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ON SCALAR FIELD IN MULTIDIMENSIONAL COSMOLOGICAL MODELS

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Abstract

The scalar field theory on the background of cosmological models with $n(n \geq 1)$ spaces of constant curvature is considered. We take the integrable case of Ricci flat internal spaces. The coupling between the scalar and the gravitational fields is described quite general in order to include the minimal coupling as well as the conformal case. In the ground state of the scalar field we find the conditions for spontaneous symmetry breaking realized for most of the possible solutions to Einstein's equations if the coupling parameter takes appropriate values. For the excited states of the scalar field we show the induction of massive modes and discuss their properties.

1 INTRODUCTION

The question for a possible cosmological scenario with a higher dimensional stage in the early time is up to now an actual problem which revived with the unification procedures of supersymmetry or string theory [1]. The main problem consists in the dynamical process leading from a stage with all dimensions developing on the same scale to the actual stage of the universe where we have only four external dimensions and all internal spaces have to be compactified and contracted to unobservable scales. To make the internal dimensions unobservable at the actual stage of the universe we have to demand their contraction to scales near to the Planck length.

In general Kaluza-Klein theories start with a N-dimensional space-time manifold. After compactification the structure of this manifold is reduced to a product manifold of the four dimensional external space-time and one or some compact internal spaces. To make the internal dimensions unobservable the internal spaces have to be reduced to scales near the Planck length [2,3,4,5]. This can be achived by contraction of the internal spaces during cosmological evolution (see e.g. [6]) or by reduction to Planck scales during compactification (see e.g. [7]).

Windows to such higher dimensional stages arise from the defects of topological compactification, e.g. massive monopoles, strings and so on, from the excitations in the internal space which occur in the external space as massive particles (pyrgons [8]), or from the amplification of perturbation modes during the contraction of internal spaces [9] . In any case the final stage has to be described by internal spaces of Planckian size in order to make internal dimensions hidden at actual cosmic time.

From this point of view it is natural to consider the model where all internal spaces are static and their scales are frozen near Planck length. The role of static internal spaces consists in determining physical constants and fundamental interactions in the four-dimensional space-time [5,10,11,12]. Physical constants appearing in four-dimensional space-time are induced by the fundamental constants during compactification [13]. This can be derived by integrating over additional dimensions.

If the internal space has a time dependent scale the values of the physical constants also vary in time. But experiments actually show that physical constants are independent on time. Therefore, all higher dimensional cosmological models have to contain a mechanism to make the internal spaces static ones.

A special type of Kaluza-Klein models contains beside our four dimensional external space-time an arbitrary number of internal spaces of constant curvature which are assumed to be Ricci flat. The advantage of this restriction consists in the possibility to solve the field equations explicitly [14, 15]. This can be also done with the inclusion of a massive scalar field as matter source. New classical solutions are found for the Lorentzian region as well as classical wormhole solutions for the Euclidean one [16]. The canonical quantization of these models is carried out and the Wheeler-DeWitt equation is solved exactly. This results e.g. in quantum wormhole solutions and solutions describing quantum tunneling [17, 18].

In the present paper we restrict these models further to the case of static internal spaces. We shall prove that the internal spaces can be static only in the case they are Ricci flat. But despite the dropping out of internal degrees of freedom the model is non trivial and of considerable predictive power. We can use the classical solutions found for these models as cosmological background.

We discuss the properties of a scalar field developing on this multidimensional cosmological background. To describe the coupling between gravity and matter the additional term ξR is introduced into the scalar field action, where R is the Ricci scalar and ξ is the coupling parameter. For three dimensional space and massless scalar field this gives for $\xi = 1/6$ the conformal coupling. The coupling of a scalar field to the gravitational field can be used to get symmetry breaking effects which are expected to dominate the early stages of the universe. The term ξR can play the same role as the negative square of mass and if a self-interaction term like $\lambda\phi^4$ is added to the Lagrangian of the scalar field the spontaneous symmetry breaking may occur [19 - 22]. Up to now this is done for Friedmann-Robertson-Walker (FRW) metrics in three dimensional spaces [19 - 21]. It was strictly stressed in these papers that spontaneous symmetry breaking takes place only in the case of spaces with negative curvature ($k = -1$) and for conformal coupling ($\xi = 1/6$). In [22] there is a briefly notice about the possibility of symmetry breaking for $\xi < 1/6$ in the cases of $k = 0, +1$ on the example of four dimensional FRW and de Sitter space-times.

We show in this paper for a scalar field with general coupling to the gravitational field (no special value of ξ) the conditions for symmetry breaking to be fulfilled under much more general conditions. In addition we consider symmetry breaking for the special cosmological solutions found for multidimensional space-times. Here we assume the internal spaces to be static ones and prove in an appendix that such internal spaces are Ricci flat. The internal spaces are taken to be compact also for this case what can be achieved by

appropriate periodicity conditions for the space coordinates [23]. We shall see that symmetry breaking is of dynamical character and may take place for most of reasonable cosmological models with appropriately choosen value of ξ. The discussion whether the corresponding state of broken symmetry is really energetically prefered will be postponed to a seperate paper (see section 4).

