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THE BENDING OF LIGHT, DUALITY, THE PHOTONIC ENERGY-STRESS TENSOR THE QUESTION OF SELF-CONSISTENCY IN GENERAL RELATIVITY AND UNIFIED POLARIZATION OF PLANE-WAVE FORMS

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Abstract

In gravity, a seemingly paradox is that although the gravity of the static electromagnetic energy is intrinsically distinct from that of massive matter, the motion of light is also a geodesic. This manifests that free radiation should have an energy-stress tensor distinct from that of electromagnetism with material charges and current. Moreover, in disagreement with the light bending calculation, existing solutions of gravity for electromagnetic waves are unbounded, in addition to violating the equivalence principle. Assuming the theoretical framework of general relativity is valid, analysis based on physical principles shows that the related Einstein equation has no physical solution *unless* another energy tensor with an anti-gravity coupling is included. Such a tensor is identified to be the energy-stress tensor for photons. It generates the geodesic equation, and has the sum of the electromagnetic and the gravitational energy. The gravitational and electromagnetic components have the same speed of propagation. For monochromatic plane-waves, a gravitational wave solution has the polarization matching that of the electromagnetic wave, while their frequency ratio is two. For a circularly polarized wave, the gravitational wave component can be zero. The existence of a distinct energy tensor for photons clarifies and explains further i) the meaning of duality; and ii) the behavior of radiative electromagnetic energy. *Then, not only is the bending of light self-consistently calculated but it would also be established as the earliest experimental proof for the existence of gravitational waves.*

I. Introduction.

Relativity suggests the existence of gravitational waves [1]. Although there are indirect observational evidences [2-4], gravitational waves have not been directly observed [5]. Theoretically, the existence of gravitational waves had been assumed a certainty to the first approximation [6]. However, Einstein himself discovered [7] in 1936 that linearized gravity is not reliable. Nevertheless, during 1950s theorists reached a consensus [8] that gravitational waves did indeed exist, although a valid proof remains to be shown [9]. Also exact physical solutions of gravitational waves are not yet available [10]. The main difficulties appear to be: 1) Einstein equation is non-linear; 2) The related physical requirements remain to be investigated (e.g., the nature of the source term has been obscure since the beginning) [11-13].

To illustrate the existence of gravitational waves theoretically, one should consider some simple situations. Analysis indicates that an electromagnetic wave would generate an accompanying gravitational wave [10,12, 14]. However, a theoretical difficulty is that if a gravitational wave carries energy, how is its energy related to the energy of the electromagnetic wave? It will be shown that the bending of light would provide meaningful suggestions. In this connection, one should explain how is the photonic geodesic equation related to the Maxwell-Einstein equations and what is the mass-density-like function for photons (see § 9)?

To this end, one must first recognize that although a transformation of light energy to mass is supported [15], the electromagnetic energy ***alone*** is not equivalent to mass [12] since unlike the massive matter, the trace of an electromagnetic energy-stress tensor is zero. This is manifested in the Reissner-Nordstrom metric for a particle of mass M with charge q [14] ($ds^2 = (1-2M/r+q^2/r^2)dt^2 - (1-2M/r+q^2/r^2)^{-1}dr^2 - r^2d\Omega^2$, where r is the distance from the particle) which shows that the portions of metric components due to mass and electric energy have ***different signs*** and r-dependences. Note that $E = mc^2$ is only for the mass m of a body and its ***total*** energy E [16]. The once prevailing interpretation [5,17,18] that this equation implies an unconditional general equivalence between any energy and mass, is actually incorrect [12,13,19].

Currently, a geodesic equation cannot be generated from the electromagnetic energy tensor. On the other hand, if the equation of motion for the electromagnetic energy were also related to a geodesic equation, the electric energy would be subjected to a gravitational force as a mass is. However, not only this disagrees with the Maxwell equation, but general relativity would not be self-consistent (Appendix A). Nevertheless, if non-

electromagnetic radiation is included in the light, its energy tensor can generate a geodesic equation, i.e., the equation of motion for the radiative energy can be different from that for the non-radiative energy.

