustrated schematically in figure 1 where each vertex corresponds to an analytic solution of the shell model. A hamiltonian of the top vertex yields uncorrelated Hartree–Fock type wave functions. This limit is reached if the single-particle energy spacings are large in comparison with a typical matrix element of the residual interaction. The transition between jj and LS coupling is controlled by the ratio of the strength of the spin–orbit coupling to that of the residual interaction. And, finally, the transition from SU(2) pairing to SU(3) rotation not only involves this same ratio but in addition requires a change of the residual interaction from pairing to quadrupole.

The triangle in figure 1 summarises the basic symmetries of the shell model as developed by Wigner, Racah and Elliott. They are briefly presented and discussed in sections 2–4. They have continued to inspire subsequent work on symmetries of the shell model some of which are discussed in sections 5–8. For full details the reader is invited to consult the original references (or the review [4]) while here only the essential features of each symmetry are recalled.

2. WIGNER'S SU(4) SUPERMULTIPLET MODEL

In 1937 Wigner [5] (and, independently of him, Hund [6]) proposed a beautiful extension of Heisenberg's idea of isospin symmetry by assuming nuclear forces to be invariant under rotations in *spin* as well as *isospin* space. This invariance is expressed by the following commutation relations:

$$[\hat{H}, \sum_{k=1}^{A} s_\mu(k)] = [\hat{H}, \sum_{k=1}^{A} t_\mu(k)] = [\hat{H}, \sum_{k=1}^{A} s_\mu(k) t_\nu(k)] = 0, \tag{1}$$

where $s_\mu(k)$ and $t_\mu(k)$ are the spin and isospin operator components respectively, of nucleon k. The 15 operators $\sum_k s_\mu(k)$, $\sum_k t_\mu(k)$ and $\sum_k s_\mu(k) t_\nu(k)$ generate the Lie algebra SU(4) and, consequently, any hamiltonian satisfying the conditions (1) has SU(4) symmetry.

To qualitatively understand the meaning of this SU(4) symmetry as a way of labelling many-particle states, it is instructive to analyse the case of two particles. Total antisymmetry of the wave function ensures that the spatial part is symmetric and the spin–isospin part antisymmetric or that the spatial part is antisymmetric and the spin–isospin part symmetric. Both cases corresponding do correspond to a different symmetry classification under SU(4). This argument can be generalised to an arbitrary number of particles and the result emerges that the SU(4) quantum numbers specify the way in which the overall antisymmetry is distributed over the spatial and spin–isospin parts of the wave function.

The physical relevance of Wigner's supermultiplet classification is connected with the short-range attractive nature of the residual interaction as a result of which states with spatial symmetry are favoured energetically. To see this point, consider an extreme form of a short-range interaction, namely a delta interaction which has a vanishing matrix element in a spatially antisymmetric two-particle state since in that case the wave function has zero probability of having $\bar{r}_1 = \bar{r}_2$. In contrast, the matrix element is attractive in a spatially symmetric state. This result can be generalised to many particles, leading to the conclusion that the energy of a state depends strongly on its SU(4) labels. This statement can be quantified (see chapter 29 of [7]) by constructing a Majorana space exchange operator [related to the Casimir operator of SU(4)] that 'measures' the symmetry of the spatial part of the wave function.

Wigner's supermultiplet model is a nuclear LS coupling scheme. With the advent of the nuclear shell model the importance of the spin–orbit coupling became clear and, as a result, the SU(4) model is now largely abandoned. In spite of its limited applicability, Wigner's idea remains important because it demonstrates the connection between the short-range character of the residual interaction and the spatial symmetry of the many-body wave function. The break down of SU(4) symmetry is a consequence of the spin–orbit term in the nuclear mean field which does not satisfy the second and third commutator in (1). The spin–orbit term breaks SU(4) symmetry and does so increasingly in heavier nuclei since the energy splitting of the spin doublets $l - \frac{1}{2}$ and $l + \frac{1}{2}$ increases with nuclear mass number A. In addition, SU(4) symmetry is also broken by the Coulomb interaction [which has a non-vanishing first and third commutator in (1)]—an effect that also increases with nuclear mass—and it is broken by spin-dependent terms in the residual interaction.