While the symmetry breaking of Kaluza-Klein models is connected with the ground state of the scalar field, the exited states lead us to a field contribution to the mass not distinguishable from the bare one. These so called pyrgons can give us a window to the higher dimensional stage of our universe. These pyrgons are derived in [24] and further discussed with their cosmological consequences e.g. in [8]. In these papers the pure Kaluza-Klein theory is considered, where an effective matter content appears in the four dimensional reduction. We consider the case of several internal spaces and an additional scalar field on the background of the corresponding classical solutions. So our model is different from those of [8] and [24], but the qualitative answer is the same.

The paper ist organized as followes. After this introduction we describe the cosmological model and derive the field equations in section 2. Excited states and effective mass are discussed in section 3. The symmtry breaking effects we consider in section 4. The conclusions, an appendix and references complete the paper.

2 SCALAR FIELD COUPLED TO GRAVITY IN HIGHER DIMENSIONS

2.1 KALUZA-KLEIN COSMOLOGICAL MODELS

Let us start with an N-dimensional space-time manifold

$$M = R \times M_0 \times \ldots \times M_n \tag{1}$$

containing n compact internal spaces M_i of d_i dimensions. M_0 denotes our three dimensional space, but we do not restrict ourself to this special value and use $d_0 > 1$ as dimension. In general the line element is given by

$$ds^2 = g_{AB} dx^A dx^B \qquad A, B = 0, \ldots, N-1 \tag{2}$$

In our case of homogeneous spaces the line element is given by

$$ds^2 = -dt^2 exp(2\gamma(t)) + \sum_{i=0}^{n} a_i^2(t) ds_i^2 \tag{3}$$

where the $a_i = e^{\beta^i}$ denote the scale factors of the different factor spaces. The spacial metrics of these spaces are given by

$$ds_i^2 = h_{m_i n_i} dx^{m_i} dx^{n_i} \qquad m_i, n_i = 1, \ldots, d_i \tag{4}$$

or with the demand of homogeneity

$$ds_i^2 = \frac{\sum (dx^{n^i})^2}{(1 + \frac{1}{4} k_i \sum (x^{n^i})^2)^2} \tag{5}$$

where $k_i = 0, \pm 1$. The Ricci scalar calculated with this metric reads

$$\begin{aligned} R = e^{-2\gamma} \Bigg(& 2 \sum_{j=0}^{\alpha} d_j \frac{d^2}{dt^2} \beta_j - 2 \frac{d}{dt} \gamma \sum_{j=0}^{\alpha} d_j \frac{d}{dt} \beta_j + \\ & + \textstyle\sum_{j=0}^{\alpha} d_j (\frac{d}{dt} \beta_j)^2 + (\sum_{j=0}^{\alpha} d_j \frac{d}{dt} \beta_j)^2 \Bigg) \\ & + \textstyle\sum_{j=0}^{\alpha} k_j d_j (d_j - 1) e^{-2\beta_j} \end{aligned} \tag{6}$$

The action of the scalar field is taken in the general form

$$S_m = -\frac{1}{2} \int \prod_{A=0}^{D} dx^A (-g)^{\frac{1}{2}} \left[g^{AB} \frac{\partial \phi}{\partial x^A} \frac{\partial \phi}{\partial x^B} - 2V(\beta_j, \phi) \right] \tag{7}$$

with

$$V(\beta_j,\phi) = V(\phi) + \frac{1}{2}\xi R\phi^2 \tag{8}$$

and the usual potential of minimal coupling with the self interaction

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4 \tag{9}$$

The scalar field ϕ may be as well as real as a complex one. In the last case we have to read all expressions $\phi^2 = \phi\phi^*$.

2.2 TOY MODELS WITH FLAT INTERNAL SPACES

In the following discussion we restrict ourself to the case of static internal spaces. In this way all internal degrees of freedom are frozen out and the internal spaces are Ricci flat: $k_j = 0, j > 0$. This model is a particular case of the integrable models with nonstatic Ricci flat internal spaces [17, 18]. Static internal spaces are considered in order to fulfil the condition that internal dimensions are frozen near the Planck scale and not observable at present stage. The flatness of static internal spaces is proven in the appendix.