Thus the light bending by following a geodesic, from the viewpoint of general relativity, necessitates that ***the energy-stress tensor of light (i.e., the totality of related free radiations) must have a form which is different from that of the electromagnetic energy tensor*** (see also §§ 4 & 5).

Experimentally electromagnetic waves and photons are inseparable although particles and waves are conceptually distinct [20]. Quantum phenomena show conclusively that the light is distinct from just a classical electromagnetic wave. On the other hand, it has never been shown conclusively that the light is identical to an electromagnetic wave in Maxwell's theory. What has been shown is only that the light is inseparably ***associated*** with an electromagnetic wave. In other words, it is possible that there are non-quantum aspects of the light, which are different from the classical electromagnetic wave.

Moreover, there is no physical principle which dictates that the distinction between the light and an electromagnetic wave is limited to quantum phenomena. Thus, a related crucial question is whether there are evidences of non-quantum nature that also make the distinction. The answer is affirmative. As discussed earlier, the lights in terms of gravity, are also not just electromagnetic waves.

In the calculations of the star light bending, wave and particle approaches [6,21] give the same deflection. This manifests that the particle-wave duality is not necessarily excluded from a classical theory. Note, however, almost all authors consider the light as consisting of massless particles, photons which obey the geodesic equation [6,9,14]. But, Einstein [22] alone maintained the derivation of light bending through the Huyghen's principle which is not directly related to the motion of a particle (although he did adapt the method of calculating the perihelion advance by others.) Perhaps, Einstein had already aware of the theoretical problem that the Einstein-Maxwell equations cannot produce the geodesic equation for the light (see also Appendix A), although his field equation would include the equation of motion for massive matter [13].

Moreover, an implicit assumption in calculating the light bending is that ***the gravity due to the light is negligible*** [6,22]. Although this is believable, self-consistency requires such an assumption to be proven within general relativity. On the other hand, if an electromagnetic wave and the related photons are distinct objects as concluded, without a distinct photonic energy-stress tensor, one would encounter that, in disagree-

ment with experiments, Einstein's equation cannot have a ***bounded*** physical solution [12]. Therefore, the question of a distinct photonic energy-stress tensor is actually ***an integral part*** of general relativity since the electromagnetic energy-stress tensor must be included in the source (see Appendix A).

In summary, it is necessary to have a distinct photonic energy-stress tensor and to show that the gravity due to light is negligible. To remain as a viable theory, general relativity must be able to provide an answer.

A distinct energy-stress tensor for photons would support that there are intrinsic connections between general relativity and quantum theory. In spite of their fundamental differences, as Bohr discovered (see § 9), there are possibly intrinsic connections among them. Note that both theories have their foundations on different aspects of a common physical phenomenon -- the velocity and quantum of light. To settle these theoretical problems, it is necessary to analyze Einstein's equation in connection with physical principles.

For simplicity, we consider the case of an electromagnetic plane-wave. Theoretically, a plane-wave is a spatial local idealization, and in practice it can also be considered as an idealization of the interior of a uniform laser beam [12]. Moreover, it leads most naturally to an interpretation in terms of the photon [6]. Our conjecture is supported by the facts [12,23] that solutions of gravity obtained by Peres [24], Penrose [25] and Bonner [26], violate physical principles and are unbounded [27,28] in disagreement with the light bending calculation. However, to show the need of a distinct energy tensor for photons, one must prove that a valid solution is impossible otherwise. This is achieved by applying the related ***physical principles.***

The principle of causality (Appendix B) implies that an electromagnetic plane-wave would generate an accompanying gravitational plane-wave. However, since the time-time component of the Einstein tensor G_{tt} is positive on the time average, there is no physical solution unless another tensor is subtracted from the source (§4). Moreover, as shown in reference [12], a physical solution requires that, in the flat metric approximation, this unknown tensor, on the time average, is the same as the electromagnetic energy-stress tensor. In terms of physics, these require that the unknown tensor is the energy-stress of the photons (§4). Thus, the energy of the photons is the ***sum*** of the electromagnetic wave energy and the gravitational wave energy.

