Spontaneous Symmetry Breaking in Quantum-Chromodynamics I. Lagrangian QCD

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Abstract: The most general conserved Noether currents (energy-momentum density tensor Θ^{μ}_{ν}) arising from the form invariance of the QCD-Lagrangian with respect to the translation group are derived. The total Hamiltonian density Θ^{0}_{0} is shown to be the sum of the standard canonical one, the conserved Noether current generating the color charge and the conserved Noether currents affiliated to the Cartan subalgebra of the flavor group. - The flavor term is identical with the "medium strong" and electromagnetic interaction Hamiltonian producing the mass splitting within hadronic flavor multiplets. - The color and flavor symmetries are spontaneously broken in the sense that the QCD-Lagrangian and the Euler-Lagrange field equations are form invariant with respect to the global color and flavor groups but not all of their generators commute with the total Hamiltonian. - The Goldstone particles which necessarily accompany the spontaneous symmetry breaking are shown to have finite masses depending on the internal quantum numbers of the currents to which they are affiliated.

I. Introduction

One cornerstone for the success of the unitary symmetry scheme in the evolution of strong interaction physics leading to flavored quantum-chromodynamics consists in bringing order to the complex problems of hadron spectroscopy. The basic philosophy (see [1]-[7] and Refs. therein) of all unitary symmetry schemes from SU(3) to SU(?) is that in an ideal universe there is only one type of interaction, the strong interaction, invariant under some internal symmetry group such that all hadrons may be grouped into unitary multiplets, that is all members of such an unitary multiplet have the same external quantum numbers like mass, spin etc. and differ by internal quantum numbers like isospin, hypercharge and so on being the eigenvalues of a suitably chosen basis for the Cartan subalgebra of the unitary group involved. The quite rough mass splitting in the real world of hadrons is attributed to a hierarchy of weaker interactions breaking the unitary symmetry: At the top a "medium strong" one causing the splitting between isospin submultiplets of different hypercharge followed by the electro-weak interaction responsible for the splitting within an isospin submultiplet.

To derive the mass splittings between isospin submultiples it has to be assumed that the effect of the medium strong interaction Hamiltonian H_{asym}. inspite of being quite stronger than the electro-weak part is small compared to the symmetry conserving strong one, so that first order perturbation theory may be applied.
Inspite of its indisputable success in bringing order to the hadron spectrum this scheme showed right from the beginning some deficiencies leaving the state of affairs a little bit artificially flavored. Some authors of textbooks state it this way: "Put more concisely, the theory would be more convincing if the symmetry-breaking was 'intrisic' to the scheme, in the sense that it emerges automatically, without it having to be put in 'by hand' ". [2, Part C, chapt. 18]. "Considering the fact that m_π is much smaller than the mass differences $m_K - m_\pi$ and $m_\eta - m_\pi$ it seems quite strange that H_{asym} can be treated as a perturbation" [1, Chapt. 12].
Indeed right at the outset the unitary scheme met the flaw, that in contrast to the baryon multiplets the masses in the splitting relations for the pseudo-scalar meson octet had - again by hand - to be replaced by their squares in order to fit the experimental data to an acceptable accuracy. Meanwhile more mesonic particles have been established by experiments [30] and it turns out, that when grouped into octets in such a way that the mass differences are all small compared to a medium mass value of the multiplet the original linear mass formulas fit the experimental values at least with the same accuracy as the quadratic ones and appart from the lowest lying pseudo-vector octet $1^{--}(\omega - \phi$ mixing) with a variation of less than 2.5%. Hence the pseudo-scalar meson octet seems to play an exceptional role for "some unknown weaker mechanism" [5] breaking the unitary flavor symmetry of the strong interaction, which still hides itself behind all plausible explanations.
In the following sections we are going to show that the symmetry breaking mechanism is indeed "intrinsic to the scheme "in the sense that the symmetry breaking interaction Hamiltonian automatically follows via Noether's Theorem from the form-invariance of the flavored QCD-Lagrangian with respect to the space-time translation group. The unitary symmetry is spontaneous broken in the sense that the QCD-Lagrangian is form invariant to the global flavor and color groups but the generators not belonging to their Cartan-subalgebras do not commute with the total Hamiltonian. Moreover the representations of the translation group which produce via Noether's Theorem the symmetry breaking interaction Hamiltonian at the same time enforce the Goldstone particles which neccessarily accompany the spontaneous symmetry breaking to take on *finite* masses depending explicitely on the internal quantum numbers of the conserved Noether currents (respectively generators) to which they are affiliated.
Hence for the (axial-)vector currents of the chiral flavor group $SU(3)_{\mathcal{F},L} \times SU(3)_{\mathcal{F},R}$ the Goldstone particles may be identified with members of the lowest lying (pseudo-) scalar meson octet. But then a subclass of the representations of the translation group dictates the well known quadratic mass splitting formula for the 0^- and 0^+ octet and in addition a further mass relation connecting the masses of the scalar octet with those of the pseudo-scalar one, which predicts the electro-weak splitting in the $\kappa = K^*(1429)$ isospin doublet to be six times larger than that of the Kaon doublet, that is 24 MeV.

II. Noether Currents of LCD

The Lagrangian of (Quantum-)Chromodynamies is build up from two $SU(3)$-color-triplets respectively $SU(n)$-flavor-multiplets $(n \geq 3)\varphi^{3,n}_{a,f}(x)$ and $\varphi^{3^*n^*}_{a,f}(x)$ of Dirac spinors called quark respectively antiquark fields and a $SU(3)$-color-octet (flavor-singlet) $\varphi^{8,1}_a(x)$ of Lorentz vektors called gluon (gauge) fields [1]-[4]. Here we will restrict the flavor group $G(3)_{\mathcal{F}}$ to be $SU(3)$ respectively some subgroup of it (isomorphie to $SU(2) \times U(1)$ or $U(1) \times U(1)$), that means consider a system with three flavors (up, down, strange). However all considerations may easily be extended to a flavor group of higher degree in order to include charmed, bottom, top and so on quarks. The two "different" $SU(3)$-groups will be marked by a suffix $\mathcal{C}$ for color respectively $\mathcal{F}$ for flavor when ever it seems necessary to avoid confusion.

For the sake of notational shortness the unitary irreducible matrix representations $D^{(p,q)}$, $(p,q) \in \boldsymbol{N}_0 \times \boldsymbol{N}_0$ of $SU(3)$ with dimension $d = 1/2[(1+p)^2(1+q) + (1+p)(1+q)^2]$ will be denoted by $R^d = D^{(p,q)}$ for $p \geq q$ respectively $R^{d*} = D^{(p,q)}$ for $p \leq q$. Then for any triple (α, u, v) from the direct product $SL(2,a) \times SU(3)_{\mathcal{C}} \times G(3)_{\mathcal{F}}$ the fields transform according to:

$$\varphi^{m,n}_{\mu,a,f}(x) \rightarrow (U(\alpha,u,v)\varphi)^{m,n}_{\mu,a,f}(x) := S^{(m,n)}(\alpha)^{\nu}_{\mu} R^m_{\mathcal{C}}(u)^b_a R^n_{\mathcal{F}}(v)^g_f \varphi^{m,n}_{\nu,b,g}(\Lambda(\alpha)x) \tag{1}$$

with

$$S^{(m,n)}(\alpha)^{\mu}_{\nu} := \begin{cases} S^{1/2}(\alpha^{-1})_{\nu\mu} \text{ for } (m,n) = (3,3) \\ \Lambda(\alpha)^{\mu}_{\nu} \text{ for } (m,n) = (8,1) \\ S^{1/2}(\alpha)_{\mu\nu} \text{ for } (m,n) = (3^*,3^*) \end{cases} \tag{2}$$

and

$$\begin{aligned} &S^{1/2}(\alpha)^{-1}\gamma^{\mu}S^{1/2}(\alpha) = \Lambda(\alpha)^{\mu}_{\nu}\gamma^{\nu}; \quad \gamma^0 S^{1/2}(\alpha)^{+}\gamma^0 = S^{1/2}(\alpha)^{-1}; \\ &\Lambda(\alpha)^{\mu}_{\nu} g_{\mu\sigma}\Lambda(\alpha)^{\sigma}_{\kappa} = g_{\nu\kappa} \end{aligned} \tag{3}$$