In this model we get the following expression for the Ricci scalar

$$R = k_0 d_0(d_0-1)e^{-2\beta_0} + e^{-2\gamma}d_0\left[2\frac{d^2}{dt^2}\beta_0 + (\frac{d}{dt}\beta_0)(d_o\frac{d}{dt}\beta_0 - 2\frac{d}{dt}\gamma) + (\frac{d}{dt}\beta_0)^2\right] \tag{10}$$

If we change to the conformal time η by the gauge condition

$$\gamma = \beta_0 \tag{11}$$

we have

$$e^{\gamma} = e^{\beta_0} = a_0 \equiv a \tag{12}$$

and ($\dot{a}$ denotes the derivative with respect to η: $da/d\eta$)

$$R = \frac{k_0 d_0(d_0-1)}{a^2} + \frac{d_0}{a^2}\left[2\frac{\ddot{a}}{a} + (d_0-3)(\frac{\dot{a}}{a})^2\right] \tag{13}$$

Taking the conformally transformed scalar field

$$\phi = \tilde{\phi}a^{\frac{1-d_0}{2}} \tag{14}$$

the action of the scalar field now takes the form (up to a divergency)

$$\begin{aligned} S_m = \frac{1}{2}\int d\eta \prod_j d^{d_j}x|\, h\,|^{\frac{1}{2}} \prod_{j>0} a_j^{d_j}\Big\{ \dot{\tilde{\phi}}^2 - h^{n_0 m_0}\partial_{n_0}\tilde{\phi}\partial_{m_0}\tilde{\phi} \\ -a^2\textstyle\sum_{j>0} a_j^{-2}h^{n_j m_j}\partial_{n_j}\tilde{\phi}\partial_{m_j}\tilde{\phi} - [m^2a^2 + \xi d_0(d_0-1)k_0 + \\ +d_0\left(\xi - \frac{d_0-1}{4d_0}\right)\left(2\frac{\ddot{a}}{a} - (d_0-3)\frac{\dot{a}^2}{a^2}\right)\Big]\tilde{\phi}^2 - \frac{\lambda}{4}a^{3-d_0}\tilde{\phi}^4\Big\} \end{aligned} \tag{15}$$

This leads to the scalar field equation

$$\begin{aligned} \ddot{\tilde{\phi}} - \Delta_2^{(d_0)}\tilde{\phi} - a^2\sum_{j=1}^{n}\frac{1}{a_j^2}\Delta_2^{(d_j)}\tilde{\phi} \\ + \left[m^2a^2 + \xi d_0(d_0-1)k_0 + d_0(\xi - \xi_{conf})\left(2\frac{\ddot{a}}{a} - (d_0-3)\frac{\dot{a}^2}{a^2}\right)\right]\tilde{\phi} \\ +\frac{\lambda}{4}a^{3-d_0}\tilde{\phi}^2\tilde{\phi} = 0 \end{aligned} \tag{16}$$

Here $\Delta_2^{(d_j)}$ denotes the Lapace-Beltrami operator in a d_j dimensional space and $\xi_{conf} = (d_0-1)/4d_0$ is the value of ξ for conformal coupling.

3 THE EXCITED STATES OF THE SCALAR FIELD AND EFFECTIVE MASS

Let us look for solutions of the scalar field equation (16). We denote by y^i the collective spacial coordinates of the ith internal space. Than we make for the scalar field the product ansatz

$$\tilde{\phi} = \varphi(\eta, y^0)Y(y^1)\ldots Y(y^n) \tag{17}$$

We assume that the scalar field on the cosmological background is situated in the eigenstates of the Laplace-Beltrami operators $\Delta_2^{(d_i)}$ for all internal spaces ($i > 0$). So we have

$$\Delta_2^{(d_i)}Y_i = -n_i^2 Y_i \tag{18}$$

We get in this case from (16)

$$-a^2\sum_{i=1}^{n}\frac{1}{a_i^2}\Delta_2^{(d_i)}\varphi + m^2a^2\varphi = a^2\left[m^2 + \sum_{i=1}^{n}\left(\frac{n_i}{a_i}\right)^2\right]\varphi \tag{19}$$

Despite the bare mass m we find the square of the effective mass

$$M_{eff}^2 = m^2 + \sum_{i=1}^{n} \left(\frac{n_i}{a_i}\right)^2 \tag{20}$$

In this way an initially massless scalar field can become a massive one due to the internal field modes. It is important to note that in our case of frozen internal spaces, as it follows from (19), the dynamical behaviour of the induced mass does not differ from that of the bare one.

This is a generalization of the pyrgons found in pure Kaluza-Klein models discussed in [8] and [24] to the case of numerous internal spaces and the presence of a scalar field.

The additional mass of the scalar field $m_{add} \approx 1/a_i$. If we assume the scale factors of the internal spaces to be frozen during compactification on scales between Planck length and a Fermi:

$$l_{pl} < a_i < l_f \tag{21}$$

with $l_{pl} = 10^{-33}cm$, $l_f = 10^{-18}cm$ and we find

$$10^{-20}g < m_{add} < 10^{-5}g \tag{22}$$

that is between 10^{-4} proton masses and the Planck mass.