However, this also means that the energy-stress tensor for photons has an ***anti-gravity coupling*** (see also §9). Although this is beyond the theory originally proposed by Einstein, Pauli [18] pointed out that such a coupling would be possible. Now, the antigravity coupling is proven to be a necessary physical feature.

Based on that photons travel along a geodesic and other physical considerations, an energy-stress tensor for photons is obtained for monochromatic electromagnetic plane-waves (see §5). Then, physical solutions of gravitational plane-waves are obtained, and the polarization of the gravitational wave matches that of the related electromagnetic wave (see §6). Thus, the anti-gravity coupling for photons is confirmed theoretically.

Nevertheless, the skeptics might argue that, in addition to singularities [29,30], the necessary existence of antigravity coupling is another evidence for the breaking down of general relativity since Newtonian gravity does not allow an antigravity coupling. To settle the question of whether the antigravity coupling exists, it seems, experimental evidences are needed. A direct measurement for the accompanying gravitational waves of electromagnetic waves would be technically very difficult. However, if the antigravity coupling is necessary for electromagneic radiation, according to the Einstein tensor, such a coupling is also necessary for a pure gravitational radiation (see also §9). Surprisingly, this indirect verification has actually been confirmed [31] by the Taylor and Hulse binary pulsar PSR 1913+16 experiment [2] almost 25 years ago!

Thus, duality is also a necessary feature of general relativity. Moreover, a distinct energy-stress tensor for photons is still compatible with quantum theory in which the gravitational effects are not considered. This calculation shows that, only when gravity is neglected, the energy-stress tensor for photons and that for the related electromagnetic wave, on the time average, are the same [12]. This suggests that classical and quantum theories may share more common features than previously assumed.

Moreover, it will be shown in §8 that there is a connection between the photonic energy tensor and the complex waves used in quantum theory. This can be interpreted as duality is implicitly contained in complex waves. Also, this connection can be used as a general method to obtain the photonic energy tensor related to an electromagnetic wave. Apparently, there is a need in understanding the complex waves further.

The antigravity coupling further manifests the intrinsic differences between general relativity and Newtonian theory which originated the notion of black holes. However, to keep the focus on duality, the profound meaning of this coupling to space-time, the existence of dynamic solutions, and the life of very massive stars will be discussed in a separated paper [32]. This paper continues an earlier paper [12] in which the complicated calculations are performed to show the necessity of a photonic energy-stress tensor. For convenience and clarity, its main arguments are briefly presented (but may not be strictly in the logical order) in §§ 2-4.

2. Causality and Duality Related to Electromagnetic Plane-Waves

Let us consider a ray of uniform electromagnetic waves (i.e. a laser beam) propagating in the z-direction. Within the ray, one can assume a strong cylindrical condition (i.e. the wave amplitude is independent of x, and y.) Thus, the electromagnetic potentials are plane-waves, and in the unit that the light velocity c = 1,

$$A_k = A_k(t - z) \ , \quad \text{where} \quad k = x, y, z, \text{ or } t \ . \tag{1}$$

Due to the principle of causality (see Appendix B), the metric g_{ab} is functions of u (= t − z), i.e.,

$$g_{ab} = g_{ab}(u) \ , \quad \text{where} \quad a, b = x, y, z, \text{ or } t \ . \tag{2}$$