3. RACAH'S SU(2) PAIRING MODEL

If the spin–orbit term in the mean-field potential is sufficiently large, a jj-coupling scheme arises. Suppose initially that the energy splitting between the different j states is large compared to the residual interaction strength, so that only nucleons in the last j shell must be considered as valence particles. Suppose, furthermore, that the residual interaction among those valence nucleons has a pairing character which is attractive for two particles coupled to angular momentum $J = 0$ and is zero otherwise,

$$\langle j^2 J M_J | \hat{V}_{\text{pairing}} | j^2 J M_J \rangle = \begin{cases} -\frac{1}{2}(2j+1)G, & \text{if } J = 0 \\ 0, & \text{if } J \neq 0 \end{cases} . \tag{2}$$

This is a schematic albeit reasonable approximation to the residual interaction between *identical* nucleons and hence can only be appropriate in semi-magic nuclei. Under the above assumptions, the shell-model hamiltonian can be diagonalised analytically in a space of n identical fermions in a j shell. This can be shown in a variety of ways (see, e.g. [8], section 7.2) but one elegant derivation relies on the existence of a quasi-spin SU(2) symmetry of the pairing hamiltonian [9]. The label s of the SU(2) algebra and its projection m_s can be put in relation to the more usual ones of seniority v [10] and (valence) particle number n. The interpretation of the seniority quantum number v is that it gives the number of nucleons not in pairs coupled to angular momentum zero.

The concepts of seniority and quasi spin have found repeated application in nuclear physics and have been the subject of fruitful generalisations. An important extension is due to Flowers [11] and concerns the seniority classification of protons *and* neutrons in jj coupling citeTAL00; in doing so the concept of *reduced isospin* t is established, which is the isospin of the nucleons not in pairs coupled to angular momentum zero. This brings about the generalisation of the quasi-spin algebra from SU(2) to SO(5). Another generalisation concerns that towards several orbits. This can be readily achieved in the case of degenerate orbits which leaves the algebraic SU(2) structure unchanged. The ensuing formalism can then be applied to semi-magic nuclei but since it requires the assumption of a pairing interaction with degenerate orbits, its applicability is limited. A much more generally valid scheme is obtained if one imposes the following condition on the shell-model hamiltonian:

$$[[\hat{H}, \hat{S}_+], \hat{S}_+] = \Delta \left(\hat{S}_+\right)^2 , \tag{3}$$

where $\hat{S}_+$ creates a two-particle eigenstate of $\hat{H}$ and Δ is a constant. This condition of *generalised seniority* [13] is much weaker than the assumption of a pairing interaction and, in particular, it does not require that the commutator $[\hat{S}_-, \hat{S}_+]$ yields (up to a constant) the number operator which is central to the quasi-spin formalism. In spite of the absence of a closed algebraic structure, it is still possible [13] to compute the exact ground-state eigenvalue of hamiltonians satisfying (3).

4. ELLIOTT'S SU(3) MODEL OF ROTATION

In Wigner's supermultiplet model the classification of the spatial part of the wave function is left unspecified. It is only assumed that the total orbital angular momentum

L is a good quantum number. The main feature of Elliott's model [14] is that it provides an orbital classification scheme which incorporates rotational characteristics. Elliott's SU(3) model of rotation presupposes Wigner's SU(4) classification and assumes in addition that the residual interaction has a *quadrupole* character. The latter is a reasonable hypothesis if the valence shell contains a sufficient number of protons and neutrons.

The proof that a shell-model hamiltonian without a one-body spin–orbit term and with a quadrupole residual interaction is analytically solvable can be found in the original papers of Elliott [14] (see also chapter 30 of [7]) and is not repeated here. Elliott's idea is important in two respects. First, it represents a genuine example (arguably the first one) of dynamical symmetry breaking: the degeneracy within a given Wigner supermultiplet is lifted (dynamically) by the quadrupole interaction. Second, it gives rise to a rotational classification of states in the context of the shell model through mixing of spherical orbits. From Elliott's analysis emerges a quantum number K associated with the projection of total orbital angular momentum on the axis of rotational symmetry. As such, the SU(3) model establishes a link between the nuclear shell model of Mayer [15] and of Jensen *et al.* [16], and the droplet model of Bohr and Mottelson [17] which up to Elliott's work existed as two separate views of the nucleus.