The Dirac matrices are assumed to have the hermiticity properties $(\gamma^{\mu})^{+} = (\bar{\gamma}^{\mu})^{t} = g^{\mu\mu}\gamma^{\mu}, (\gamma^0)^{t} = \gamma^0$. With respect to the group $SU(3)$ and its representations we adopt the conventions and (apart from minor changes) the notations of de Swart [5]-[7]. The color/flavor indices are triples $a = (I, I^3, Y)$ running over the index set Γ_d defined by the Okubo-rule [6]:

$$\begin{aligned} &\Gamma_d(p,q) := \{(I, I^3, Y) | I = 1/2(\lambda - \rho); I^3 = -I, ..., I; \\ &Y = \lambda + \rho - 1/3(2p + 4q); \lambda, \rho \in \boldsymbol{N}_0; 0 \leq \rho \leq q\} \end{aligned} \tag{4}$$

I... is interpreted to be the total "color/flavor isorpin", I^3... its third component and Y... the "color/flavor hypercharge". An immediate consequence of the definition of T_d is:

$$(I, I^3, Y) \in \Gamma_d(\Gamma_{d^*}) \Leftrightarrow (I, -I^3, -Y) \in \Gamma_{d^*}(\Gamma_d) \tag{5}$$

In accordance with this relation the negative of an index triple $a = (I, I^3, Y)$ is by definition the triple $-a := (I, -I^3, -Y)$. Hence the "color/flavor charge"

$$Q(a)... = Q(I, I^3, Y)... := I^3... + \frac{1}{2}Y... \tag{6}$$

satisfies $Q(-a)... = -Q(a)...$. Finally there exists an important connection between the complex conjugate representation $\overline{R^d}(\overline{R^{d^*}})$ and the representation $R^{d^*}(R^d)$. Adopting the notational convention $d^{**} = (d^*)^* = d$ it reads for all d and d^*:

$$R^{d^*}(u)^b_a = (-1)^{Q(b)-Q(a)}\overline{R^d(u)}^{-b}_{-a} \tag{7}$$

$a, b \in \Gamma_{d^*}, u \in SU(3)$.
In case of the adjoint (octet) representation as well as all other ("diagonal") representations with $R^d = D^{(p,p)} = R^{d^*}$ the condition (7) is an equation between different elements of the some matrix, which is called self-conjugateness. As an immediate consequence the gluon fields form a self-conjugate color octet:

$$\overline{\varphi^{8,1}_{\mu,a}(x)} = (-1)^{Q(a)c}\,\varphi^{8,1}_{\mu,-a}(x) \tag{8}$$

Dirac indices will often be dropped by the usual conventions and $\varphi^{3,3}_{a,f}$ is considered to be a column whereas $\varphi^{3^*,3^*}_{a,f}$ a row. Then the Dirac adjoint of $\varphi^{3,3}_{a,f}$ is connected to $\varphi^{3^*,3^*}_{a,f}$ by:

$$\tilde{\varphi}^{3,3}_{a,f}(x) := \varphi^{3,3}_{a,f}(x)^t\gamma^0 = (-1)^{-Q(a)c-Q(f)\mathcal{F}-2/3}\varphi^{3^*,3^*}_{-a,-f}(x) \tag{9}$$

In terms of these fields the Lagrangian of chromodynamics reads:

$$\begin{aligned} L(\varphi(x), B(x)) = -1/4(-1)^{Q(a)c}\,G_{\mu\nu,-a}(x)G^{\mu\nu}{}_{,a}(x) + \tilde{\varphi}^{3,3}_{a,f}(x)[\tfrac{i}{2}\gamma^\mu\{\delta^b_a\overleftrightarrow{\partial}_\mu + ig(\tau_c)^b_a\varphi^{8,1}_{\mu,c}(x)\} \\ -\delta^b_a\varrho_f]\delta^e_f\varphi^{3,3}_{b,e}(x) + (-1)^{Q(a)c}[\varphi^{8,1}_{\mu,-a}(x)\partial^\mu B^{8,1}_a(x) - \tfrac{\alpha}{2}B^{8,1}_{-a}(x)B^{8,1}_a(x)] \end{aligned} \tag{10}$$

with the gluon field tennor

$$G_{\mu\nu,a}(x) := \partial_\mu\varphi^{8,1}_{\nu,a}(x) - \partial_\nu\varphi^{8,1}_{\mu,a}(x) + ig\sqrt{3}\left(\begin{matrix} 8 & 8 & 8_A \\ b & c & a \end{matrix}\right)\varphi^{8,1}_{\mu,b}(x)\varphi^{8,1}_{\nu,c}(x) \tag{11}$$

being obviously a self-conjugate color octet (flavor singlet).
$\tau_a, a \in \Gamma_8$ denote the traceless (3,3)-matrices of the self-conjugate fundamental representation of the (complex) Lie-algebra $su(3)$ satisfying the relations [5]-[7]:

$$[\tau_a, \tau_b]_- = -2\sqrt{3}\left(\begin{matrix} 8 & 8 & 8_A \\ a & b & c \end{matrix}\right)\tau_c, \quad tr.(\tau_a \cdot \tau_b^+) = 2\delta_{ab}, \quad \tau_a^+ = (-1)^{Q(a)}\tau_{-a} \tag{12}$$

The gauge fixing field $B_a^{8,1}$ is related to the gluon octet via the Euler-Lagrange field equation $\partial^\nu \varphi_{\nu,a}^{8,1}(x) + \alpha B_a^{8,1}(x) = 0$.

Usually there is added a further term to the Lagrangian describing $FP-$ghosts [8]. In order to keep the explicite formulas as short as possible we dropped it, since it would only add some further terms to the explicite expression of the Noether currents in terms of the basic fields without being in any way tangent to the general structure of these currents which we are going to use in our considerations.

The members of the set $\{\varphi_{\mu,a,f}^{3,3}, \tilde{\varphi}_{\mu,a,f}^{3,3}, \varphi_{k,b}^{8,1}, B_b^{8,1} | \mu = 0, ..., 3; k = 1, 2, 3; a, f \in \Gamma_3; b \in \Gamma_8\}$ are taken as independent canonical variable.

By construction the Lagrangian is form invariant under the global color group $SU(3)_c$ and the corresponding eight conserved Noether-currents read $(c \in \Gamma_8)$:

$$\begin{aligned} j_c^{8,\mu}(x)_C &:= -\tfrac{1}{2}\{\tilde{\varphi}_{a,f}^{3,3}(x)\gamma^\mu(\tau_c)_a^b \varphi_{b,f}^{3,3}(x) + i2\sqrt{3}(-1)^{Q(a)c}\begin{pmatrix} 8 & 8 & 8_A \\ c & b & a \end{pmatrix}\times \\ &[G_{-a}^{\mu\epsilon}(x)\varphi_{\epsilon,b}^{8,1}(x) + \varphi_{\lambda,-a}^{8,1}(x) g^{\mu\lambda} B_b^{8,1}(x)]\} = \tfrac{1}{g}(-1)^{Q(a)c}\{\delta_c^a \partial_\lambda G_{-a}^{\mu\lambda}(x) \\ &+ g^{\mu\lambda}[\delta_c^a \partial_\lambda + ig\sqrt{3}\begin{pmatrix} 8 & 8 & 8_A \\ c & d & a \end{pmatrix}\varphi_{\lambda,d}^{8,1}(x)] B_{-a}^{8,1}(x)\} \end{aligned} \tag{13}$$