4 THE GROUND STATE OF THE SCALAR FIELD AND SYMMETRY BREAKING

Symmetry breaking is an effect of the ground state of the scalar field. If we denote the vacuum state $\mid 0 >$ we find from the homogeneity of the spaces that a nonzero vacuum expectation value of the scalar field may depend only on time [19 - 21]

$$< 0 \mid \phi(\eta, \mathbf{x}) \mid 0 > = < 0 \mid \phi(\eta, 0) \mid 0 > \equiv g(\eta) \tag{23}$$

Therefore, taking the vacuum expectation value in equation (16) the Laplace-Beltrami operators drop out. This is equivalent to the statement that the ϕ-field is situated in the ground state. As an result we find an equation for $g(\eta)$ which is the same for the models with and without internal spaces. The influence of internal spaces gives only different behaviour of the scale factor $a_0(\eta)$.

In this paper we only analyze the condition for symmetry breaking. This is fulfilled if for $g(\eta)$ nonzero solution can be exist. The necessary condition is a negative square of the effective mass in the equation (16). Therefore, conditions for symmetry breaking are given by the behaviour of the mass term

$$M^2(\eta, a) = m^2a^2 + \xi d_0(d_0 - 1)k_0 + d_0(\xi - \xi_{conf})\left(2\frac{\ddot{a}}{a} - (d_0 - 3)\frac{\dot{a}^2}{a^2}\right) \tag{24}$$

If this square of the mass term becomes negative we have realized the condition for spantaneous symmetry breaking of global gauge invariance (see [19 - 22]).

Because we reduce our considerations to the case of static internal spaces which do not appear in (24) explicitly only parameters of the external space occur in (24) and we can drop the index "0" without misunderstanding.

Now we look for the conditions in which the square of the effective mass (24) becomes negative. We put $m = 0$ for simplicity. A discussion including a nonzero bare mass does not change the results qualitatively.

We first mention that the usual values for the curvature k and the relation $\frac{\ddot{a}}{a}$ are of opposite sign. Therefore we start with the model of a closed universe and consider a particular case:

I. $k = 1, \frac{\ddot{a}}{a} = -1$

This case is realized for example for the radiation filled closed FRW universe ($d = 3$). With the notation

$$A_d(\eta) = d(d-3)\frac{\dot{a}^2}{a^2} \tag{25}$$

we get

$$A_d(\eta) = \begin{cases} \geq 0 & \text{for } d \geq 3 \\ < 0 & \text{for } d = 2 \end{cases} \tag{26}$$

Than we have the square of the effective mass

$$M^2 = \xi d(d-3) + \frac{1}{2}(d-1) - A_d(\eta) \tag{27}$$

We distinguish three cases (in all of them we take $\xi \geq 0$)

1. $d = 3$
   Here we have $M^2 = 1$ and there is no symmetry breaking at all.

2. $d = 2$
   Symmetry breaking is given for

$$\xi \geq 2\xi_{conf} + \frac{\dot{a}^2}{a^2} = \frac{1}{4} + \frac{\dot{a}^2}{a^2}$$

3. $d > 3$
   The condition for symmetry breaking is given by

$$\xi < \frac{\dot{a}^2}{a^2} - \frac{1}{2}\frac{d-1}{d(d-3)} = \frac{\dot{a}^2}{a^2} - \frac{2\xi_{conf}}{d_0-3}$$

Now we consider the case of an open universe

II. $k = -1, \frac{\ddot{a}}{a} = 1$

This particular case is realized e.g. for an open radiation filled FRW universe ($d = 3$) or for a Milne universe (d arbitrary). The square of the effective mass reads

$$M^2 = -\xi d(d-3) - \frac{1}{2}(d-1) - A_d(\eta) \tag{28}$$

The three cases are (we again take $\xi \geq 0$)

1. $d = 3$
   Here we have $M^2 = -1$ and symmetry breaking for all ξ.

2. $d = 2$
   Symmetry breaking is given for

$$\xi < 2\xi_{conf} - \frac{\dot{a}^2}{a^2} = \frac{1}{4} - \frac{\dot{a}^2}{a^2}$$

3. $d > 3$
   Symmetry breaking takes place for all ξ

More realistic models correspond to $\frac{\ddot{a}}{a} \neq const.$ In this analysis we consider explicitly the most important case $d = 3$. Here we have $\xi_{conf} = 1/6$. The square of the effective mass (24) takes the form

$$M^2(\eta, \xi) = -\frac{\ddot{a}}{a} + \frac{\xi}{\xi_{conf}}\left(k + \frac{\ddot{a}}{a}\right) \tag{29}$$

Let us start with closed models:

III. $k = 1$
The square of the effective mass (29) reads

$$M^2(\eta, \xi) = -\frac{\ddot{a}}{a} + \frac{\xi}{\xi_{conf}}\left(1 + \frac{\ddot{a}}{a}\right) \tag{30}$$

We mention two special properties:

1. In the case of conformal coupling $\xi = \xi_{conf}$ we have $M^2 > 0$ and therefore no symmetry breaking for arbitrary values of $\frac{\ddot{a}}{a}$.

