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Universal Fermi Constant and Final State Interaction in Weak Decays based on Null Instanton Model

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Universality of Fermi constant in weak leptonic, semileptonic decays of mesons and baryons have been studied, by taking the final state interaction based on a null-instanton model into account.

$.1 Introduction

A universality of Fermi constant is one of the main ingredients in weak interaction. The parity violation in weak interaction of Lee-Yang and the V-A type proposed by Feynman-Gell-Mann and Marshak-Sudershan have been unified with electromagnetic interaction by Weinberg-Salam as an electroweak interaction, known as, the standard model. The predicted Z and W boson states are well established experimentally and a detailed information on particle interaction is available. The weak mixing angle introduced by Cabibbo, Kobayashi and Maskawa (CKB) in relation to the CP and T violation is in a current study. Its lower and upper bound as well as those for quark masses have been tabulated by Particle Data Group.[1] The details of theoretical development of quantum field theory in this century and some typical applications will be found in a recent books written by Weinberg.[2]

The purpose of this paper is to seek how much the universality of the Fermi constant is certified by assuming the final state interaction based on a null-instanton model under a proposed interaction.[3]

The radiative correction to Fermi constant in baryon semileptonic decays and relevant ideas are learnt from the Lee's book[4] and the calculational details are based on the Källén's one.[5] The processes studied include leptonic and semileptonic decays of mesons and the latter ones for baryons and the Fermi type beta decays. We have studied corrections arising from the intra- and inter-nucleons in the null-instanton model as a pre and a final

state interaction, respectively, so as to guarantee the universality of the Fermi constant..

In order to find ps-meson leptonic decay parameters we have extended the effective field theories based on chiral $SU_3 \times SU_3$ symmetry, which is recommended to apply usually under the pions as Goldstone bosons based on Nambu's idea, to the $SU_6 \times SU_6$ symmetry straightforwardly. Here the use is made of SU_4, SU_5 and SU_6 generators,[6),7)] and additional parameters F_D, F_B and F_T are introduced for c, b and t associated ps-mesons. Three mass sum rules each for the three higher symmetry groups have derived after eliminating inverse square of these parameters joined linearly with the common constant arising from vacuum expectation value of quark bilinear fields and linear sum of relevant quark masses in the mass square of individual ps-meson masses. It is found that except for $SU_4 \times SU_4$, one each in the mass formulae in $SU_5 \times SU_5$ and $SU_6 \times SU_6$ is satisfied extremely good. In the second case we put all masses of T mesons with different quark partners to the t-quark mass. The other mass sum rule predict the η_b meson mass in the range 6985-7310 MeV which becomes 6496-6993 MeV if we use the η_b and B^0 mass ratio and quark masses as inputs. In the former the observed B_C mass[8] is used by neglecting experimental error as one of the inputs. The predicted η_b mass is anomalously lower than the one expected from QCD. The mass of B_C becomes 5733-6277 MeV from bounds of quark masses which stays within an experimental error. The upper and lower bounds F_X (X=D, B) fixed from quark masses explains consistently the observed ps-meson leptonic decay rates with the observed ones. If we rely on these predictions, even if their partial success, there will be some physical reason to support our approach beyond taking the pions as Goldstone bosons.

In both semileptonic decays of baryons and the pure Fermi type beta transitions of the nuclei the universality of the Fermi constant can be realized by the correction terms arising from the final state interaction in individual decay. Here the experimental information on beta decays on nuclear states[9] and the available maximal energies[10] are estimated by a joint use of available tables in them.

The null instanton effect which arises from an exchange interaction between a pair of quarks in intrahadron and between

that in internucleon ones. Here we assumed a single quark carries the null-instanton number, while an each member of a pair of quarks can carry non-zero instanton number by keeping the total nullity of it in addition. It is known that the null-instanton interaction has a strong mutual distance dependence and diminishes quickly with it.[1)] The main purpose of this paper is to estimate that effect from the typical transient processes.

$.2 is devoted to the derivation of the mass formulae, the associated sum rules and leptonic decay parameters for ps-mesons. In $.3 the leptonic and the semileptonic decays of ps-mesons are discussed in terms of the parameters found in $.2 and correction arising from final state interaction, respectively. $.4 a D and F parameters on baryon semileptonic decays in SU_3 symmetry are fixed first from the observed ratio g_A/g_V at zero momentum transfer, then they are used to find out correction from estimated baryon matrix element in each decay process. In $.5 the pure Fermi type beta decay is treated so as to find out the correction arising from final state interaction between quarks in the parent nucleus and those belonging to the residual nucleus. We are succeeded to separate out the effect arising from each subshell nucleons in an appropriate approximation. Finally $.6 summarizes the results. In Appendix the numerical results of integrals relevant to meson semileptonic decays are summarized for convenience.

$. 2 PS-Meson Mass Formulae and relevant Parameters

A straightforward generalization of effective $SU_3 \times SU_3$ field theories of Weinberg[2)] can be made to a $SU_6 \times SU_6$ ones, where we have used the same convention as far as possible. It is too big to spell out the full matrix

$$\sum_a \lambda_a \xi_a = (\sqrt{2}/F)\, B \tag{2.1}$$

of interests for a larger group, so we shall list first the new diagonal matrix elements, viz., 11, 22, 33, 44, 55, 66 elements and new upper half non-diagonal ones of B, viz.,14, 15, 16, 24, 25, 26, 34, 35, 36, 45, 46, 56 ones for convenience. The diagonal parts of B are given by

$$B_{11} = \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 + \frac{1}{\sqrt{12}}\eta_c + \frac{1}{\sqrt{20}}\eta_b + \frac{1}{\sqrt{30}}\eta_t\ , \tag{2.2}$$