Given that Elliott's SU(3) model uses Wigner's supermultiplet classification as a starting point, it likewise breaks down as a result of the spin–orbit coupling in the nuclear mean field and it cannot be applied to heavy nuclei. Since in the s and p shells the SU(3) model reduces to Wigner's supermultiplet theory, the first test case where orbital mixing and associated deformation may occur, is for nuclei in the sd shell. Elliott's SU(3) model has thus found its main application in sd-shell nuclei [18,19]. Over the years several schemes have been proposed with the aim of transposing the SU(3) scheme such that it becomes applicable to heavier nuclei. An example of such modification—named quasi-SU(3)—can be found in [20]; others are discussed in sections 6 and 7.

5. THE SO(8) MODEL

The pairing interaction considered in section 3 acts between identical nucleons in a j shell coupled to total angular momentum $J = 0$ and isospin $T = 1$. If protons and neutrons are considered, the isospin of a pair of nucleons is not necessarily $T = 1$ but can also be $T = 0$, and the pairing interaction can be generalised correspondingly. One way of doing so is to consider the LS coupling limit with a set of degenerate l shells and a pairing interaction between pairs of nucleons coupled to total *orbital* angular momentum $L = 0$. Overall antisymmetry of a two-particle state implies, for $L = 0$, that $S = 0, T = 1$ or $S = 1, T = 0$. Thus the pairing interaction can be of spin-scalar, isovector or of spin-vector, isoscalar type and contains two parameters: an overall strength and a parameter which determines the relative strengths of isovector and isoscalar pairing.

The algebraic structure of this generalised pairing problem comes about in the same way as the SU(2) quasi-spin algebra discussed in section 3. Pair creation and annihilation operators are complemented with particle-number conserving operators to obtain a closed algebra which in this case corresponds to SO(8). Furthermore, an analysis of the subalgebra structure shows that three meaningful limits or dynamical symmetries of the SO(8) model can be considered [21,22]: two are obtained in case of pure isoscalar

and of pure isovector pairing, respectively. The third limit is obtained with equal pairing strengths in which case it turns out that the pairing hamiltonian has SU(4) symmetry [i.e., satisfies (1)]. The entire range of pairing strengths can thus be simulated and the benchmark situations are analytically solvable. These properties make that SO(8) is a schematic model that can be used to study the question of $T = 0$ versus $T = 1$ pairing and therefore it receives currently renewed attention in relation to $Z \approx N$ nuclei (see, e.g. [23]). It can, however, only have limited applicability since it assumes an LS classification and no spin–orbit coupling. Once a spin–orbit term is added to the pure pairing hamiltonian, the subspace constructed out of $L = 0$ pairs no longer is decoupled and one again is forced to solve the eigenvalue problem in the full shell-model space. Alternatively, approximate solutions can be found through boson mappings [24].

6. PSEUDO-SPIN SYMMETRY

A very successful way of extending applications of the SU(3) model to heavy nuclei (and to high angular momentum) is based upon the concept of pseudo-spin symmetry. To understand the nature of this symmetry, consider the unitary transformation

$$\hat{u} = 2i\frac{\bar{s}\cdot\bar{r}}{r}. \tag{4}$$

If this transformation is applied to a single-particle hamiltonian that includes a harmonic-oscillator mean field with a spin–orbit and an orbit–orbit term, one finds [25] (up to a constant) a transformed hamiltonian with a modified spin–orbit coupling. In particular, it can be shown that the spin–orbit term disappears entirely if in the original hamiltonian the strength of the orbit–orbit coupling is four times that of the spin–orbit coupling. This results from the single-particle levels with $j = l + \frac{1}{2}$ and $j = (l+2) - \frac{1}{2}$ being degenerate for all values of l. These single-particle levels can be considered as originating from a pseudo-orbital angular momentum $\tilde{l} = l + 1$, in the presence of zero pseudo-spin–orbit splitting $\tilde{l}\cdot\tilde{s}$. This is illustrated in figure 2 for the sdg shell where the levels $(2d_{5/2}, 1g_{7/2})$ and $(3s_{1/2}, 2d_{3/2})$ occur in doublets and can be considered as originating from $\tilde{l} = 3$ and $\tilde{l} = 1$, respectively. Note, however, that there is no partner for $1g_{9/2}$ in this scheme.