In the last row the Euler-Lagrange equations have been used.
Here we are especially interested in the combination $j_{(1,0,0)}^{8,\mu}(x) + \frac{1}{\sqrt{3}} j_{(0,0,0)}^{8,\mu}(x)$ because it will reoccur in the Noether energy momentum tensor below. From the explicite connection between the real self-conjugate traceless matrices $\tau_c, c \in \Gamma_8$ and the hermitian traceless matrices $\lambda_j, j = 1, ..., 8$ of Gell-Mann

$$\tau_{(0,0,0)} = \lambda_8, \quad \tau_{(1,0,0)} = \lambda_3, \quad \tau_{(1,1,0)} = -\frac{1}{\sqrt{2}}(\lambda_1 - i\lambda_2)$$

$$\tau_{(1/2,1/2,1)} = -\frac{1}{\sqrt{2}}(\lambda_4 - i\lambda_5), \quad \tau_{(1/2,1/2,1)} = -\frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7)$$

respectively multiplying the commutation relations (12) by $(\tau_e)^+$ and taking the trace it follows at once:

$$\begin{aligned} &(\tau_{(1,0,0)} + \tfrac{1}{\sqrt{3}}\tau_{(0,0,0)})_a^b = 2Q(a)\delta_a^b \\ &2\sqrt{3}[\begin{pmatrix} 8 & 8 & 8_A \\ (1,0,0) & b & a \end{pmatrix} + \tfrac{1}{\sqrt{3}}\begin{pmatrix} 8 & 8 & 8_A \\ (0,0,0) & b & a \end{pmatrix}] = 2Q(a)\delta_{ba} \end{aligned} \tag{14}$$

and therefore:

$$j^{8,\mu}_{(1,0,0)}(x)_c + \frac{1}{\sqrt{3}} j^{8,\mu}_{(0,0,0)}(x)_c = -\{Q(a)_c \bar{\varphi}^{3,3}_{a,f}(x)\varphi^{3,3}_{a,f}(x)$$
$$+i(-1)^{Q(a)_c} Q(a)_c [G^{\mu,\epsilon}_{-a}(x)\varphi^{8,1}_{\epsilon,a}(x) + g^{\mu\epsilon}\varphi^{8,1}_{\epsilon,-a}(x)B^{8,1}_a(x)]\} \tag{15}$$

Concerning the flavor transformations the subgroup of $SU(3)$ with respect to which the CD-Lagrangian is form invariant is determined alone by the "mass" matrix of the quarks

$$\varrho = \begin{pmatrix} \varrho_u & 0 & 0 \\ 0 & \varrho_d & 0 \\ 0 & 0 & \varrho_s \end{pmatrix} = \begin{pmatrix} \varrho_{(1/2,1/2,1/3)} & 0 & 0 \\ 0 & \varrho_{(1/2,-1/2,1/3)} & 0 \\ 0 & 0 & \varrho_{(0,0,-2/3)} \end{pmatrix},$$

which is not observable, but can only be estimated indirectly. If all three quark "masses" are equal $G(3)_{\mathcal{F}}$ is of course again $SU(3)$. If $\varrho_u = \varrho_d \neq \varrho_s$ form invariance holds only for the subgroup

$$G(3)_{\mathcal{F}} = \{\begin{pmatrix} \mathcal{G} & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{pmatrix} \exp[i\frac{t}{2\sqrt{3}}\tau_{0,0,0)}]|\mathcal{G} \in SU(2) \wedge t \in \mathbb{R}\} \tag{16}$$

isomorphic to $SU(2) \times U(1)$. Finally if all three "masses" are pairwise different their remains only the subgroup.

$$G(3)_{\mathcal{F}} = \{\exp i/2(\tau_{(1,0,0)}\rho + \frac{1}{\sqrt{3}}\tau_{(0,0,0)}t)|\rho, t \in \mathbb{R}\} \tag{17}$$

isomorphic to $U(1) \times U(1)$.
Corresponding to these three cases the Noether Theorem delivers the existence of eight, four respectively two locally conserved flavor currents:

$$j^{8,\mu}_c(x)_{\mathcal{F}} = -1/2\{\bar{\varphi}^{3,3}_{a,f}(x)\gamma^\mu(\tau_c)^e_f\varphi^{3,3}_{a,e}(x)\} \tag{18}$$

with

$$c \in . \begin{cases} \Gamma_8 \text{ if } G(3)_{\mathcal{F}} = SU(3) \\ \{(0,0,0),(1,0,0),(1,1,0),(1,-1,0)\} \quad \text{if } G(3)_{\mathcal{F}} \sim SU(2) \times U(1) \\ \{(0,0,0),(1,0,0)\} \quad \text{if } G(3)_{\mathcal{F}} \sim U(1) \times U(1) \end{cases}$$

It is generally accepted [1]-[4] that $\varrho_d \approx \varrho_u \approx 0$ and $\varrho_u + 100MeV \leq \varrho_s \leq \varrho_u + 200MeV$, which reflects the mass splitting in the hadron spectrum with the splitting within an isospin submultiplit due to electromagnetism. In the last two cases, where the Noether theorem delivers only four or two instead of eight conserved currents one can define four respectively six further currents - the missing ones with respect to the full $SU(3)$ symmetry - by the right hand side of equation (18). Of course they are not conserved. Important for the following is that the two currents $j^{8,\mu}_{(1,0,0)}(x)_{\mathcal{F}}$ and $j^{8,\mu}_{(0,0,0)}(x)_{\mathcal{F}}$ which will reoccur in the Noether energy-momentum tensor are locally conserved in all three cases above.

Next we consider the translation group $(\boldsymbol{R}_4, +)$. For any fixed complex four-vector $m \in C_4$ and any fixed triple (μ, κ, w) of real four-vectors a representation $\Gamma^{m,M}_{\kappa,w}$ of the translation group may be defined by:

$$\begin{aligned}(T^{m,M}_{\kappa,\omega}(y)\varphi^{n,d}_{\mu,a,f}(x) &:= \exp[Q(a)_C(m\cdot y) + Q(f)_{\mathcal{F}}(M\cdot y)]\times \\ &\times R^d(\exp i/2[\tau_{(1,0,0)}(\kappa\cdot y) + \tfrac{1}{\sqrt{3}}\tau_{(0,0,0)}(\omega\cdot y)])^e_f\varphi^{n,d}_{\mu,a,e}(x+y)\end{aligned} \tag{19}$$

with $(n,d) \in \{(3,3),(3^*,3^*),(8,1)\}$ and $B^{8,1}_a$ transforming in the same way as the gluon field $\varphi^{8,1}_{\dots,a}$ (i.e. color octet and flavor singlet). By means of the equation (7) and (9) it follows for the Dirac adjoint spinor $\tilde{\varphi}^{3,3}$:

$$\begin{aligned}(T^{m,M}_{\kappa,\omega}(y)\tilde{\varphi})^{3,3}_{a,f}(x) &= (-1)^{-Q(a)_C - Q(f)_{\mathcal{F}} - 2/3}(T^{m,M}_{\kappa,\omega}(y)\varphi)^{3^*,3^*}_{-a,-f}(x) \\ &= \exp[-Q(a)_C(m\cdot y) - Q(f)_{\mathcal{F}}(M\cdot y)]\times \\ &\times(\exp(-i/2)[\tau_{(1,0,0)}(\kappa\cdot y) + \tfrac{1}{\sqrt{3}}\tau_{(0,0,0)}(\omega\cdot y)])^e_f\tilde{\varphi}^{3,3}_{a,e}(x+y)\end{aligned} \tag{20}$$