$$B_{22} = -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta^0 + \frac{1}{\sqrt{12}}\eta_c + \frac{1}{\sqrt{20}}\eta_b + \frac{1}{\sqrt{30}}\eta_t\ , \tag{2.3}$$

$$B_{33} = -\sqrt{\frac{2}{3}}\eta^0 + \frac{1}{\sqrt{12}}\eta_c + \frac{1}{\sqrt{20}}\eta_b + \frac{1}{\sqrt{30}}\eta_t\ , \tag{2.4}$$

$$B_{44} = -\frac{\sqrt{3}}{2}\eta_c + \frac{1}{\sqrt{20}}\eta_b + \frac{1}{\sqrt{30}}\eta_t\ , \tag{2.5}$$

$$B_{55} = -\frac{2}{\sqrt{5}}\eta_b + \frac{1}{\sqrt{30}}\eta_t\ , \tag{2.6}$$

and

$$B_{66} = -\sqrt{\frac{5}{6}}\eta_t\ , \tag{2.7}$$

while the nondiagonal ones become

$$(B_{14}, B_{15}, B_{16}) = (D^0, B^+, T^0)\ , \tag{2.8}$$

$$(B_{24}, B_{25}, B_{26}) = (D^+, B^0, T^+)\ , \tag{2.9}$$

$$(B_{34}, B_{35}, B_{36}) = (D_s^+, B_s^0, T_s^+)\ , \tag{2.10}$$

$$(B_{45}, B_{46}) = (B_c^+, T_c^0)\ , \tag{2.11}$$

and

$$B_{56} = T_b^+\ . \tag{2.12}$$

Here the symbols D, B and T on the right hand side mean the corresponding meson wave functions. Their charge conjugated states are obtained by changing order of subscripts on the left hand side.

The Lagrangian of the mass term is written in terms of quark field $\bar{q}(x)$ by[2)]

$$L_{mass} = -\bar{q}M_q q = -\bar{\tilde{q}}\ \exp(-i\sqrt{2}\gamma_5 B/F_\pi)\ M_q\ \exp(-i\sqrt{2}\gamma_5 B/F_\pi)\ \tilde{q}\ , \tag{2.13}$$

where M_q is the diagonal matrix specified by quark masses:

$$\mathrm{diag}[M_q] = (m_u, m_d, m_s, m_c, m_b, m_t). \tag{2.14}$$

According to Weinberg, Eq.(2.12) contains a purely bosonic part, by replacing the bilinear quark fields with its vacuum expectation value, given by the unbrolen SU_3 and parity symmetries as

$$<\bar{\tilde{q}}_n \gamma_5 \tilde{q}_m>_0 = 0, \qquad <\bar{\tilde{q}}_n \tilde{q}_m>_0 = -v\delta_{nm}\ . \tag{2.15}$$

The meson mass term in the Lagrangian is then obtained by evaluating

$$-\frac{v}{F^2}Tr\{B,\{B,M_q\}\} = -v(\frac{1}{F_\pi^2}\ SU_3\times SU_3 \text{associated term} + \frac{1}{F_D^2}\ \text{charm-associated term} + \frac{1}{F_B^2}\ \text{bottom-associated term} + \frac{1}{F_T^2}\ \text{top-associated term})\ , \tag{2.16}$$

where the mixes are not spelled out explicitly (see, later discussion). For the sake of completeness, we shall list first $SU_3\times SU_3$ mass formula in addition to new mass formula with adding correction Δ arising from electromagnetism into account. They become

$$m_{K^+}^2 = 4v(m_u + m_s)/F_\pi^2 + \Delta\ ,\quad m_{K^0}^2 = 4v(m_d + m_s)/F_\pi^2\ ,$$

$$m_{\pi^+}^2 = 4v(m_d + m_u)/F_\pi^2 + \Delta\ ,\quad m_{\pi^0}^2 = 4v(m_d + m_u)/F_\pi^2\ ,$$

$$m_\eta^2 = 4v(m_u + m_d + 4\ m_s)/F_\pi^2\ . \tag{2.17}$$

The additional mass formulas in $SU_4\times SU_4$ symmtry with Coulomb correction become

$$m_{D^+}^2 = 4v(m_d + m_c)/F_D^2 + \Delta\ ,\quad m_{D^0}^2 = 4v(m_u + m_c)/F_D^2\ ,$$

$$m_{D_s^+}^2 = 4v(m_s + m_c)/F_D^2 + \Delta\ ,\quad m_{\eta_c}^2 = \frac{2v}{3F_D^2}(m_u + m_d + m_s + 9m_c). \tag{2.18}$$

The $SU_5\times SU_5$ symmetry adds 5 more formulae as

$$m_{B^+}^2 = 4v(m_u + m_b)/F_B^2 + \Delta\ ,\quad m_{B^0}^2 = 4v(m_d + m_b)/F_B^2\ ,$$

$$m_{B_s^0}^2 = 4v(m_s + m_b)/F_B^2\ ,\quad m_{B_c^+}^2 = 4v(m_c + m_b)F_B^2 + \Delta\ ,$$

$$m_{\eta_b}^2 = \frac{2v}{5F_B^2}(m_u + m_d + m_s + m_c + 16m_b). \tag{2.19}$$

Finally $SU_6\times SU_6$ adds 6 more mass formulae, viz.,

$$m_{T^+}^2 = 4v(m_d + m_t)/F_T^2 + \Delta\ ,\quad m_{T^0}^2 = 4v(m_u + m_t)/F_T^2\ ,$$

$$m_{T_s^+}^2 = 4v(m_s + m_t)/F_T^2 + \Delta\ ,\quad m_{T_c^0}^2 = 4v(m_c + m_t)/F_T^2\ ,$$

$$m_{T_b^+}^2 = 4v(m_b + m_t)/F_T^2 + \Delta\ ,\quad \text{and}$$

$$m_{\eta_t}^2 = \frac{4v}{15F_t^2}(m_u + m_d + m_s + m_c + m_b + 25m_t). \tag{2.20}$$