2. For $\frac{\ddot{a}}{a} = -1$ we have $M^2 > 0$ for arbitrary ξ (no symmetry breaking for all ξ).

Therefore, it is natural to make the following decomposition $\frac{\ddot{a}}{a} = -1 \pm \alpha, \alpha > 0$ (where α depends on time) and to discuss the following two cases:

a) $\frac{\ddot{a}}{a} = -1 - \alpha, \alpha > 0$

We find

$$M^2 = \frac{\alpha}{\xi_{conf}}\left(\xi_{conf}\left(1 + \frac{1}{\alpha}\right) - \xi\right) \tag{31}$$

From this equation we get the condition for symmetry breaking

$$\xi > \xi_{conf}\left(1 + \frac{1}{\alpha}\right) \tag{32}$$

and find a lower boundary for ξ in the models with broken symmetry

$$\inf \xi = \xi_{conf}\left(1 + \frac{1}{\alpha}\right) \tag{33}$$

We mention two limits:

$$\begin{array}{lcl} \text{if } \alpha \to 0 & \Longrightarrow & \inf \xi \to \infty, (\xi \gg \xi_{conf}) \\ \text{if } \alpha \to \infty & \Longrightarrow & \inf \xi \to \xi_{conf}(+0) \end{array}$$

b) $\frac{\ddot{a}}{a} = -1 + \alpha, \alpha > 0$

The square of the effective mass is given by

$$M^2 = \frac{\alpha}{\xi_{conf}}\left(\xi_{conf}\left(\frac{1}{\alpha} - 1\right) + \xi\right) \tag{34}$$

The condition for symmetry breaking takes place for

$$\xi < \xi_{conf}\left(1-\frac{1}{\alpha}\right) \tag{35}$$

and the upper boundary for ξ in the models with broken symmetry is

$$\sup\xi = \xi_{conf}\left(1-\frac{1}{\alpha}\right) \tag{36}$$

We consider two different regions for α

1. $0 < \alpha \leq 1$

$$\begin{array}{llll} \text{if } \alpha \to 0 & \left(\frac{\ddot{a}}{a} \to -1\right) & \Longrightarrow & \sup\xi \to -\infty \\ \text{if } \alpha \to 1(-0) & \left(\frac{\ddot{a}}{a} \to -0\right) & \Longrightarrow & \sup\xi \to (-0) \end{array}$$

2. $1 < \alpha$

$$\begin{array}{llll} \text{if } \alpha \to 1(+0) & \left(\frac{\ddot{a}}{a} \to +0\right) & \Longrightarrow & \sup\xi \to +0 \\ \text{if } \alpha \to \infty & \left(\frac{\ddot{a}}{a} \to \infty\right) & \Longrightarrow & \sup\xi \to \xi_{conf}(+0) \end{array}$$

Now, let us consider the case of open space:

IV. $k = -1$

The square of the effective mass (29) reads

$$M^2(\eta,\xi) = -\frac{\ddot{a}}{a} + \frac{\xi}{\xi_{conf}}\left(\frac{\ddot{a}}{a}-1\right) \tag{37}$$

We mention again two special parameter values: $\frac{\ddot{a}}{a} = 1$ and $\xi = \xi_{conf}$. We have the condition $M^2 < 0$ fulfilled in both of these cases. If $\frac{\ddot{a}}{a} = 1$ we have symmetry breaking for all ξ and if $\xi = \xi_{conf}$ we have symmetry breaking for all a. Therefore, we use the decomposition $\frac{\ddot{a}}{a} = 1 \pm \alpha, \alpha > 0$ and discuss the following two cases:

a) $\frac{\ddot{a}}{a} = 1 + \alpha$

We find for M^2

$$M^2 = \frac{\alpha}{\xi_{conf}}\left(\xi - \xi_{conf}\left(1+\frac{1}{\alpha}\right)\right) \tag{38}$$

and get the condition for symmetry breaking

$$\xi < \xi_{conf}\left(1+\frac{1}{\alpha}\right) \tag{39}$$

that means for

$$\sup\xi = \xi_{conf}\left(1+\frac{1}{\alpha}\right) \tag{40}$$

We mention two limits:

$$\begin{array}{llll} \text{if } \alpha \to 0 & \left(\frac{\ddot{a}}{a} \to 1\right) & \Longrightarrow & \sup\xi \to \infty \\ \text{if } \alpha \to \infty & \left(\frac{\ddot{a}}{a} \to \infty\right) & \Longrightarrow & \sup\xi \to \xi_{conf}(+0) \end{array}$$

We find the conditions for symmetry breaking fulfilled for all coupling parameters ξ for which

$$\xi_{conf} \leq \sup\xi < \infty \tag{41}$$

that is for all reasonable theories.