It should be stated very clearly, that the equations (19) are nothing else than the definition of a set of operations $\{T^{\dots}_{\dots}(y)|y \in \boldsymbol{R}_4\}$ satisfying the (Abelian) composition law $T^{\dots}_{\dots}(x) \circ T^{\dots}_{\dots}(y) = T^{\dots}_{\dots}(x+y)$ of the translation group and therefore generating a representation of it. (For $M = m = \kappa = \omega = 0$ it is just the standard "transformation law" of fields under translations well known from any textbook). The essential point comes now:
The CD-Lagrangian as given by the equations (10)-(11) is form invariant, that means $L(\hat{\varphi}(\hat{x}), \hat{B}(\hat{x})) = L(\varphi(x), B(x))$, *under the following group of substitutions:*

$$\begin{aligned}&x \to \hat{x} = x + y, \quad y \in \boldsymbol{R}_4 \\ &\varphi^{n,d}_{\mu,a,f}(x) \to \hat{\varphi}^{n,d}_{\mu,a,f}(\hat{x}) := (T^{m,M}_{\kappa,\omega}(-y)\varphi)^{n,d}_{\mu,a,f}(x+y), (n,d) \in \{(3,3),(3^*,3^*),(8,1)\} \\ &B^{8,1}_a(x) \to \hat{B}^{8,1}_a(\hat{x}) := (T^{m,M}_{\kappa,\omega}(-y)B)^{8,1}_a(x+y).\end{aligned} \tag{21}$$

The proof for the flavor part is obvious since the Lagrangian is diagonal in the flavor indices. The proof of the color part is given in the appendix of [9] and consists mainly in showing that $(\tau_c)^b_a = 0$ unless $Q(a) = Q(b) + Q(c)$.

Due to the form invariance of the Lagrangian under the (four parameter) group of translations above Noether's Theorem [10] dictates the existence of four locally conserved currents $\Theta^{m,M}_{\kappa,\omega}(x)^\mu_\nu, (\nu = 0,1,2,3, \partial_\mu\Theta^{m,M}_{\kappa,\omega}(x)^\mu_\nu = 0)$ which are considered as the components of the energy-momentum density tensor, provided they possess the necessary reality properties of an observable. In the case at hand they read explicitely:

$$\begin{aligned}\Theta^{m,M}_{\kappa,\omega}(x)^\mu_\nu &= \Theta(x)^\mu_\nu + i(m_\nu Q(a)_C + M_\nu Q(f)_{\mathcal{F}})\tilde{\varphi}^{3,3}_{a,f}(x)\gamma^\mu\varphi^{3,3}_{a,f}(x) \\ &-\tfrac{1}{2}\tilde{\varphi}^{3,3}_{a,f}(x)\gamma^\mu[\kappa_\nu(\tau_{(1,0,0)})^e_f + \omega_\nu(\tau_{(0,0,0)})^e_f]\varphi^{3,3}_{a,e}(x) \\ &-Q(a)_C(-1)^{Q(a)_C}[G^{\mu\epsilon}_{-a}(x)\varphi^{8,1}_{\epsilon,a}(x) + g^{\mu\epsilon}\varphi^{8,1}_{\epsilon,-a}(x)B^{8,1}_a(x)]m_\nu\end{aligned} \tag{22}$$

Here $\Theta(x)^\mu_\nu = \Theta^{0,0}_{0,0}(x)^\mu_\nu$ is the (standard) *canonical* energy-momentum density tensor

$$\begin{aligned}\Theta(x)^\mu_\nu &= -\delta^\mu_\nu L(\varphi(x), B(x)) + \tfrac{i}{2}\bar{\varphi}^{3,3}_{a,f}(x)\gamma^\mu \overleftrightarrow{\partial}_\nu \varphi^{3,3}_{a,f}(x) \\ &-(-1)^{Q(a)}[G^{\mu\epsilon}_{-a}(x)\partial_\gamma \varphi^{8,1}_{\epsilon,a}(x) + g^{\mu\epsilon}\varphi^{8,1}_{\epsilon,-a}(x)\partial_\gamma B^{8,1}_a(x)].\end{aligned} \tag{23}$$

Especially $\Theta(x)^0_0$ is just the *canonical* Hamiltonian density.
Comparing the right hand side of the equation (22) with the equations (15) and (18) we see that due to the relation (14) the terms occurring in addition to the canonical part are just special components of the locally conserved flavor respectively color currents. Hence the final result for the locally conserved Noether tensor obtained from translational invariance reads:

$$\begin{aligned}\Theta^{m,M}_{\kappa,\omega}(x)^\mu_\nu &= \Theta(x)^\mu_\nu - i\, m_\nu[\dot{j}^{8,\mu}_{(1,0,0)}(x)_C + \tfrac{1}{\sqrt{3}}\dot{j}^{8,\mu}_{(0,0,0)}(x)_C] \\ &+(\kappa_\nu - i\, M_\nu)\dot{j}^{8,\mu}_{(1,0,0)}(x)_{\mathcal{F}} + \tfrac{1}{\sqrt{3}}(\omega_\nu - i\, M_\nu)\dot{j}^{8,\mu}_{(0,0,0)}(x)_{\mathcal{F}}\end{aligned} \tag{24}$$

with $m \in C_4$ and $M, \kappa, \omega \in \mathbb{R}_4$.
All further considerations and results will be based on this formula without any reference to the dependence of the currents on the basic quark and gluon fields with the exception of their properties under color and flavor transformations originating from them.

In classical field theories as well as in quantized ones using a non covariant gauge (Coulomb gauge for instance) which admits a (positive definite) Hilbert space $(\mathcal{H}, (\cdot|\cdot))$ spanned by physical states the integral

$$P_\nu(\kappa, \omega, m, M) = \lim_{V_3 \to \infty} \int_{V_3} d^3\mathbf{x}\Theta^{m,M}_{\kappa,\omega}(x)^0_\nu \tag{25}$$

being interpreted as the total energy momentum (operator) represents an observable. Hence $\Theta^{m,M}_{\kappa,\omega}(x)^0_\nu$ or at least those parts of it, which contribute to the integral have to be real respectively an essentially self-adjoint operator. Therefore in these cases M_ν has to be zero and m_ν purely imaginary, if the color term contributes to the integral at all. (In these non covariant gauges the color current is a (identically conserved) Gauß current and the integral may vanish.) In other words all representations of the translation group which do not meet these conditions would be excluded via physical interpretation. Besides loosing explicite Lorentz-covariance and Einstein-causality (locality) in the intermediate steps within these gauges the fact that only color singlets are realized in nature and hence admitted in the physical Hilbert space has to be put in as an additional condition "by hand".
The situation is different (for the color coupling m_ν) in a covariant gauge of the Gupta-Bleuler type and especially in the case of QCD at hand with colored basic fields and only color singlets admitted to the physical Hilbert space. In such a gauge [11]-[22] the physical Hilbert space $(\mathcal{H}^{\kappa,\omega}_{phys}, < \cdot, \cdot >)$ is detached from the Hilbert space $(\mathcal{H}, (\cdot|\cdot))$ in

which the basic fields, symmetry transformations, currents etc. act as operators. It is obtained via factorization $H^{m,M}_{\kappa,\omega}/\overset{\circ}{H}{}^{m,M}_{\kappa,\omega}$ of two subspaces $\overset{\circ}{H}{}^{m,M}_{\kappa,\omega} \subset H^{m,M}_{\kappa,\omega} \subset \mathcal{H}$ generated by an indefinite metric $< \cdot,\cdot >= (\cdot|\eta^{m,M}_{\kappa,\omega}\cdot)$ on $\mathcal{H}$ which is positive semi-definite on $H^{m,M}_{\kappa,\omega}$ and zero on $\overset{\circ}{H}{}^{m,M}_{\kappa,\omega}$. Moreover the scalar product on $\mathcal{H}^{\kappa,\omega}_{phys} = \overline{H^{m,M}_{\kappa,\omega}/\overset{\circ}{H}{}^{m,M}_{\kappa,\omega}}$ is again given by the metric $< \cdot,\cdot >$ and of course is positive definite on this space. The local gauge invariance of the theory is reflected in the requirement [13],[21]