In the derivation of mass sum rule we neglect contribution arising from the non-diagonal mass matrix, viz., the mixes. The maximal effects from them become

$$m_{\eta\eta_c}^2 = \frac{2\sqrt{2}v}{3F_\pi F_D}(m_u + m_d - 2m_s),\quad m_{\eta_c\eta_b}^2 = \frac{2v}{\sqrt{15}F_D F_B}(m_u + m_d + m_s - 3m_c)$$

and

$$m_{\eta_b\eta_t}^2 = \frac{2\sqrt{2}v}{5\sqrt{6}F_B F_T}(m_u + m_d + m_s + m_c - 4m_b)\ , \tag{2.21}$$

in $SU_4 \times SU_4$, $SU_5 \times SU_5$ and $SU_6 \times SU_6$ symmetry, respectively. These mixes contribute solely to the diagonal neutral meson mass part which does not concern to the leptonic decays of interest.

The mass sum rules in $SU_4 \times SU_4$ can be obtained by eliminating m_u, m_d, m_s, m_c, Δ, v/F_π^2 and v/F_D^2 from first 4 equations in Eqs.(2.16) and a set of 4 ones in Eqs.(2.17). They are

$$(5m_{K^0}^2-4m_{K^+}^2+4m_{\pi^+}^2-8m_{\pi^0}^2)m_{D_s^+}^2 = -(4m_{K^+}^2+4m_{K^0}^2-4m_{\pi^+}^2-m_{\pi^0}^2)m_{D^0}^2+ 6(m_{K^0}^2-m_{\pi^0}^2)m_{\eta_c}^2+(m_{\pi^+}^2-m_{\pi^0}^2)(-4m_{K^+}^2+5m_{K^0}^2+4m_{\pi^+}^2-8m_{\pi^0}^2),$$

$$(7.583\times10^{11} = 5.725\times10^{12})MeV^4, \qquad (2.22a)$$

$$(4m_{K^+}^2-4m_{\pi^+}^2+3m_{\pi^0}^2)m_{D^+}^2 = (m_{K^+}^2-m_{\pi^+}^2)(6m_{\eta_c}^2-m_{D_s^+}^2)-(m_{K^+}^2-m_{\pi^+}^2)(4m_{D^0}^2-5m_{\pi^+}^2) +(3m_{D_s^+}^2-5m_{K^+}^2+5m_{\pi^+}^2),$$

$$(3.325\times10^{12} = 8.172\times10^{12})MeV^4, \qquad (2.22b)$$

and

$$5(m_{K^0}^2-m_{\pi^0}^2)m_{D^+}^2 = 6(m_{K^0}^2-m_{\pi^0}^2)m_{\eta_c}^2-(m_{K^+}^2-m_{\pi^+}^2-3m_{\pi^0}^2)m_{D_s^+}^2-(4m_{K^0}^2-m_{K^+}^2)m_{D^0}^2- (m_{\pi^+}^2-m_{\pi^0}^2)(m_{D^0}^2-m_{K^+}^2+m_{\pi^+}^2+8m_{\pi^0}^2)-5m_{K^0}^2m_{\pi^0}^2,$$

$$(4.009\times10^{12} = 8.949\times10^{12})MeV^4 . \qquad (2.22c)$$

The numerical result shows that the sum rules are not good in this intermediate symmetry region as one expects from the beginning. This anticipation does not seem be continued absolutely to higher symmetries as we shall show below.

Joining $\Delta = m_{\pi^+}^2 - m_{\pi^0}^2$ with 5 mass formulae in Eqs.(2.18) with the first 3 ones in (2.17) one can get mass sum rules by eliminating 7 parameters for quark u, d, s, c, b masses and v/F_D^2 and v/F_B^2. One finds

$$(m_{D_s^+}^2 - m_{D^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2)m_{B^0}^2 = (m_{D_s^+}^2 - m_{D^+}^2)m_{B^+}^2 + (m_{D^+}^2 - m_{D^0}^2)m_{B_s^0}^2 - (m_{\pi^+}^2 - m_{\pi^0}^2)(m_{B_s^0}^2 + m_{D_s^+}^2 - m_{D^+}^2),$$

$$(1.10643\times10^{13} = 1.10783\times10^{13})MeV^4 , \qquad (2.23a)$$

where the l.h.s. and the r.h.s. coincides up to 4 significant numbers. Two more sum rules are obtained by the above procedure. The first simplest one becomes

$$0 = (-7m_{D^+}^2 + m_{D^0}^2)m_{B^+}^2 + (7m_{D^0}^2 - m_{D^+}^2)m_{B^0}^2 + (m_{\pi^+}^2 - m_{\pi^0}^2)(7m_{B^+}^2 + m_{B^0}^2 - 6m_{\pi^+}^2 + 6m_{\pi^0}^2),$$

$$(0 = -3.5960\times10^{12})MeV^4, \qquad (2.23b)$$

where it looks inconsistent numerically. One finds that the large numerical value on the r.h.s. is a little more than two order of magnitude reduced value after a cancellation between the first and second term on the first equality. It contains a reduction of an electromagnetic order. The third one is

$$7(m_{D_s^+}^2 - m_{D^0}^2 - m_{\pi^+}^2 - m_{\pi^0}^2)m_{B_c^+}^2 = (m_{D^+}^2 - 8m_{D_s^+}^2 + m_{D^0}^2)m_{B^+}^2 + (8m_{D^0}^2 - m_{D^+}^2 - m_{D_s^+}^2)m_{B_s^0}^2 + 10(m_{D_s^+}^2 - m_{D^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2)m_{\eta_b}^2 + (m_{\pi^+}^2 - m_{\pi^0}^2)(7m_{B^+}^2 + 2m_{B_s^0}^2 + 8m_{D_s^+}^2 - m_{D^+}^2 - m_{D^0}^2 - 7m_{\pi^+}^2 + 7_{\pi^0}^2) \qquad (2.23c)$$