Pseudo-spin symmetry has a long history in nuclear physics. The existence of nearly degenerate pseudo-spin doublets in the nuclear mean-field potential was noted already thirty years ago by Hecht and Adler [26] and, independently, by Arima *et al.* [27]. These authors also realised that, because of the small pseudo-spin–orbit splitting, pseudo-LS (or $\tilde{L}\tilde{S}$) coupling should be a reasonable starting point in medium-mass and heavy nuclei where LS coupling becomes unacceptable. With $\tilde{L}\tilde{S}$ coupling as a premiss, an SU(3) model can be constructed in the same way as Elliott's SU(3) model can be defined in LS coupling. The ensuing pseudo-SU(3) model was investigated seriously for the first time in [28] with many applications following afterwards (for a review, see [29]). Although the pseudo-SU(3) model is probably the most important emanation of pseudo-spin symmetry, it must be emphasised that the latter is a broader concept than just pseudo-SU(3), as is illustrated in section 8 which deals with pseudo-SU(4) symmetry. The formal definition of the pseudo-spin transformation (4) in terms of a helicity operator was given by Bohr *et al.* [25] and later generalised by Castaños *et al.* [30], to include transformations that not only act on the spin–angular part of the wave function—as does (4)—but also on its

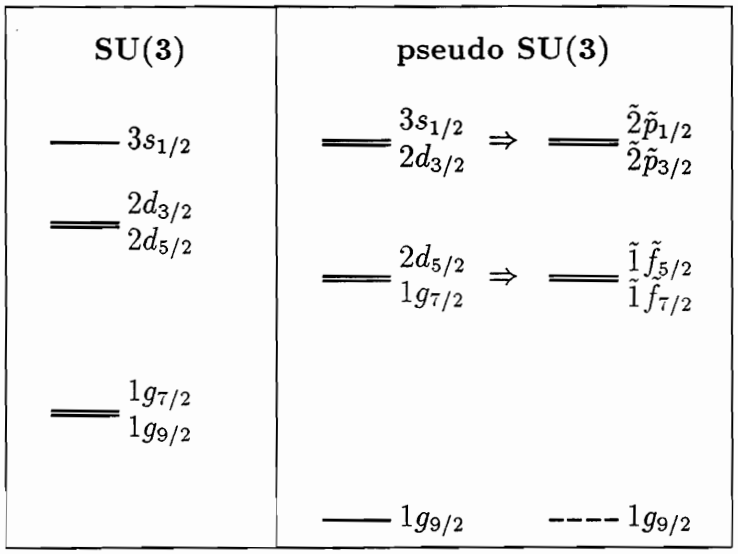

Figure 2. Single-particle spaces of SU(3) and pseudo-SU(3) for the example of the *sdg* shell. In pseudo-SU(3) the level degeneracies can be interpreted in terms of a pseudo-spin symmetry.

orbital part. Finally, it is only recently that an explanation of pseudo-spin symmetry was suggested in terms of the relativistic mean-field model of the nucleus. First, the relation between the strength of the spin–orbit and the orbit–orbit coupling (necessary for pseudo-spin symmetry) was found to be approximately valid in numerical calculations [31] and later the pseudo-spin symmetry was proven to be a symmetry of the Dirac equation which occurs if the scalar and vector potentials are equal in size but opposite in sign [32,33].