$$< \Psi, (j^{8,\mu}_a(x)_C - \partial_\nu G^{\mu\nu}_a(x))\Phi >= 0 \tag{26}$$

for all Φ from a dense subspace of $H^{m,M}_{\kappa,\omega}$ and all $\Psi \in H^{m,M}_{\kappa,\omega}$. Similarly to the case of the non covariant gauge the color current may not contribute to the space integral (25) sandwiched between these states provided the behavior at infinity is nice enough. Moreever it has been proved in Ref.[9] that in case the vacuum state is an eigenstate of the metric operator $\eta^{m,M}_{\kappa,\omega}$ then every representation of the translation group $T^{m,M}_{\kappa,\omega}$ with $Re.m \neq 0$ being isometric with respect to the metric $< \cdot,\cdot >$ forces all colored states of $\mathcal{H}$ to be neutral and therefore the physical Hilbert space $\mathcal{H}^{\kappa,\omega}_{phys}$ is spaned by color singlets only (quark-gluon confinement). In addition it lifts to an unitary representation $U_{\kappa,\omega}$ independent of m and M on $(\mathcal{H}^{\kappa,\omega}_{phys}, < \cdot,\cdot >)$. Hence in any case <u>no</u> restriction is enforced on $m \in C_4$, since the conserved Noether currents respectively their space integrals both acting in $\mathcal{H}$ are not observables themselves. The corresponding observables are constructed from them in $\mathcal{H}^{\kappa,\omega}_{phys}$ and there the color terms do not contribute. All representations with non vanishing $m \neq 0$ are physically equivalent in the sense, that they lead to the same (up to unitary equivalence) physical Hilbert space, observables etc.

On the other hand the generally accepted physical picture behind the confinement mechanism, namely that the forces between the quarks grow in strength with the distance between them [1]-[4] demands the translation group to be represented by non-isometric operators in the non-physical Hilbert space $(\mathcal{H}, (\cdot|\cdot))$ because otherwise there would arise a contradiction to the cluster properties of the Wightman functions [22]. However it is well known [11, Theorem 4.2] that in gauge quantum field theories with the vacuum being an eigenstate of the metric operator an $\eta-$isometric representation of the translation group is non-unitary in $(\mathcal{H}, (\cdot|\cdot))$ if and only if $Re.m \neq 0$. (We note in passing, that the vacuum being an eigenstate of the metric operator is only a sufficient condition but <u>not</u> a necessary one).

In contrast to the complex color coupling m the real flavor coupling M however has to be zero, because otherwise the same mechanism as in the case of color confinement would ban all flavor multiplets except singlets from $\mathcal{H}^{\kappa,\omega}_{phys}$.

In accordance with the discussion above in the following M_ν is taken to be zero for al $\nu = 0,1,2,3$ whereas m_ν can be arbitrary complex (for the covariant gauges). Then the energy density $E^m_{\kappa,\omega}(x) := \Theta^{m,0}_{\kappa,\omega}(x)^0_0$ reads:

$$E^m_{\kappa,\omega}(x) = H(x) - i\, m_0[j^{8,0}_{(1,0,0)}(x)_C + \tfrac{1}{\sqrt{3}} j^{8,0}_{(0,0,0)}(x)_C] + \kappa_0 j^{8,0}_{(1,0,0)}(x)_{\mathcal{F}} + \tfrac{\omega_0}{\sqrt{3}} j^{8,0}_{(0,0,0)}(x)_{\mathcal{F}} \tag{27}$$

with $m_0 \in C$ and $\kappa_0, \omega_0 \in \mathbb{R}$ and $H(x)$ the *canonical* Hamiltonian density.

III. Spontaneous Symmetry Breaking.

Of particular interest are the two flavor $SU(3)-$octet terms which occur on the right hand side of (27) in addition to the flavor singlet canonical Hamitonian and the flavor singlet color terms in the first row. They are just identical with the flavor symmetry breaking "medium strong interaction Hamiltonian" ($j^{8,0}_{(0,0,0)}$) respectively the eletromagnetic interaction ($j^{8,0}_{(1,0,0)}$) which have been successfully used to calculate the mass-splitting within hadron multiplets [1]-[7] and especially derive the famous Gell-Mann-Okubo mass formulas [24],[25]. Up to now either these terms have to be added by hand to the flavor singlet interaction Hamiltonian or the first few terms have to be selected again by hand from the expansion of a mass operator into "$SU(3)-$multipoles" which preserve the selection rules for isospin and hypercharge. The medium strong flavor term $j^{8,0}_{(0,0,0)}(x)_{\mathcal{F}}$ does not touch total isospin and hypercharge conservation and is responsible for the mass splitting between the different isospin multiplets within a flavor multiplet, whereas $j^{8,0}_{(1,0,0)}(x)_{\mathcal{F}}$ leaves only the third component of the isospin besides the hypercharge conserved and causes the splitting within an isospin multiplet.

The first main new result of our investigation is that the form invariance of the $CD-$Lagrangian with respect to (non-standard/$\kappa_0 \neq 0, \omega_0 \neq 0$) representations of the translation group produces exactly these mass splitting (flavor symmetry breaking) terms in the corresponding conserved Noether current (energy momentum density tensor); in short they are derived from translational invariance. Hence the "unknown medium strong interaction Hamiltonian" has come to be known (to those who want to know it).

Since the representations of the (abelian) translation group occurring in the equation (19) are the most general ones ($\lambda_3 = \tau_{(1,0,0)}$ and $\lambda_8 = \tau_{(0,0,0)}$ span the Cartan algebra of $su(3)$) which leave the $CD-$Lagrangian form invariant our result explains why the octet model, i.e. the assumption that all the higher flavor tensor operators $j^{27}_{(0,0,0)}, j^{64}_{(0,0,0)} \cdots$ respectively $j^{27}_{(1,0,0)}, j^{64}_{(1,0,0)}, \cdots$ do not contribute to the hadron mass splitting, works so well. In other words the symmetry breaking is intrinsic to the scheme, in the sense that it emerges automatically, without having to be put in by hand.

One may even turn the argument around and take the hadron mass splitting as an indication that nature prefers to select (for the strong interaction) a representation with $\omega_0 \neq 0$ and $\kappa_0 \neq 0$ instead of the standard one with $(\omega_0, \kappa_0) = (0,0)$. Because whenever one adds the flavor symmetry breaking interaction Hamiltonian - by hand or otherwise - to the strong symmetry conserving canonical one, the resulting states of the system can not tranform any more with the standard representation of the translation group.

From the equation (27) all the hadron mass formulas known in the literature [1]-[7], [24],[25] can easily be reproduced by means of the usual and generally accepted rules of the game to be found in any textbook on this topic. They consist in taking the expectation value with respect to the scalar product $< \cdot, \cdot >$ in $\mathcal{H}^{\kappa,\omega}_{phys}$ or, what leads to the same result, with respect to the semi-definite one $< \cdot, \cdot > = (\cdot|\eta\cdot)$ in $H^{m,0}_{\kappa,\omega}$ for the various one particle hadron states from $\mathcal{H}^{\kappa,\omega}_{phys}$ respectively $H^{m,0}_{\kappa,\omega} \backslash \overset{\circ}{H}{}^{m,0}_{\kappa,\omega}$ and adding the following two assumptions: (i) The flavor $SU(3)$ breaking interaction Hamiltonian, that means in the present scheme the two flavor currents in equation (27), are members of an irreducible