From this one finds m_{η_b} = 6985 MeV. From a relation

$$\frac{m_{\eta_b}^2}{m_{B^0}^2} = \frac{m_u + m_d + m_s + m_c + 16m_b}{5(m_d + m_b)} \qquad (2.24)$$

the lower and upper bound of m_{η_b} is found to be 6496 and 6993 MeV, respectively, which are consistent with the one estimated from the sum rule. However, it should be observed close to the lowest Υ mass if one rely on QCD. One more interesting test is the observed B_c meson mass. From

$$\frac{m_{B_c^+}^2}{m_{B^0}^2} = \frac{m_c + m_b}{m_d + m_b} \qquad (2.25)$$

the lower and upper bound of $m_{B_c^+}$ are estimated to be 5733 and 6277 Mev. Experimentally it[8] is reported to be 6.40± 0.39 (stat.)± 0.13 (syst.) GeV.

Replacing Δ by pion masses 6 equations in Eqs.(2.19) and the first 3 equations in Eqs.(2.18) have been coupled in order to remove 8 parameters, 6 u, d, s, c, b, t quark masses, v/F_T^2 and v/F_B^2 in order to find additional mass sum rules in $SU_6 \times SU_6$. Three sum rules are found to be

$$(m_{B_s^0}^2 - m_{B^-}^2 + m_{\pi^-}^2 - m_{\pi^0}^2)m_{T^-}^2 = (m_{\pi^-}^2 - m_{\pi^0}^2)(m_{B_s^0}^2 - m_{B^0}^2 + m_{T_s^-}^2) + (m_{B^-}^2 - m_{B^0}^2)m_{T_s^-}^2 + (m_{B_s^0}^2 - m_{B^0}^2)m_{T^0}^2 ,$$

$$(3.1229 \times 10^{10} - 3.1024 \times 10^{10}) GeV^2 . \qquad (2.26a)$$

In this numerical work all top associated meson masses are assumed to be degenerated with the observed t-quark mass. The sum rule is satisfied unexpectedly well as one did not expect at the beginning. Two more analogous sum rules as Eqs.(2.22b) and (2.22c) are given by

$$0 = (m_{\pi^-}^2 - m_{\pi^0}^2)(-23m_{B^-}^2 + 13m_{B^0}^2 - m_{T_c^0}^2 - 11m_{T^-}^2 - m_{T_s^-}^2 - m_{T^0}^2 + 23m_{\pi^-}^2 - 23m_{\pi^0}^2) + (m_{T_c^0}^2 + 11m_{T^-}^2 + m_{T^0}^2 + m_{T_s^-}^2)m_{B^-}^2 - (m_{T_c^0}^2 + 11m_{T^0}^2 + m_{T^-}^2 + m_{T_s^-}^2)m_{B^0}^2 , \qquad (2.26b)$$

where the nullity is supported by an electromagnetic order relative to the m_t^2, and

$$(m_{B_s^0}^2 - m_{B^-}^2 + m_{\pi^-}^2 - m_{\pi^0}^2)(15m_{\eta_b}^2 - 11m_{T_b^-}^2) = (m_{\pi^-}^2 - m_{\pi^0}^2)(m_{T_c^0}^2 + 12m_{T_s^-}^2 + m_{T^0}^2 + 23m_{B^-}^2 - 12m_{B_s^0}^2 - m_{B^0}^2 - 23m_{\pi^-}^2 + 23m_{\pi^0}^2) + (m_{T_c^0}^2 + 12m_{T^0}^2 + m_{T_s^-}^2)m_{B_s^0}^2 - (m_{T_c^0}^2 + 12m_{T_s^-}^2 + m_{T^0}^2)m_{B^-}^2 . \qquad (2.26c)$$

This will give also anomalously small m_{η_t}. We conclude, however, that two good sum rules, Eqs. (2.23a) and (2.26a), found in this paper should be taken seriously. The effect of mixes will become importent for η_c, η_b and η_t masses which does not affect to the present study so we shall not study them in this work.

In the next section we shall compare some of the relations found in this section joining with CKM angles through ps-meson leptonic decays..

$. 3 Leptonic and Semileptonic Decays of ps-Mesons

Let us estimate first the lower and upper bound of F_D, F_B and F_T in units of F_π. They are estimated from

$$\frac{F_D^2}{F_\pi^2} = \frac{m_{K^0}^2}{m_{D^-}^2 - m_{\pi^-}^2 + m_{\pi^0}^2} \frac{m_d + m_c}{m_d + m_s} = 0.436822 - - - 1.58545, \qquad (3.1)$$

$$\frac{F_B^2}{F_\pi^2} = \frac{m_{K^0}^2}{m_{B^0}^2} \frac{m_d + m_b}{m_d + m_s} = 0.203704 - - - 0.621942, \qquad (3.2)$$

and

$$\frac{F_T^2}{F_\pi^2} = \frac{m_{K^0}^2}{m_{T^-}^2 - m_{\pi^-}^2 + m_{\pi^0}^2} \frac{m_d + m_t}{m_d + m_s} = 0.007961 - - - 0.022621, \qquad (3.3)$$

where the numerical values on the r.h.s. equalities correspond to those combined lowest and highest quark masses.