b) $\frac{\ddot{a}}{a} = 1 - \alpha$

The square of the effective mass is given by

$$M^2 = \frac{\alpha}{\xi_{conf}}\left(\xi_{conf}\left(1-\frac{1}{\alpha}\right) - \xi\right) \tag{42}$$

and symmetry breaking takes place for

$$\xi > \xi_{conf}\left(1-\frac{1}{\alpha}\right) \tag{43}$$

that means for

$$\inf\xi = \xi_{conf}\left(1-\frac{1}{\alpha}\right) \tag{44}$$

We mention two limits:

$$\begin{array}{llll} \text{if } \alpha \to 0 & \left(\frac{\ddot{a}}{a} \to 1\right) & \Longrightarrow & \inf\xi \to -\infty \\ \text{if } \alpha \to \infty & \left(\frac{\ddot{a}}{a} \to -\infty\right) & \Longrightarrow & \inf\xi \to \xi_{conf}(-0) \end{array}$$

We find the conditions for symmetry breaking fulfilled for all coupling parameters ξ for which

$$-\infty < \inf \xi \leq \xi_{conf} \tag{45}$$

that is for a large class of possible coupling parameters. With $0 \leq \alpha \leq 1$ we have $-\infty < \inf \xi \leq 0$ and this includes all reasonable theories.

In this general discussion we found the condition for symmetry breaking realized for almost all theoretical models with appropriate value of the coupling parameter ξ. This investigation is based on the analysis of the conformal scalar field. In order to check whether the state of broken symmetry is really realized we would have to show that this state is energetically preferred. To do this we have to calculate the 00-component of the energy-momentum-tensor of the scalar field and to show that the energy of the state with broken symmetry is less than that with unbroken symmetry. This procedure depends on the choice of the energy-momentum-tensor (metric or canonical one) and may lead to a time shift for the state of broken symmetry (see [19]). This has to be done for any model given by a solution $a(\eta)$, seperately.

In the present paper we only show the realization of the conditions for symmetry breaking for large class of models. The energy content we consider only for the two special cases of a massless scalar field in an open $k = -1$ three dimensional space $d = 3$:

$$1. \quad \frac{\ddot{a}}{a} = 1 \quad \text{and} \quad 2. \quad \xi = \xi_{conf}$$

It is easy to show that in the first case for arbitrary ξ and in the second case for arbitrary value of $\frac{\ddot{a}}{a}$ the energy density in the state of broken symmetry derived from metric energy-momentum-tensor is equal to

$$\varepsilon(\eta) = -\frac{1}{2\lambda a^4} < 0 \tag{46}$$

in agreement with [19]. For all times this state is prefered with respect to the state of unbroken symmetry for which the energy density $\varepsilon(\eta) = 0$.

After this general consideration we discuss the properties of $M^2(\eta, \xi)$ for different classical solutions $a(\eta)$. We do this for two cases. At first we take the usual Friedmann model without internal spaces. After that we consider the model with an arbitrary number of static and Ricci flat internal spaces and use the solutions specific for this model derived in [17, 18].

4.1 SYMMETRYBREAKING FOR FRIEDMANN TYPE UNIVERSES

We now consider Friedmann models and start with the case of a closed dust filled universe (case III). In this case the Friedmann solution is given by

$$a(\eta) = a_0(1 - \cos\eta), \qquad 0 \leq \eta \leq 2\pi \tag{47}$$

and we have

$$\frac{\ddot{a}}{a} = \frac{\cos\eta}{1 - \cos\eta} \tag{48}$$

We have the following range of parameters:

$$0 \leq \eta \leq 2\pi \qquad \Longrightarrow \qquad -\tfrac{1}{2} \leq \tfrac{\ddot{a}}{a} \leq \infty$$

where the singularity ($\frac{\ddot{a}}{a} \to \infty$) corresponds to $\eta = 0$ and the value $\frac{\ddot{a}}{a} = -\frac{1}{2}$ corresponds to the point of maximal expansion $\eta = \pi$. Therefore we have the case III.b) of the previous section:

$$\frac{\ddot{a}}{a} = -1 + \alpha \qquad \Longrightarrow \qquad \alpha = \frac{1}{1 - \cos\eta} \geq \frac{1}{2} \tag{49}$$

For realistic models we have to assume

$$\sup \xi > 0 \tag{50}$$

This is given for $\alpha > 1$ what means $\frac{\ddot{a}}{a} > 0$. To realize these values we need $\eta < \frac{\pi}{2}$ or $\eta > \frac{3}{2}\pi$. The actual state of the universe is described by the following value [25]:

$$\eta_0 = 1,975 rad = 0,63\pi > 0.5\pi$$

It follows that in all models with reasonable coupling symmetry is restored at present stage of the universe.