octet tensor operator of the $SU(3)$ flavor group. (ii) The one particle hadron states may be grouped together into and approximately treated as exact $SU(3)$ flavor multiplets.
Now a second important result within our scheme is that the property (i) must not be put in as an additional assumption but follows via Noether's Theorem provided all three quark "masses" in the CD-Lagrangian (10) are equal ($\rho_u = \rho_d = \rho_s$). However similarly to the case of the physical hadrons above the observed splitting of the phenomenological quark masses is obtained via equation (27) starting with equal and even zero quark mass parameters in the CD-Lagrangian and taking the expectation values for the (color- and flavor-) quark triplet with the (positive definite) scalar product $(\cdot|\cdot)$ in the unphysical Hilbert space $\mathcal{H}$. (In every representation with $Im.m_0 \neq 0$ there emerges also a contribution from the color charge density $\dot{j}^{8,0}_{(1,0,0)}(x)c + \frac{1}{\sqrt{3}}\dot{j}^{8,0}_{(0,0,0)}(x)c$.) Hence there exists no physical reason which can prevent one from starting with a CD-Lagrangian which is $SU(3)$ form invariant with respect to both the color and the flavor group and even with vanishing quark mass parameters ($\rho_u = \rho_d = \rho_s = 0$).
The physical hadron mass splitting as well as the phenomenological quark masses then emerge via a *spontaneous symmetry breaking* of both the global flavor group and the global subgroup of the local gauge color group in the sense that the CD-Lagrangian and hence the Euler-Lagrange field equations are form invariant with respect to these groups but their infinitesimal generators τ_a with $a \neq (1,0,0),(0,0,0)$ do <u>not</u> commute with the energy operator (i.e. the total Hamiltonian consisting of the canonical plus the symmetry breaking one). Since they do not commute with the generator of the time translations they are not conserved in time.
However as it was already mentioned in the introduction, in view of the fact that the mass of the pions is much smaller than the mass differences $m(K) - m(\pi)$ and $m(\eta) - m(\pi)$ it seems very strange that in this case the symmetry breaking part of the Hamiltonian can be treated as a perturbation in the way it is usually done and indicated above. The question is: Is the (quadratic) mass formula for the pseudo-scalar mesons only of phenomenological value or can it be derived and understood on a more solid basis than a quite doughtful perturbation treatment ? Indeed, one can ! It will be shown below, that the representations of the translation group which produce via Noether's Theorem the spontaneous symmetry breaking currents in the total Hamiltonian force the corresponding Goldstone particles to take on a *finite* mass depending on the isospin and hypercharge quantum numbers. Moreover a subclass of these representations dictates the quadratic Gell-Mann-Okubo mass splitting formulas for the lowest lying (pseudo-) scalar meson octet.
Postponing the Goldstone problem for a moment, let us shortly consider the chiral flavor symmetry. With all $\rho_f, f = u, d, s$ being zero the quark triplet may be decomposed into a left-handed $\varphi^{3,3,L}_{\dots} := (1+\gamma_5)\varphi^{3,3}_{\dots}$ and a right-handed $\varphi^{3,3,R}_{\dots} := (1-\gamma_5)\varphi^{3,3}_{\dots}$ one ($\gamma_5 := i\gamma^0\gamma^1\gamma^2\gamma^3$). Then the Lagrangian (10) reads:

$$
\begin{aligned}
&L(\varphi(x), B(x)) = -1/4(-1)^{Q(a)} G_{\mu,\nu,-a}(x) G^{\mu\nu}{}_{,a}(x) + \\
&\sum_{h=L,R} \tilde{\varphi}^{3,3,h}_{a,f}(x) \tfrac{i}{2} \gamma^{\mu} \{ \delta^{b}_{a} \overleftrightarrow{\partial}_{\mu} + ig(\tau_c)^{b}_{a} \varphi^{8,1}_{\mu,c}(x) \} \varphi^{3,3,h}_{b,f}(x) \\
&+(-1)^{Q(a)} [\varphi^{8,1}_{\mu,-a}(x) \partial^{\mu} B^{8,1}_{a} - \tfrac{\alpha}{2} B^{8,1}_{-a}(x) B^{8,1}_{a}(x)]
\end{aligned}
\tag{28}
$$

and is now form invariant under the chiral flavor group $SU(3)_{\mathcal{F},L} \times SU(3)_{\mathcal{F},R}$. Performing everywhere the replacements

$$
\varphi^{3,3}_{\dots}(x) \to \varphi^{3,3,h}_{\dots}(x), \quad j^{8,\mu}_{\dots}(x)_{\mathcal{F}} \to j^{8,h,\mu}_{\dots}(x)_{\mathcal{F}}, \quad (\kappa, \omega, M) \to (\kappa^h, \omega^h, M^h) \tag{29}
$$

$$
(\Theta^{m,M}_{\kappa,\omega}, E^{m}_{\kappa,\omega}, \eta^{m,M}_{\kappa,\omega}, \overset{(\circ)}{H}{}^{m,M}_{\kappa,\omega}, \mathcal{H}^{\kappa,\omega}_{phys}) \to (\Theta^{m,(M^L,M^R)}_{(\kappa^L,\kappa^R),(\omega^L,\omega^R)}, E^{m}_{(\kappa^L,\kappa^R),(\omega^L,\omega^R)}, \dots) \tag{30}
$$

and summing in the right hand side of the equations (13),(15),(22)-(24) and (27) all terms carrying the new chiral index over $h = L, R$, everything that has been said before remains true also in the chiral case.

The spontaneous symmetry breaking mechanism above can also be treated in a more general frame bejond the Lagrangian formalism used here. Especially the Goldstone problem deserves a detailed and rigerous reinvestigation, since some changes concerning the mass of the Goldstone particles are to be expected due to the difference in the representations of the translation group considered here compared to the usually applied (standard) one. The detailed presentation would go far bejond the scope of the present note and will be given in Part II.

The generalizations of the Goldstone theorem from classical (non-gauge) quantum field theories to gauge theories with indefinite metric (see e.g. Refs. [26]-[29] respectively [13],[21] and references therein) state, that the spontaneous breaking of an algebraic (global) symmetry group whose generators are associated to conserved currents is necessarily accompanied by the occurrence of $\delta(p^2)$ singularities (Goldstone particles), which are created by the corresponding conserved currents from the vaccum and are pushed via the local gauge condition (26) into the unphysical sections of the Hilbert space in case the (global) symmetry group is the subgroup with constant elements of a local gauge group like the color group in QCD [13],[21]. In case the spontaneous broken (global) symmetry group is not considered to be a subgroup of a local gauge group like the flavor group in QCD the massless Goldstone states may be lying either in the unphysical sections or in the physical one $H \setminus \overset{\circ}{H}$ and then occur as real particles in $\mathcal{H}_{phys}$ (genuine Goldstone mechanism).

However as far as the zero mass of the Goldstone states is concerned these results may not hold true anymore in the case at hand, since they are strongly tied to the standard representation of the translation group (with $\omega = \kappa = m = 0$) and the four-momentum spectrum of the states allowed in both the physical and the unphysical sections of the total Hilbert space $\mathcal{H}$ (For details see Refs. [13],[21]). Obbiously the exponential factors in the representations (19) and (20) which make the difference to the standard one will displace and even deform the support of the spectral functions in the Jost-Lehmann-Dyson representation of the commutator matrix elements involved, thereby moving the light cone of the Goldstone particles and even edge them away from the light cone.

Indeed our second main result is that the Goldstone particles which necessarily accompany the spontaneous symmetry breaking of the chiral flavor group posses the finite mass

$$p(a)_h^2 = I^3(a)^2(\kappa^h)^2 + \frac{1}{4}Y(a)^2(\omega^h)^2 + I^3(a)Y(a)(\kappa^h\omega^h), \tag{31}$$

$a \in \Gamma_8\backslash\{(1,0,0),(0,0,0)\}$, and may be identified with the corresponding members of the lowest lying meson octets (0^- respectively 0^+) of the hadronic mass spectrum - as physicists want it since two to three decades and claim it verbally in lectures and written up in any text book.

The proof follows in almost the classical way [13],[21],[27]-[29] from a generalized Jost-Lehmann-Dyson representation for the commutator matrix element of the current $j_a^{8,h,\mu}(x)_{\mathcal{F}}$ involved and an element $\phi(f)$ from the polynomial $*$-algebra generated by the basic fields which incorporates automatically besides locality (Einstein causality) and the spectrum condition also the current conservation. A sketch of the proof which shows the essential features is given in section IV below. The detailed and rigorous proof, which is burdened with a lot of gymnasties in distribution theory due to the non-isometry or even unboundedness of the translation operators in the unphysical Hilbert space $\mathcal{H}$, will be presented in Part II.