One of the main purpose of this study is to keep the universality of the Fermi constant

$$G_F = 1.16639 \times 10^{-11} MeV^{-2} , \qquad (3.4)$$

in various weak lepton associated processes. In meson leptonic decay the weak vertex will not require any vertex correction by the strong ibteraction under the conserved current hypothesis. The decay rate is specified by the parent, charged lepton masses and CKM angles with an appropriate expression, which is given in standard text.[4,5]

The numerical result is tabulated in Table I.

Table I. Leptonic Decay Rate of ps-meson Leptonic Decay (sec^{-1})

process	lower b.	upper b.	exp. value
$\pi^+ \to \mu^+ \nu_\mu$	3.829×10^7	3.841×10^7	$(3.8408 \pm 0.0007) \times 10^7$
$\pi^+ \to e^+ \nu_e$	4.914×10^3	4.929×10^3	$(4.725 \pm 0.015) \times 10^3$
$K^+ \to \mu^+ \nu_\mu$	4.851×10^7	5.169×10^7	$(5.128 \pm 0.015) \times 10^7$

$K^+ \to e^+\nu_e$	1.246×10^3	1.328×10^3	$(1.25\pm0.056)\times10^3$
$D^+ \to \mu^+\nu_\mu$	6.06×10^7	2.34×10^8	$<6.8\times10^8$
$D^+ \to \tau^+\nu_\tau$	1.60×10^8	6.18×10^8	----------
$D^+ \to e^+\nu_e$	1.43×10^3	5.51×10^3	----------
$D_s^+ \to \mu^+\nu_\mu$	1.28×10^9	4.68×10^9	$(8.6\pm\frac{4.7}{4.3})\times10^9$
$D_s^+ \to \tau^+\nu_\tau$	1.25×10^{10}	4.56×10^{10}	$(1.5\pm0.9)\times10^{11}$
$B^+ \to \mu^+\nu_\mu$	5.52×10^3	1.05×10^4	$<1.3\times10^7$
$B^+ \to \tau^+\nu_\tau$	1.23×10^6	2.34×10^7	$<3.5\times10^8$
$B^+ \to e^+\nu_e$	0.129	2.47	$<9.1\times10^6$
$T^+ \to \mu^+\nu_\mu$	3.51×10^4	1.05×10^6	----------
$T^+ \to \tau^+\nu_\tau$	9.93×10^6	2.98×10^8	----------
$T^+ \to e^+\nu_e$	0.821	24.6	----------

The first column lists the various leptonic decay processes of mesons, and the second, third and fourth column give the estimated lower, upper bound and experimental decay rates, respectively. The parameters F_π and F_K are found to be 131.4 and 158 MeV, respectively. In the remaing three heavy mesons, the decay parameters are used those given at the begining of this section. There does not seem to exist any inconsistency between the theory and experiment. These results will give an additional support to the extension of the theory to the higher chiral symmtery, although a further experimental study is needed.

Let us turn our attention to the weak semileptonic decay of mesons. The formulation for $\pi^+ \to \pi^0 e^+\nu_e$ decay has been studied

in detail by Källén[5] with appropriate approximation for the integral involved in the estimate of the total decay rate. Here the parent and daughter meson mass difference is small. In general many processes going to treat, this approximation does not hold. Our main task is purely to do it as a good approximation as possible for the processes discussed. The decay rate for any process under discussion may be given by

$$\frac{1}{\tau} = \frac{G_F^2}{192\pi^4 mpa} V_{xy}^2 \, \mathrm{I}, \tag{3.5}$$

where mpa is the parent meson mass, V_{xy} the CKM mixing angle for quark x and y, and I the integral of interest. It is given by

$$\mathrm{I} = \frac{\pi}{2mpa^2} \int_{m^2}^{(mpa-mda)^2} du f(-u)\sqrt{\lambda(u, mpa^2.mda^2)}, \tag{3.6}$$

where mda and m are the associated daughter meson and charged lepton masses, respectively, $\lambda(x,y,z) = x^2+y^2+z^2-2xy-2yz-2zy$ is the well-known Källén factor and

$$f(-u) = (1-\frac{m^2}{u})^2[4\lambda(u, mpa^2, mda^2) + \frac{6m^2}{u}(mpa^2 - mda^2)^2]. \tag{3.7}$$

Substituting

$$u = x\Delta^2, \quad \varepsilon = \frac{m^2}{\Delta^2}, \quad \text{where } \Delta = mpa - mda \tag{3.8}$$

into (3.6), (3.7) and (3.5), one finds

$$\frac{1}{\tau} = \frac{G_F^2}{192\pi^3}(1+\frac{mda}{mpa})^3 \Delta^5 \, V_{xy}^2 \, I_1, \tag{3.9}$$

where

$$I_1 = \int_\varepsilon^1 \frac{dx}{x}\sqrt{1-x}\sqrt{1-\kappa x}(1-\frac{\varepsilon}{x})^2[(2x+\varepsilon)(1-x)(1-\kappa x)+3\varepsilon]. \tag{3.10}$$

Here

$$\kappa = \frac{\Delta^2}{(mpa + mda)^2}. \qquad (3.11)$$

In the numerical study $\sqrt{1-\kappa x}$ is expanded up to 20th and 30th power of κx when κ takes values between 0.2 to 0.3 and that between 0.4 to 0.73, respectively. They are summarized in Appendix for convenience. The result estimated without making a null-instanton correction is tabulated in Table II.