A vanishing pseudo spin–orbit splitting does not necessarily imply the validity of $\tilde{L}\tilde{S}$ coupling which can be broken by pseudo-spin dependent terms in the residual interaction. The assumption of $\tilde{L}\tilde{S}$ coupling can be verified by analysing the wave functions of a standard shell-model calculation. An example of such an analysis is given in [35] and leads to the conclusion that $\tilde{L}\tilde{S}$ coupling is a reasonable *ansatz* for nuclei in the mass $A \approx 60$ region. Intuitively, this result is a combined effect of the short-range nature of the residual interaction and the fact that nucleons interact predominantly at the surface of the nucleus. Because of the latter property, matrix elements of the residual interaction are not very sensitive to the radial structure of the wave function at the interior of the nucleus. As a result, the problem of n nucleons in a pseudo harmonic oscillator shell is approximately equivalent to that of n nucleons in a normal harmonic oscillator shell. In fact, this equivalence is exact for the surface delta interaction [36,37] which is known to be a reasonable approximation to the true effective interaction in nuclei.

7. THE FERMION DYNAMICAL SYMMETRY MODEL

An important property of Racah's pairing hamiltonian is that it does not couple states constructed out of S pairs to the rest of the shell-model space. The question now arises

whether this property of decoupling can be generalised to more realistic scenarios. In particular, given the important rôle of the quadrupole degree of freedom in nuclei, one would like an extension of this idea towards a decoupled space in terms of monopole S and quadrupole D pairs. A systematic procedure for constructing such hamiltonians was devised by Ginocchio [38], drawing on earlier ideas by Hecht *et al.* [39] and using a method which resembles that of pseudo-spin [26,27]. The theory was later developed under the name of *fermion dynamical symmetry model* or FDSM by Wu and others [40,41].

The starting point of the method is the separation of the nucleon angular momentum $\bar{j}$ into a pseudo-orbital part $\bar{k}$ and a pseudo-spin part $\bar{\imath}$: $\bar{j} = \bar{k} + \bar{\imath}$. This is similar to the pseudo-spin scheme of section 6. By convention, k is taken integer and i half-integer but no other condition is imposed *a priori*. This separation leads to the definition of the particle creation operators

$$a^\dagger_{ki,jm_j} = \sum_{m_k m_i} \langle km_k\, im_i | jm_j \rangle a^\dagger_{km_k im_i}, \tag{5}$$

where $a^\dagger_{km_k im_i}$ is related as follows to the usual $(l\frac{1}{2})j$-coupled creation operators:

$$a^\dagger_{km_k im_i} = \sum_{jm_j} \langle km_k\, im_i | jm_j \rangle a^\dagger_{l\frac{1}{2},jm_j}. \tag{6}$$

Pairs of particles can now be defined in a KI-coupled instead of a jj-coupled basis, the two being related through

$$\begin{aligned} \hat{B}_+(KI, JM_J) &\equiv (a^\dagger_{ki} \times a^\dagger_{k'i'})^{(KI,J)}_{M_J} \\ &= \sum_{jj'} \hat{K}\hat{I}\hat{j}\hat{j}' \left\{ \begin{array}{ccc} k & k' & K \\ i & i' & I \\ j & j' & J \end{array} \right\} (a^\dagger_{l\frac{1}{2},j} \times a^\dagger_{l'\frac{1}{2},j'})^{(J)}_{M_J}, \end{aligned} \tag{7}$$

where $\hat{s} = \sqrt{2s+1}$ and the coefficient between curly brackets is a nine-j symbol [7]. Any two-body nucleon interaction can be written equivalently in terms of either jj-coupled or KI-coupled pair creation and annihilation operators since these are connected by the unitary transformation (7). In addition to the two-particle creation operators (7) and their hermitian conjugates, a closed algebraic structure requires multipole operators of the form

$$\hat{P}(KI, JM_J) \equiv (a^\dagger_{ki} \times \tilde{a}_{k'i'})^{(KI,J)}_{M_J}, \tag{8}$$

where the annihilation operators $\tilde{a}_{km_k im_i}$ have the correct transformation properties in pseudo-orbital and pseudo-spin space,

$$\tilde{a}_{km_k im_i} \equiv (-)^{k+m_k+i+m_i} a_{k-m_k i-m_i}. \tag{9}$$

It is now a matter of straightforward algebra to work out the commutation relations between the operators $\hat{B}_+(KI, JMJ)$, their hermitian conjugates $\hat{B}_-(KI, JMJ)$ and $\hat{P}(KI, JM_J)$. These will depend on the choice of k and i, and their coupled values K and I, and a judicious choice will define a set of pairs that decouple from all others.