In the case of the axial currents $j_{a,A}^{8,\mu}(x)_{\mathcal{F}} = j_a^{8,L,\mu}(x)_{\mathcal{F}} - j_a^{8,R,\mu}(x)_{\mathcal{F}}$ we obtain with $\kappa_A := 1/2(\kappa^L - \kappa^R)$ and $\omega_A := 1/2(\omega^L - \omega^R)$ from equation (31):

$$\begin{aligned} p(1/2,+/-1/2,+/-1)_A^2 &= \tfrac{1}{4}\kappa_A^2 + \tfrac{1}{4}\omega_A^2 + 1/2(\kappa_A\omega_A) = m(K_{+/-})^2 \\ p(1/2,-/+1/2,+/-1)_A^2 &= \tfrac{1}{4}\kappa_A^2 + \tfrac{1}{4}\omega_A^2 - 1/2(\kappa_A\omega_A) = m(K_{0/\bar{0}})^2 \\ p(1,+/-1,0)_A^2 &= \kappa_A^2 = m(\pi_{+/-})^2 \end{aligned} \tag{32}$$

The identification of the Goldstone particles of the axial currents with the pseudo-scalar mesons fixes in part the representations of the translation group which seems to be selected by nature for QCD. Solving the equations (32) for the representation parameters we get:

$$\begin{aligned} \omega_A^2 &= 2m(K_0)^2 + 2m(K_+)^2 - m(\pi_+)^2 \approx 3m(\eta)^2 \\ \kappa_A^2 &= m(\pi_{+/-})^2; \quad (\omega_A\kappa_A) = m(K_+)^2 - m(K_0)^2 \end{aligned} \tag{33}$$

In the same way we obtain in the case of the vector currents $j_{a,V}^{8,\mu}(x)_{\mathcal{F}} = j_a^{8,L,\mu}(x)_{\mathcal{F}} + j_a^{8,R,\mu}(x)_{\mathcal{F}}$ and the scalar meson multiplet 0^+ with $\kappa_V := 1/2(\kappa^L + \kappa^R)$ and $\omega_V := \frac{1}{2}(\omega^L + \omega^R)$ from equation (31), [30]:

$$\begin{aligned} \omega_V^2 &= 2m(\kappa_0)^2 + 2m(\kappa_+)^2 - m(\delta_+)^2 \approx 3m(f_0)^2 \\ \kappa_V^2 &= m(\delta_{+/-})^2; \quad (\omega_V\kappa_V) = m(\kappa_+)^2 - m(\kappa_0)^2 \end{aligned} \tag{34}$$

The equations (33) and (34) strongly suggest to express the four four-vectors $\kappa_A^\mu, \kappa_V^\mu$ and $\omega_A^\mu, \omega_V^\mu (\mu = 0,1,2,3)$ in terms of two four- vectors κ^μ, ω^μ of unit length $\kappa^2 = 1$ respectively (almost unit length) $\omega^2 \approx 1$ in the following way:

$$\kappa_A^\mu = m(\pi_{+/-})\kappa^\mu; \quad \omega_A^\mu = \sqrt{3}m(\eta)\omega^\mu$$
$$\kappa_V^\mu = m(\delta_{+/-})\kappa^\mu; \quad \omega_V^\mu = \sqrt{3}m(f_0)\omega^\mu \tag{35}$$

Then the first of the equations (33) and (34) are identical with those which follow from the Gell-Mann-Okubo (quadratic) mass formula applied to the 0^- respectively 0^+ meson octet. Moreover from the equations (35) and the last ones of the two systems (33) and (34) a further mass relation is obtained, which connects the masses of the scalar octet with those of the pseudo-scalar one:

$$m(\delta_{+/-})m(\not{f})[m(K_0)^2 - m(K_+)^2] = m(\pi_{+/-})m(\eta)[m(\kappa_0)^2 - m(\kappa_+)^2] \tag{36}$$

Inserting the known particle masses [30] it predicts the electromagnetic splitting of $\kappa = K^*(1429)$ to be six times larger than that of the kaons, that is 24 MeV.

IV Generalized Goldstone Theorem

In this section we sketch the proof of the generalization of the Goldstone Theorem to the case of non-standard representations of the translation group leaving aside the distribution theoretical subtleties in the derivation of the Jost-Lehmann-Dyson representation arising from the non-isometry of the translation operators in the unphysical Hilbert space $\mathcal{H}$. For this we have to specity some notations and general assumptions (the usual ones).
Let $\phi(f) := \int dx_1 ... dx_n f(x_1, ..., x_n)\phi_{i_1}...\phi_{i_n}(x)$ with f from S_{4n} denote an element from the polynomial algebra $\mathcal{P}$ generated by the basic fields, that means ϕ_i is one of the basic fields including besides the quark and gluon fields $\varphi^{\cdots}_{\cdots}$ also the conserved currents $j^{\cdots}_{\cdots}$. The basic fields are assumed to satisfy Einstein causality, that is any two of them either commute or anticommute at space-like separated points. Furthermore it is assumed that the conserved flavor currents $j^{8,\mu}_{a,\cdots}(f)_{\mathcal{F}}$ create from the vacuum Ψ_0 only states with non-negative energy and non-negative four momentum squared (spectrum condition). With the abbreviations $f_y(x) := f(x-y)$ and $p_\mu(a) := I^3(a)\kappa_\mu + \frac{1}{2}Y(a)\omega_\mu$ and dropping the index h describing left- and right-handedness respectively vector and axial vectors the spectrum condition reads explicitely:

$$\int d^4y \exp\{-i(qy)\} < \Psi, T^{m,0}_{\kappa,\omega}(y) j^{8,\mu}_{a,\cdots}(\not{f})_{\mathcal{F}}\Psi_0 >=$$
$$\int d^4y \exp\{-i([q-p(a)]y)\} < \Psi, j^{8,\mu}_a(f_y)_{\mathcal{F}}\Psi_0 >= 0 \tag{37}$$

for all $(q-p(a)) \notin \bar{V}_-$ (closed backward cone) and all $\Psi \in \mathcal{H}$.
Finally let $f_R(|\boldsymbol{x}|)$ denote a $C^\infty-$ function which besides being equal to 1 for $0 \leq |\boldsymbol{x}| \leq R$ and zero for $|\boldsymbol{x}| > R + \Delta R$ satisfies the bound $|R\frac{d}{dr}f_R(r)| \leq c < \infty$ for all $R > 0$. Then the generalization of the Goldstone Theorem [13], [21], [26]-[29] reads:

(Generalized) Goldstone Theorem: The flavor symmetry generated by the conserved current $j^{8,\mu}_{a,\ldots}(x)_{\mathcal{F}}$ is *not* spontaneously broken, that is

$$\lim_{R\to\infty} < \Psi_0, \int d^3\boldsymbol{x} f_R(|\boldsymbol{x}|)[j^{8,0}_{a,\ldots}(x)_{\mathcal{F}}, \phi(f)]_- \Psi_0 >_{|x^0=0} = 0 \tag{38}$$

for all $\phi(f) \in \mathcal{P}$, unless there exists a particle of mass

$$M(a)^2 = p(a)^2 = I^3(a)^2\kappa^2 + 1/4Y(a)^2\omega^2 + I^3(a)Y(a)(\kappa\omega)$$