Table II. Semileptonic Decay Rates of Mesons (sec^{-1})

process	lower b.	upper b.	exp. value
$\pi^+ \to \pi^0 e^+ \nu_e$	0.386408	0.387599	0.39373 ± 0.0131
$K^+ \to \pi^0 e^+ \nu_e$	3.47087×10^6	3.69841×10^6	$(3.89 \pm 0.05) \times 10^6$
$K^+ \to \pi^0 \mu^+ \nu_\mu$	2.46048×10^6	2.61697×10^6	$(2.56 \pm 0.08) \times 10^6$
$K_S^0 \to \pi e \nu$	7.03008×10^6	7.49095×10^6	$(7.50 \pm 0.08) \times 10^6$
$K_S^0 \to \pi \mu \nu$	4.53486×10^6	4.83215×10^6	$(5.25 \pm 0.07) \times 10^6$
$K_L^0 \to \pi e \nu$	7.03008×10^6	7.49095×10^6	$(7.50 \pm 0.02) \times 10^6$
$K_L^0 \to \pi \mu \nu$	4.53486×10^6	4.83215×10^6	$(5.26 \pm 0.05) \times 10^6$
$D^+ \to \bar{K}^0 e^+ \nu_e$	2.23596×10^{11}	2.24331×10^{11}	$(6.3 \pm 0.8) \times 10^{10}$
$D^+ \to \bar{K}^0 \mu^+ \nu_\mu$	2.16603×10^{11}	2.17315×10^{11}	$(6.6 \pm \frac{2.8}{1.9}) \times 10^{10}$
$D^0 \to K^- e^+ \nu_e$	2.22075×10^{11}	2.22806×10^{11}	$(8.82 \pm 0.43) \times 10^{10}$
$D^0 \to K^- \mu^+ \nu_\mu$	2.15119×10^{11}	2.15827×10^{11}	$(7.78 \pm 0.41) \times 10^{10}$
$B^+ \to \bar{D}^0 e^+ \nu_e$	3.74413×10^{10}	5.09617×10^{10}	total three modes

$B^+ \to \bar{D}^0 \mu^+ \nu_\mu$	3.72528×10^{10}	5.07052×10^{10}	$(1.13 \pm 0.20) \times 10^{10}$
$B^+ \to \bar{D}^0 \tau^+ \nu_\tau$	9.50403×10^9	1.29360×10^{10}	
$B^0 \to D^- e^+ \nu_e$	3.72876×10^{10}	5.07528×10^{10}	same as above
$B^0 \to D^- \mu^+ \nu_\mu$	3.70995×10^{10}	5.04966×10^{10}	$(1.28 \pm 0.16) \times 10^{10}$
$B^0 \to D^- \tau^+ \nu_\tau$	9.43013×10^9	1.28354×10^{10}	

The specification of Table II is almost the same as that in Table I, except for B^+ and B^0 decays where only upper bound on total three decay modes for each decay is known. In this tabulation an $e-\mu-\tau$ universality is applied.

The correction term proportional to decay amplitude arises from quark-antiquark exchange between the parent (daughter) and spectator quark in the initial (magnetic moment type) and final state (transient type) interactions. We specify that interaction by the assumed null-instanton effect.[3] The interaction is quark exchange type, viz.,

$$c\,[\frac{3+\sigma_i.\sigma_j}{4} + \frac{1-\sigma_i.\sigma_j}{4}]\; P_{ij}\;, \qquad (3.12)$$

where c is a dimensionless strength operator $\vec{\sigma}_i$ the Pauli spin operator for the ith quark and P_{ij} is the exchange operator between ith and jth quark including their colors. The correction is evaluated as the eigenvalue problem of this operator. The weak vertex correction and self-energy type exchange is avoided by the assumption of conserved current and unovservable effect, respectively. Let us specify the eigenvalues of c as a0, b0, c0, d0 for the initial q-q, q-s, q-c, q-b (q= u, d) and the corresponding final interaction as f1, g1, h1, k1, respectively. Here, the number attached to the eigenvalue indicates the exchanged quark pair spin. The correction term can be written in a factored form in amplitude as

$$(1-3a0)(1-3f1)^2, \qquad (3.13a)$$

$$(1-3b0)(1-3f1)(1-3g1), \qquad (3.13b)$$

$$(1-3c0)(1-3g1)(1-3h1), \qquad (3.13c)$$

and

$$(1-3d0)(1-3g1)(1-3k1), \qquad (3.13d)$$

respectively, for the pion, kaon, D-meson and B-meson semileptonic decays. Here a factor3 for each strength comes from a color degree of freedom. Here the Pauli exclusion principle is applied explicitly. From Table II one sees that one needs a small corrections for the first seven processes, while the remaning heavy meson decays require big corrections. It may be better to wait an improvement of data for the latters before making a detailed numerical fit and a refinement of the theory is left for future.