The search is now on for sets of pair operators that include a monopole S pair and a quadrupole D pair, and close under commutation to an algebra which is smaller than

the one that spans the entire shell-model space. Two such algebras can be defined [38]: SO(8) and Sp(6), obtained from K-scalar pairs with $i = \frac{3}{2}$ or I-scalar pairs with $k = 1$, respectively.

Two conditions are required for the occurrence of the above algebraic realisations and the associated decoupling from the rest of the shell-model space. First, it needs the appropriate single-particle orbits which should be consistent with the representation in terms of k and i. All combinations $j = \frac{1}{2}, \frac{3}{2}, \ldots, j_{\max}$ are possible, sometimes in two different ways, but note that this sequence does not include the unnatural-parity orbit which should thus be dealt with separately. Second, a far more restrictive condition concerns the decoupling of part of the shell-model space. This requires that the hamiltonian be written in terms of generators of a single algebra, either SO(8) or Sp(6). This condition, in fact, may lead to unrealistic hamiltonians. More details can be found in [29] and references therein.

A valuable aspect of the FDSM is that the Pauli principle is correctly treated without any approximation. On the negative side in the evaluation of the FDSM is that the structure of the pairs is algebraically imposed. This is clear from the expression (7) for $\hat{B}_+(KI, JM_J)$ which gives a well-defined expansion in terms of the usual jj-coupled pairs. In exceptional cases the algebraic pairs might correspond to those favoured by the nuclear interaction; more often they will not. In such situations the FDSM can only be viewed as a useful shell-model truncation scheme.

8. PSEUDO-SU(4) SYMMETRY

Just as the SU(2) symmetries of spin and isospin can be combined to yield the larger SU(4) symmetry, one can equally well combine the SU(2) symmetries of pseudo-spin and isospin to give what can be called a pseudo-SU(4) [or $\widetilde{\mathrm{SU}}(4)$] symmetry [34,35]. A hamiltonian with pseudo-SU(4) symmetry satisfies the following commutations relations:

$$[\hat{H}, \sum_{k=1}^{A} \tilde{s}_\mu(k)] = [\hat{H}, \sum_{k=1}^{A} t_\mu(k)] = [\hat{H}, \sum_{k=1}^{A} \tilde{s}_\mu(k) t_\nu(k)] = 0, \tag{10}$$

where $\tilde{s}_\mu$ are the transformed spin operator components, $\tilde{s}_\mu = \hat{u}^{-1} s_\mu \hat{u}$. The hamiltonian in (10) conserves the total pseudo-orbital angular momentum $\tilde{L}$ and the total pseudo-spin $\tilde{S}$, which result from the separate coupling of all individual pseudo-orbital angular momenta $\tilde{l}(k)$ and pseudo-spins $\tilde{s}(k)$.

The existence of a pseudo-SU(4) symmetry requires minimally that the valence shell coincides with a pseudo-oscillator shell. A region where this is possibly the case concerns $Z \approx N$ nuclei at the beginning of the 28–50 shell where the dominant orbits are $2p_{1/2}$, $2p_{3/2}$ and $1f_{5/2}$ which can be considered as a pseudo-sd shell.

It is instructive to compare the predictions of pseudo-SU(4) symmetry concerning Gamow–Teller β decay with those of SU(4). A typical example of the latter, the β^+ decay of ${}^{18}_{8}\mathrm{Ne}_{10}$, is shown in figure 3: the Gamow–Teller decay proceeds to two 1^+ states in ${}^{18}_{9}\mathrm{F}_9$ with $\log(ft)$ values of 3.1 and 4.5, respectively. In SU(4) these correspond to transitions within the same supermultiplet; the first occurs without change of orbital structure—and hence is fast—while the second is forbidden. The SU(4) classification thus provides a qualitative understanding of Gamow–Teller β decay in ^{18}Ne.