Let P^k be the momentum of a photon. Then, based on Einstein's equation (§ 3), one obtains the conditions,

$$P^z = P^t, \quad P^x = P^y = 0, \quad \text{and} \quad P^m g_{mk} = P_k = 0 \ , \tag{3a}$$

for k = x, y, and v (≡ t + z) (see reference [12] for details). Eq. (3a) is equivalent to

$$g_{xt} + g_{xz} = 0 \ , \quad g_{yt} + g_{yz} = 0 \ , \text{ and } \quad g_{tt} + 2g_{tz} + g_{zz} = 0 \ , \tag{3b}$$

or

$$g^{xt} - g^{xz} = 0 \ , \quad g^{yt} - g^{yz} = 0 \ , \text{ and } \quad g^{tt} - 2g^{zt} + g^{zz} = 0 \ . \tag{3c}$$

To obtain eq. (3), Einstein's notion of weak gravity has been used [12,17]. The wave transversality implies

$$P^m A_m = 0 \ , \qquad \text{or equivalently} \qquad A_z + A_t = 0 \ . \tag{4}$$

Eqs. (2) to (4) imply that not only the geodesic equation, the Lorentz gauge, but also Maxwell's equation are satisfied. Moreover, the Lorentz gauge becomes equivalent to a covariant expression.

The above analysis suggests that an electromagnetic plane-wave can be an exact solution in a non-flat manifold. In a coordinate system where P_m are ***constants***, the scalar $\int P_m dx^m$ would equal to $P_m x^m$.

3. The Reduced Field Equation for the Gravity of an Electromagnetic Plane-Wave

For this case, Einstein [33] believed the field equation is $G_{ab} = KT(E)_{ab}$ where the Einstein tensor $G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$ (R_{ab} is the Ricci curvature tensor, and $R = g^{mn}R_{mn}$), K is the coupling constant, $T(E)_{ab} = -g^{mn}F_{ma}F_{nb} + \frac{1}{4} g_{ab}F_{mn}F^{mn}$, and F_{ab} is the electromagnetic field tensor. Thus, R = 0. It follows eq. (2) that

$$R_{tt} = - R_{tz} = R_{zz} , \tag{5a}$$

because $F_{mn} F^{mn} = 0$ due to eq. (3). The other components give eq. (3), and are zero [12]. Then,

$$R_{tt} \equiv -\frac{\partial \Gamma^m{}_{tt}}{\partial x^m} + \frac{\partial \Gamma^m{}_{mt}}{\partial t} - \Gamma^m{}_{mn}\Gamma^n{}_{tt} + \Gamma^m{}_{nt}\Gamma^n{}_{mt} = -K\,T(E)_{tt} = K\,g^{mn}F_{mt}F_{nt} . \tag{5b}$$

After some lengthy algebra [12], eq. (5b) is simplified to a differential equation of u as follows:

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 - G'(g'/2g) = 2K\,(F_{xt}{}^2 g_{yy} + F_{yt}{}^2 g_{xx} - 2F_{xt}F_{yt}\, g_{xy}) \tag{6}$$

$$= 2GR_{tt} , \qquad \text{where} \qquad G \equiv g_{xx}\, g_{yy} - g_{xy}{}^2, \text{ and } g = |g_{ab}|$$

is the determinant of the metric. The metric elements are connected by the following relation:

$$-\,g = G\, g_t{}^2 , \quad \text{where} \quad g_t \equiv g_{tt} + g_{tz}. \tag{7}$$

Note that eq. (35.31) in reference [14] and eq. (2.8) in reference [34] are special cases of eq. (6).

Equations (3), (4), (6), and (7) allow A_t, g_{xt}, g_{yt}, and g_{zt} to be set to zero (or equivalently $g_{uk} = 0$ for k = x, y, u). In any case, these assigned values have little effect in subsequent calculations.