Now we discuss the open Friedmann model (case IV. of previous section) for which the solution is given by

$$a(\eta) = a_0(\cosh\eta - 1), \qquad -\infty \leq \eta \leq \infty \tag{51}$$

For this model we have

$$\frac{\ddot{a}}{a} = \frac{\cosh\eta}{\cosh\eta - 1} = 1 + \alpha \tag{52}$$

with

$$\alpha = \frac{1}{\cosh\eta - 1} \tag{53}$$

and the range of values

$$0(\eta \to \pm\infty) \leq \alpha < \infty(\eta \to 0)$$

This corresponds to case IV.a) of the general discussion and we get for the coupling parameter

$$\xi_{conf} \leq \sup\xi < \infty \tag{54}$$

As a result we find that the class of theories for which the conditions of symmetry breaking is fulfilled increases with proceeding expansion. Shortly after starting expansion almost all reasonable theories are included.

For the present stage we have

$$\eta_0 = 1,975 rad = 0,63\pi \qquad \Longrightarrow \qquad \alpha_0 = 0.37$$

and symmetry breaking will take place for the theories with

$$-\infty < \xi_0 < 3.7\xi_{conf}$$

That means that in this case almost all models fulfill the condition for symmetry breaking at present time. The upper limit tends to infinity with proceeding expansion.

4.2 MODELS WITH RICCI FLAT INTERNAL SPACES

Let us now consider the case of our multidimensional model with static internal spaces. As it is shown in the appendix the scale factors of the internal space may be static only in the case they are Ricci flat.

In [16, 17] the solution of the model with d-dimesnional external space of positive constant curvature and Ricci flat internal spaces is found to be

$$a = a_0\,[sin(d-1)\eta]^{\frac{1}{d-1}} \tag{55}$$

From this solution we find the expression

$$\frac{\ddot{a}}{a} = -1 + \frac{2-d}{[sin(d-1)\eta]^2} \tag{56}$$

For $d = 2$ we have the case I.2. In the case $d = 3$ equation (56) is reduced to

$$\frac{\ddot{a}}{a} = -1 - \frac{1}{[sin2\eta]^2} \tag{57}$$

We are here in the situation of case III.a) of the previous subsection. The region for the change of α is given by

$$1 \leq \alpha < \infty \tag{58}$$

From the condition for symmetry breaking

$$\xi > \xi_{conf}\left(1 + \frac{1}{\alpha}\right) \tag{59}$$

we find it fulfilled for arbitrary α if

$$\xi > 2\xi_{conf} \tag{60}$$

but for α tending to infinity we have $\xi > \xi_{conf}$. Using for the minimal possible value of ξ the notation ξ_{inf} we can write

$$\xi_{conf} < \inf\xi < 2\xi_{conf} \tag{61}$$

For all models with coupling parameter $\inf\xi < \xi < \infty$ we find the conditions for symmetry breaking fulfilled.

Let us now turn to open models (case IV. of the preveous section). Here we have two different solutions. One solution has a cosmological singularity. The other one has no singularity but a turning point and dexcribes a universe with tunneling process [16,18].

Multicomponent model with singularity:

The solution is given by

$$a = a_0\,[\sinh(d-1)\eta]^{\frac{1}{d-1}}, \qquad 0 \leq \eta \leq \infty \tag{62}$$

In the case $d = 2$ this corresponds to II.2. For $d = 3$ we have

$$a = a_0\,[\sinh 2\eta]^{\frac{1}{2}} \tag{63}$$

From this solution we get

$$\frac{\ddot{a}}{a} = 1 - \frac{1}{(\sinh 2\eta)^2} \tag{64}$$

and we have an example of case IV.b) of the previous section with

$$\alpha = \frac{1}{(\sinh 2\eta)^2} \tag{65}$$

and the range of values

$$0(\eta \to \infty) \leq \alpha \leq \infty(\eta \to 0)$$

This gives us the result that the conditions for symmetry breaking are fulfilled for all models with

$$-\infty < \inf \xi \leq \xi_{conf}$$

After the moment of time with $\alpha = 1 \Longrightarrow \sinh 2\eta = 1$ symmetry breaking is possible for all reasonable theories because of $-\infty < \inf \xi \leq 0$.

Multicomponent model with turning point:

The solution is given by

$$a = a_0 \left[\cosh(d-1)\eta\right]^{\frac{1}{d-1}}, \qquad -\infty \leq \eta \leq \infty \tag{66}$$

The case $d = 2$ is again described by II.2. For $d = 3$ we have

$$a = a_0 \left[\cosh 2\eta\right]^{\frac{1}{2}} \tag{67}$$

From this solution we get

$$\frac{\ddot{a}}{a} = 1 + \frac{1}{(\cosh 2\eta)^2} \tag{68}$$

and we have an example of case IV.a) of the previous section with

$$\alpha = \frac{1}{(\cosh 2\eta)^2} \tag{69}$$

and the range of values

$$0(\eta \to \pm\infty) \leq \alpha \leq 1$$

This gives us the result that the conditions for symmetry breaking are fulfilled for all models with

$$2\xi_{conf} \leq \sup \xi < \infty(\eta \to \pm\infty)$$

As a result we find that for all values of η the conditions for symmetry breaking are fulfilled for all reasonable theories.