÷

We present the proof only for testfunction f with compact support. The generalization to all f from the class S is easily achieved along the same line as in the classical case [see Chap. IV, § 1.1 in Ref.[29]].
Following the pattern of Wightman [31] and Vladimirov [32] it can be shown that in consequence of Einstein causality, spectrum condition and translational covariance the vacuum expectation value of the commutator may be represented by the following Jost-Lehmann-Dyson representation:

$$\begin{aligned} &< \Psi_0, [j^{8,\mu}_{a,\ldots}(x)_{\mathcal{F}}, \phi(f)]_- \Psi_0 >= \exp\{-i(p(a)x)\} \times \\ &\times \int d^3\boldsymbol{u} \int_0^\infty ds[\rho^\mu_a(\boldsymbol{u}, s)_1 \Delta_s(x^0, \boldsymbol{x} - \boldsymbol{u}) + \rho^\mu_a(\boldsymbol{u}, s)_2 \tfrac{\partial}{\partial x^0} \Delta_s(x^0, \boldsymbol{x} - \boldsymbol{u})], \end{aligned} \tag{39}$$

with

$$\Delta_s(x) = \frac{1}{i(2\pi)^3} \int d^4k \exp\{-i(kx)\} \epsilon(k^0) \delta(k^2 - s) \tag{40}$$

being the solution of the initial value problem

$$(\Box_x + s)\Delta_s(x) = 0, \quad \Delta_s(x)_{|x^0=0} = 0, \frac{\partial}{\partial x^0}\Delta_s(x)_{/x^0=0} = -\delta(\boldsymbol{x}), \quad s \geq 0 \tag{41}$$

The structure of this representation is determined by the spectrum condition (38) via a solution of a five dimensional wave equation in configuration space. The spectralfunctions $\rho^{\ldots}_{\ldots}(\boldsymbol{u}, s)\ldots$ are strongly decreasing distributions in the variable s and $C^\infty-$ functions in $\boldsymbol{u}$ with compact support as a consequence of Einstein causality. The exponential factor in equation (39) arises from translational covariance.
The most important point for the following is now the application of current conservation $\partial_\mu j^{8,\mu}_{a,\ldots}(x)_{\mathcal{F}} = 0$. By straight forward calculations it leads to the following two relations between the spectral functions:

$$\begin{aligned} &\rho^0_a(\boldsymbol{u}, s)_1 = ip_0(a)\rho^0_a(\boldsymbol{u}, s)_2 + (\tfrac{\partial}{\partial u^j} + ip_j(a))\rho^j_a(\boldsymbol{u}, s)_2 \\ &(\tfrac{\partial}{\partial u^j} + ip_j(a))[\rho^j_a(\boldsymbol{u}, s)_1 + ip_0(a)\rho^j_a(\boldsymbol{u}, s)_2] - (\Delta_{\boldsymbol{u}} - s + p_0(a)^2)\rho^0_a(\boldsymbol{u}, s)_2 = 0 \end{aligned} \tag{42}$$

Here and in the following summation over the Latin double index j from 1 to 3 is understood. Introducing instead of $\rho_a^j(\boldsymbol{u},s)_1$ the new spectral function $\sigma_a^j(\boldsymbol{u},s)$ via

$$\rho_a^j(\boldsymbol{u},s)_1 + ip_0(a)\rho_a^j(\boldsymbol{u},s)_2 = -(\frac{\partial}{\partial u^j} - ip^j(a))\rho_a^0(\boldsymbol{u},s)_2 + \sigma_a^j(\boldsymbol{u},s) \tag{43}$$

the second of the equations (42) reduces to the much simpler one

$$(s - p(a)^2)\rho_a^0(\boldsymbol{u},s)_2 = (\frac{\partial}{\partial u^j} + ip_j(a))\sigma_a^j(\boldsymbol{u},s) \tag{44}$$

with the most general solution [33]

$$\rho_a^0(\boldsymbol{u},s)_2 = (\frac{\partial}{\partial u^j} + ip_j(a))(s - p(a)^2)^{-1}\sigma_a^j(\boldsymbol{u},s) + \delta(s - p(a)^2)E_a(\boldsymbol{u}) \tag{45}$$

$E_a(\boldsymbol{u})$ is a C^∞−function with compact support since all other spectralfunctions share these properties. Feeding the equations (43) and (45) together with the first one of the system (42) back into the representation (39) and integrating by parts over the variables u_j we end up with the following representation for the zeroth component of the commutator matrix element:

$$\begin{aligned}
&< \Psi_0, [j^{8,0}_{a,\ldots}(x)_{\mathcal{F}}, \phi(f)]_- \Psi_0 >= \exp\{-i(p(a)x)\}\times \\
&\times \int d^3\boldsymbol{u}\{E_a(\boldsymbol{u})(ip_0(a) + \tfrac{\partial}{\partial x^0})\Delta_{p(a)^2}(x^0, \boldsymbol{x} - \boldsymbol{u}) - \\
&\int_0^\infty ds[\sigma_a^j(\boldsymbol{u},s)_1 + \sigma_a^j(\boldsymbol{u},s)_2\tfrac{\partial}{\partial x^0}](\tfrac{\partial}{\partial x^j} - ip_j(a))\Delta_s(x^0, \boldsymbol{x} - \boldsymbol{u})\}
\end{aligned} \tag{46}$$

Here the abbreviations

$$\begin{aligned}
\sigma_a^j(\boldsymbol{u},s)_1 &:= \rho_a^j(\boldsymbol{u},s)_2 + ip_0(a)(s - p(a)^2)^{-1}\sigma_a^j(\boldsymbol{u},s) \\
\sigma_a^j(\boldsymbol{u},s)_2 &:= (s - p(a)^2)^{-1}\sigma_a^j(\boldsymbol{u},s)
\end{aligned} \tag{47}$$

have been used and for the partial integration the compact support of the spectral functions. By means of a further partial integration we get from equation (46)

$$\begin{aligned}
&< \Psi_0, \int d^3\boldsymbol{x} f_R(|\boldsymbol{x}|)[j^{8,0}_{a,\ldots}(x)_{\mathcal{F}}, \phi(f)]_- \Psi_0 > - \int d^3\boldsymbol{u} E_a(\boldsymbol{u})\times \\
&\times \int d^3\boldsymbol{x} \exp\{-i(p(a)x)\} f_R(|\boldsymbol{x}|)(ip_0(a) + \tfrac{\partial}{\partial x^0})\Delta_{p(a)^2}(x^0, \boldsymbol{x} - \boldsymbol{u}) = \\
&\int d^3\boldsymbol{x} \exp\{-i(p(a)x)\}(\tfrac{\partial}{\partial x^j} f_R(|\boldsymbol{x}|))\times \\
&\times \int d^3\boldsymbol{u} \int_0^\infty ds[\sigma_a^j(\boldsymbol{u},s)_1 + \sigma_a^j(\boldsymbol{u},s)_2\tfrac{\partial}{\partial x^0}]\Delta_s(x^0, \boldsymbol{x} - \boldsymbol{u})
\end{aligned} \tag{48}$$

Now the further arguments are the same as in the standard case. For all $|x^0|$ smaller than some $\epsilon > 0$ the right hand side vanishes for all R larger than some fixed $R_0(\epsilon)$ as a consequence of the support properties of the spectral functions and the commentator

function $\Delta_s(x)$, since the derivative of f_R is unequal to zero only in the ring region between R and $R + \Delta R$. By the same reasons the second term on the left hand side becomes independent from f_R for $R > R_0(\epsilon)$, that is only the ball $|\boldsymbol{x}| \leq R_0(\epsilon)$ contributes to the $x-$integrations. Then from the initial values (41) we obtain for $x^0 = 0$ and all $R > R_0(\epsilon)$

$$\begin{aligned} &< \Psi_0, \int d^3\boldsymbol{x} f_R(|\boldsymbol{x}|)[j^{8,0}_{a,\ldots}(x)_{\mathcal{F}}, \phi(f)]_- \Psi_0 >_{|x^0=0} = \\ &- \int d^3\boldsymbol{u} \exp\{+i(\boldsymbol{p}(a)\boldsymbol{u})\} E_a(\boldsymbol{u}) = -\tilde{E}_a(\boldsymbol{p}(a)) \end{aligned} \tag{49}$$

Hence the only contribution to the limit (39) comes from the one particle intermediate state in the commutator with the internal quantum numbers $a = (I, I^3, Y)$ of the current and the mass $p(a)^2$.

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