$. 4 Semileptpnic Decays of Baryons

Let us begin with the determination of D and F parameters in baryon semileptonic decays.[4] The observed g_A/g_V's at zero momentum transfer for four processes $n \to pe^-\bar{\nu}_e$, $\Lambda \to pe^-\bar{\nu}_e$, $\Sigma^- \to ne^-\bar{\nu}_e$ and $\Xi^- \to \Lambda e^-\bar{\nu}_e$ are utilized to determine D and F by the least-squares fit under the assumed time reversal invariance, viz., $\phi_{AV} = 180°$. The result is given by

$$D = 0.81047 \text{ and } F = -0.45653 \quad \text{with } \chi^2 = 0.35183. \qquad (4.1)$$

A radiative correction to the baryon semileptonic decays is taken as[4]

$$1-\frac{\alpha}{4\pi}(6\ln[\frac{m_Z}{mpa}]+3\ln[\frac{mpa}{2E_{max}}]+\frac{207}{20}-\frac{4\pi^2}{3}), \qquad (4.2)$$

where m_Z and mpa are the Z-boson mass and the charged baryon mass involved in the process, respectively, and E_{max} is the energy difference between the parent and daughter baryon. We are interested in the lower and upper bound of the effective coupling

constant observed in units of G_F^2 which is represented D, F and a numerical constant. For this purpose we have taken a non-relativistic approximation for baryon matrix element[5] for simplicity. The numerical results are found to be

$$1+3(D-F)^2 = 3.50248 - - - 3.51328 \text{ for } n \to pe^-\bar{\nu}_e, \qquad (4.3a)$$

$$2D^2 = 1.03662 - - - 1.03982 \text{ for } \Sigma^- \to \Lambda e^-\bar{\nu}_e, \qquad (4.3b)$$

$$2D^2 = 1.12724 - - - 1.13072 \text{ for } \Sigma^+ \to \Lambda e^+\nu_e, \qquad (4.3c)$$

$$\frac{3}{2}+\frac{1}{2}(D-3F)^2 = 3.33036 - - - 3.54868 \text{ for } \Lambda \to pe^-\bar{\nu}_e, \qquad (4.3d)$$

$$1+3(D+F)^2 = 1.11454 - - - 1.18761 \text{ for } \Sigma^- \to ne^-\bar{\nu}_e, \qquad (4.3e)$$

$$\frac{3}{2}+\frac{1}{2}(D+3F)^2 = 1.72776 - - - 1.84103 \text{ for } \Xi^- \to \Lambda e^-\bar{\nu}_e, \qquad (4.3f)$$

and

$$1+3(D-F)^2 = 2.78754 - - - 2.97028 \text{ for } \Xi^- \to \Sigma^0 e^-\bar{\nu}_e. \qquad (4.3g)$$

Here the lower and upper bound correspond to upper and lower bound of CKM angles, respectively. The reduction rates of the Fermi constant are obtained by taking the ratio of the square root of them by substituting the D and F found in Eq.(4.1) to the square root on left hand side of Eqs.(4.3) for each process.

To keep the universality of Fermi constant, the required corrections for these decays are given by

(1-a0-a1)(1-2f0-2f1) for (4.3a) ,
(1-b0-a1)(1-2g0-2f1) for (4.3 b, c)
(1-b0-b1)(1-f0-g0-f1-g1) for (4.3 d, e)
and
(1-b0-c1)(1-f0-g0-g1-h1) for (4.3 f, g),

(4.4)

where a new parameter c1 and h1 corresponds to the s-s exchange parameters in the initial and final state one, respectively. We shall leave numerical details except for

$$f = (1-a0-a1)(1-2f0-2f1) = 0.776034 - - - 0.777042, \quad (4.5)$$

$. 5 Fermi Type Beta Decay

The Fermi type nuclear beta decay is treated in this section.[12),13)] In all examples, the transitions occur through the isospin $T = 1 \rightarrow 1$ and the spin-parity $0^+ \rightarrow 0^+$ so that the matrix element through a pure vector current becomes $\sqrt{2}$. We shall examine four data[9)]

$$^{14}O(g.s.) \rightarrow {}^{17}N(1stex.s.)e^+\nu_e(99.33\%), \quad (5.1a)$$

$$^{18}Ne(g.s.) \rightarrow {}^{18}F(2ndex.s.)e^+\nu_e(7.7\%), \quad (5.1b)$$

$$^{26}Al(1stex.s.) \rightarrow {}^{26}Mg(g.s.)e^+\nu_e(100\%), \quad (5.1c)$$

and

$$^{34}Cl(g.s.) \rightarrow {}^{34}S(g.s.)e^+\nu_e(100\%), \quad (5.1d)$$

where the relevant nuclear state is specified after each nucleus symbol and the decay branching ratio is given in a last bracket for each process. Here E_{max} is the mass difference between the parent and daughter nuclear states. The use is made of E_{max} = 2.32058, 2 .89498, 3.72068 and 4.97188 Mev for Eqs.(5.1a-d), respectively.[9),10)] In the following a Coulomb correction for the positron was neglected, the radiative correction is done to the Källén's formula. The min-max of numerical values corresponding to the CKM angle in matrix element of interest in units of the pure Fermi one become

$$0.954518 - - - 0.955989 \quad \text{for (4.5a)}, \quad (5.2a)$$

$$0.948934 - - - 0.950395 \quad \text{for (4.5b)}, \quad (5.2b)$$

$$0.910590 - - - 0.911992 \quad \text{for (4.5c)} \quad (5.2c)$$

and

$$0.882224 - - - 0.883584 \quad \text{for (4.5d)} \quad (5.2d)$$

The corresponding additional corrections from the final state interaction between the host (and spectator) quark and associated quarks in the parent and daughter nucleus are multiplied to the common factor f=(1-a0-a1)(1-2f0-2f1) for each process as

$$[1-30(f0(p)+f1(p))-14(f0(s)+f1(s))]f, \quad (5.3a)$$

$$[1-6(f0(d)+f1(d))-36(fo(p)+f1(p))-12(f0(s)+f1(s))]f, \quad (5.3b)$$

$$[1-30(f0(d)+f1)d))-36(f0(p)+f1(p))-12(f0(s)+f1(s))]f \quad (5.3c)$$

and

$$[1-12(f0(2s)+f1(2s))-42(f0(d)+f1(d))-36(f0(p))+f1(p))-12(f0(s)+f1(s))]f, \quad (5.3d)$$