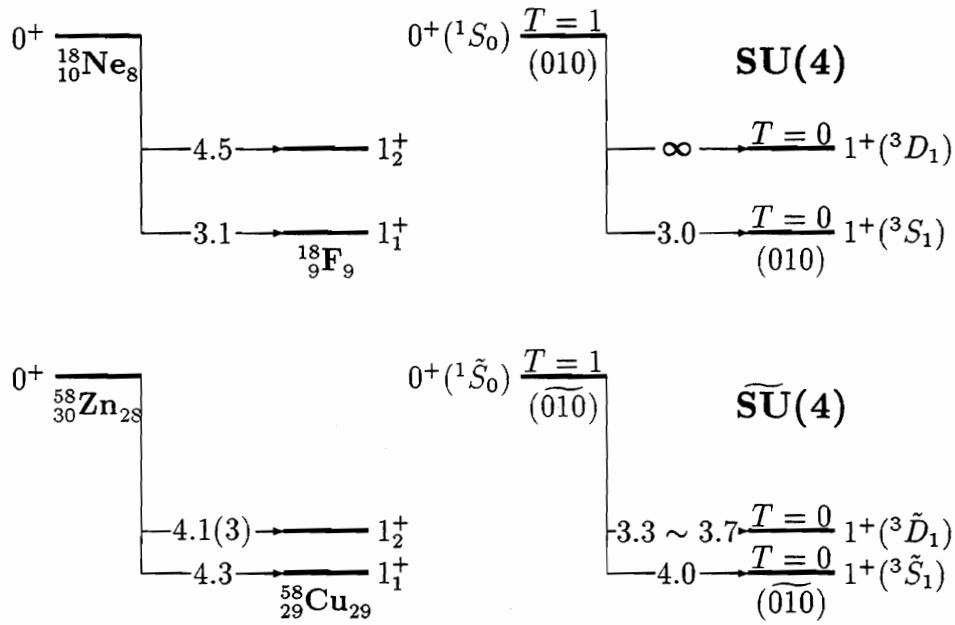

Figure 3. The $\log(ft)$ values for Gamow–Teller matrix elements in the β^+ decay of ^{18}Ne and ^{58}Zn as contrasted with the corresponding values in standard and pseudo SU(4).

The analysis of the analogous problem in pseudo-SU(4) is more complicated because the Gamow–Teller operator is not a generator of pseudo-SU(4). The situation is also illustrated in figure 3. The transition without change in orbital structure is about one order of magnitude weaker than the corresponding one in SU(4) (4.0 versus 3.0). Furthermore, the second transition no longer is forbidden. Its matrix element depends upon the amount and character of orbital mixing; the numbers shown in the figure ($3.3 \sim 3.7$) represent a typical range between prolate and oblate deformation. It is also seen that the observed ft values in the decay of $^{58}_{30}$Zn$_{28}$ [42] strongly differ from those for ^{18}Ne, and are more akin to the pseudo-SU(4) prediction.

9. SUMMARY

The applications of spectrum generating algebras and of dynamical symmetries in the nuclear shell model are many and varied. They stretch back to Wigner's early work on the supermultiplet model and encompass important landmarks in our understanding of the structure of the atomic nucleus such as Racah's SU(2) pairing model and Elliott's SU(3) rotational model. One of the aims of this contribution has been to show the historical importance of the idea of dynamical symmetry in nuclear physics. Another has been to indicate that, in spite of being old, this idea continues to inspire developments that are at the forefront of today's research in nuclear physics.

It has been argued in this contribution that the main driving features of nuclear struc-

ture can be represented algebraically but at the same time the limitations of the symmetry approach must be recognised. It should be clear that such approach can only account for gross properties and that any detailed description requires more involved numerical calculations of which we have seen many fine examples during this symposium. In this way symmetry techniques can be used as an appropriate starting point for detailed calculations. A noteworthy example of this approach is the pseudo-SU(3) model which starting from its initial symmetry *ansatz* has grown into an adequate and powerful description of the nucleus in terms of a truncated shell model.

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