5 CONCLUSIONS

As the result of the above discussion we find that the special results found so far for a scalar field conformally coupled to gravity in a three dimensional cosmological background [19 - 22] can be generalized to arbitrary values of the coupling parameter and to multidimensional cosmological models. We found that for open spaces there are practically no restrictions for the conditions for symmetry breaking to be fulfilled. For closed spaces the coupling parameter has to take appropriate values. These values may depend on time and symmetry breaking appears as a dynamical process. So we found the symmetry restored for the actual stage of a dust filled closed Friedmann universe.

A completion of this discussion has to contain the prove that symmetry breaking is really realized in all cases the conditions for it are fulfilled. This is demonstrated for two particular cases. In all other cases this analysis should be performed separately. This paper was devoted to show the general possibility of symmetry breaking for a large class of cosmological models.

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6 APPENDIX: STATIC INTERNAL SPACES AF RICCI FLAT

This appendix is denoted to the proof of the following

THEOREM:
The Einstein gravitational field equations in vacuum for multidimensional cosmological models with an external space of constant curvature and $n \geq 1$ static internal spaces described by the metric (3) ($\beta_i = const, i = 1 \ldots n$) are compatible only if these internal spaces are Ricci flat.

The gravitational action S for the metric frw can be written in harmonic

time gauge ($\gamma = 0$) in the form [14]

$$S = \int \mathcal{L} dt \tag{70}$$

where

$$\mathcal{L} = \frac{\mu}{2} G_{ij} \dot{\beta}^i \dot{\beta}^j - \frac{\mu}{2} V \tag{71}$$

The metric of minisuperspace reads

$$G_{ij} = d_i \delta_{ij} - d_i d_j \tag{72}$$

and the potential is given by

$$V = -e^{2\sum_{i=0}^{n} d_i \beta^i} \sum_{i=0}^{n} \Theta_i e^{-2\beta^i} \tag{73}$$

μ is proportional to the product of the volumes of spaces with constant curvature. By $\Theta_i = k_i d_i (d_i - 1)$ we denote the curvature of the ith internal space. The Einstein field equations read

$$G_{ij} \ddot{\beta}^j = -\frac{1}{2} \frac{\partial}{\partial \beta^i} V \tag{74}$$

With our potential (73) we find

$$G_{ij} \ddot{\beta}^j = e^{2\sum_j d_j \beta^j} \left\{ (d_i - 1) \Theta_i e^{-2\beta^i} + d_i \sum_{k \neq i} \Theta_k e^{-2\beta^k} \right\} \tag{75}$$

With static internal spaces we have to put $\ddot{\beta}^i = 0, i \geq 1$. Then we have $n+1$ equations for β^0 which have to be compatible. The equations are

for $i = 0$

$$\ddot{\beta}^0 = e^{2\sum_j d_j \beta^j} \left\{ -\frac{\Theta_0}{d_0} e^{-2\beta^0} - \frac{1}{d_0 - 1} \sum_{k \neq i} \Theta_k e^{-2\beta^k} \right\} \tag{76}$$

and for $i \neq 0$

$$\ddot{\beta}^0 = e^{2\sum_j d_j \beta^j} \left\{ -\frac{(d_i - 1)}{d_0 d_i} \Theta_i e^{-2\beta^i} - \frac{1}{d_0} \sum_{k \neq i} \Theta_k e^{-2\beta^k} \right\} \tag{77}$$

The factors in front of $e^{-2\beta^j}, j = 1, \ldots, n$ have to coincide in all equations. But these factors are

from eqn. (76): $-\frac{1}{(d_0-1)} \Theta_j$
from eqn. (77): $-\frac{1}{d_0} \Theta_j$ for $i \neq j$
from eqn. (77): $-\frac{(d_i-1)}{d_0 d_i} \Theta_j$ for $i = j$

These three coefficients can be equal only in the case $\Theta_j = 0$, that means all internal spaces are Ricci flat.

But the demand to all internal spaces to be Ricci flat does in general not lead to the static case ($\dot{\beta}^i = 0, i \geq 1$). To show this we put all $\Theta_j = 0$ and find the field equation with $i, j \neq 0$

$$\ddot{\beta}^0 + \frac{G_{ij}}{G_{j0}} \ddot{\beta} j = -\frac{\Theta_0}{d_0} e^{2[(d_0-1)\beta_0 + \sum_{j=1}^{n} d_j \beta^j]} \tag{78}$$

This gives us the equation

$$\ddot{\beta}^j = 0 \tag{79}$$

with the solution $\beta^j = c^j \tau + d^j$ but the static solution is contained in this set.

The conditions of this theorem do not change if we minimally couple to the gravitational field a massless scalar field as a source. The solutions discussed above as cosmological background contain also this case. But we have to distinguish between the scalar field as matter source of the cosmological model and the scalar field developing like perturbations on this cosmical background.

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