where numerical factor for each correction term in square bracket is the total quark pairs , viz., a quark number 3 times the relevant nucleon number. Here the s, p, d and 2s indicate nuclear shell states to which they belong. If we assume the strengths described by the same notations are approximately equal we can estimate them separately by taking the appropriate combination of them with numbers given in Eqs.(4.6). This is done by making successive subtraction of neighboring equations in Eqs.(4.7) and by doing the corresponding procedures to Eqs.(5.2). One finds easily that

$$(1-12f0(s)-12f1(s))\, f = 0.882224 - - - 0.883584, \quad (5.4)$$

$$(f0(p)+f1(p))\, f = (-0.0006678) - - - (-0.0006670), \quad (5.5)$$

$$f(f0(d)+f1(d)) = 0.0015977 - - - 0.0016001, \quad (5.6)$$

$$f(f0(2s)+f1(2s)) = 0.0007706 - - - 0.0007672. \quad (5.7)$$

If we rely on Eqs.(4.5) and (5.4) strength parameter from s-shell should be negative. The same negative sign is found from p-shell correction, Eq.(5.5). It is interesting to point out that the anti-quenching of nuclear magnetic moments from Schmidt lines occurs in $s_{1/2}$- and $p_{1/2}$-shell nuclei. We found a relatively stronger effect from d-shell, Eq.(5.6), to those in p- and 2s-shell, Eqs.(5.5) and (5.7), which is consistent with the property of null-instanton force stated at the introduction. These results should be remind when one makes the model check from a different source. It is important to notice that the correction from the full nucleon effect in the final state interaction become important to the transient process. This is not curious, since the total energy difference of the initial and final nuclear states concerns to the transition, viz., the full nucleons affect to the process.

It is instructive to compare the numerology between magnetic moment of nucleus and Fermi type weak decay studied in this paper. Here we have confined only the null-instanton effect in initial configuration to a parent nucleon due to the short range character of the weak interaction. This is an approximation which made a numerical work easier. Secondly the correction from the final state interaction is written in a factorized form. We have developed carefully the theory of the nuclear magnetic moment so as to admit the corrections arising from the intra- and inter-nucleon in a factorized form which was not shown explicitly in an original manuscript. A $\Sigma^0 - \Lambda$ transient moment should have the additional correction from the final state interaction. The detail of them will be reported in an appropriate place.

$. 6 Summary and Conclusion

In this paper a null-instanton model is tested from the leptonic and semileptonic decays of mesons, baryons and nuclear Fermi beta decays.

In order to find the decay constants for leptonic decays of ps-mesons, the meson mass formulae in chiral $SU_3 \times SU_3$ symmetry are extended to $SU_6 \times SU_6$ one as a somewhat bold try. Two of the new mass formulae belonging to $SU_5 \times SU_5$ and $SU_6 \times SU_6$ are well satisfied numerically. In the latter fit it is assumed that all T meson masses are assumed to be degenerated to t-quark one. The leptonic decay rates estimated by making use of the decay constants determined by above procedure appear to be consistent

with the available experimental information. The semilptonic decays of light ps-mesons require the small correction from the null-instanton model. However, the large corrections required to the ps-mesons belonging to the higher symmetry which may require a more experimental study and an improvement of the theory.

The null-instanton effect in the neutron and the nuclear Fermi type decays are compared so as to fix the parameters arising from final state interaction between parent and the nucleons in the daughter nucleus. The sign anomaly found from s- and p-shell nucleon effect will give an additional support on the observed anti-quanching of magnetic moments from Schmidt lines for the $s_{1/2}$- and $p_{1/2}$-shell nuclei. A comparison of corrections found from s-, p-, d- and 2s-shells give a support on the property suggestted from the null-instanton model.

The corrections from the final state interaction found from the semileptonic decays of hyperons is small relative to that for the neutron which will be expected from the fact that the effect arises from s-quark associated ones in the former.

Appendix Integral I_1

It will be convenient to summarize the numerical values of the integral I_1, Eq.(3.10), appeared for semileptonic decays of ps-mesons for a future study for one who needs them by a similar approach. The process is specified in a bracket just after I_1.

$$I_1(\pi^+ \to \pi^0 e^+ \nu_e) = 0.752833,$$

which becomes 0.752926 in Källén's approximation.

$$I_1(K^+ \to \pi^0 e^+ \nu_e) = 0.692537, \quad I_1(K^+ \to \pi^0 \mu^+ \nu_\mu) = 0.446997,$$

$$I_1(K_s^0 \to \pi e \nu) = 0.695637, \quad I_1(K_s^0 \to \pi \mu \nu) = 0.448731,$$

$$I_1(D^+ \to \bar{K}^0 e^+ \nu_e) = 0.689305, \quad I_1(D^+ \to \bar{K}^0 \mu^+ \nu_\mu) = 0.667747,$$

$$I_1(D^0 \to K^- e^+ \nu_e) = 0.688631, \quad I_1(D^0 \to K^- \mu^+ \nu_\mu) = 0.667061,$$

$$I_1(B^+ \to \bar{D}^0 e^+ \nu_e) = 0.723708, \quad I_1(B^+ \to \bar{D}^0 \mu^+ \nu_\mu) = 0.720065,$$

$I_1(B^+ \to \bar{D}^0\tau^+\nu_\tau) = 0.183705$, $I_1(B^0 \to D^-e^+\nu_e) = 0.724002$,

$I_1(B^0 \to D^-\mu^+\nu_\mu) = 0.720065$ and $I_1(B^0 \to D^-\tau^+\nu_\tau) = 0.183102$.

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