Now, there are four metric elements (g_{xx}, g_{xy}, g_{yy}, and g_{tt}) to be determined. Although there is only one differential equation, to show that there is no physical solution, eq. (6) is sufficient. (No electromagnetic wave is a special case; and this means no gravitational plane-wave [12].) In other words, ***in contrast to Einstein's belief [33], the difficulty of this equation is not limited to mathematics.***

4. Physical Solution and Necessities on the Source Tensor.

For an electromagnetic plane-wave, it has been shown in general that the required periodic nature of the metric is due to causality (see Appendix B). However, if one assumes only that the metric is a function of t and z, then the Einstein equation implies $g(u)_{ab}$ by utilizing Einstein's notion of weak gravity [12]. Thus, the principle of causality supports the Einstein tensor which satisfies the covariant divergence $\nabla_c G^{cb} \equiv 0$. On the other hand, since such a R_{tt} , on the time average, is non-negative, there is no solution for eq. (6).

Here, it will be shown further that, for special cases, the Einstein tensor $G(u)_{ab}$ implies that $g(u)_{ab}$ is periodic. Let us consider a circularly polarized monochromatic electromagnetic plane-wave,

$$A_x = \frac{1}{\sqrt{2}} A_0 \cos \omega u \ , \text{ and } \ A_y = \pm \frac{1}{\sqrt{2}} A_0 \sin \omega u \ . \tag{8}$$

Then $P_t = \omega$ (since $\hbar = 1$). The rotational invariants with respect to the z-axis are constants. These invariants are: R_{tt} , $T(E)_{tt}$, G , $(g_{xx} + g_{yy})$, g_{tz} , g_{tt} , g , and etc. Let us assume the invariant,

$$g_{xx} + g_{yy} = -2 - 2C, \text{ then } g_{xx} = -1 - C + B \ , \text{ and } g_{yy} = -1 - C - B. \tag{9}$$

Thus,

$$B^2 + g_{xy}{}^2 = (1+C)^2 - G \ , \text{ and } (B')^2 + (g_{xy}')^2 = 2GR_{tt} \geq 0 \tag{10}$$

obtained from $G \equiv g_{xx}\, g_{yy} - g_{xy}{}^2$ and eq. (6), are constants. It follows that eq. (10) implies

$$B = B_\alpha \cos(\omega_1 u + \alpha), \quad \text{and} \quad g_{xy} = \pm B_\alpha \sin(\omega_1 u + \alpha) \ , \tag{11a}$$

where

$$\omega_1{}^2 = 2R_{tt}G/B_\alpha{}^2, \quad \text{and} \quad B_\alpha{}^2 = (1+C)^2 - G \geq 0. \tag{11b}$$

Thus, it is proven that the metric is a periodic functions. Also, as implied by causality, the metric is <u>not</u> an invariant under a rotation (since a transverse electromagnetic wave is not such an invariant).

Since $T(E)_{tt}$ is a constant, it is necessary to have

$$\omega_1 = 2\omega, \quad \text{and} \quad T(E)_{tt} = \frac{1}{2G}\,\omega^2 A_0{}^2(1 + C - B_\alpha \cos\alpha) > 0. \tag{12}$$

Eq. (11) implies that the metric is a circularly polarized wave with the same direction of polarization as the electromagnetic wave (8). However, if the photonic energy tensor were zero, it is not possible to satisfy Einstein's equation because $T(E)_{tt}$ and R_{tt} $(= 2\omega^2 B_\alpha^2/G)$ have the same sign. (Note that $G > 0$; and the equation of motion of a charged particle does not allow changing the sign of the coupling constant K [13].)

This calculation illustrates that there is ***no possibility***, within the current theory, to construct an acceptable metric representing the accompanying gravitational wave. On the other hand, since physical influences can be propagated at most with a light speed, the influence of an electromagnetic wave on its accompanying gravitational wave would essentially be ***spatially local***. This means that the electromagnetic plane-wave, a well-tested spatial local idealization, is a valid modeling in physics. In practice, a plane-wave would model the interior of a laser beam. Thus, if general relativity is fundamentally correct, there must be a way to modify the equation such that a physical solution can be obtained for an electromagnetic plane-wave.

Since the Einstein tensor is supported by causality, it would be sufficient to modify the source tensor [12]. The additional energy term should be a constant of different sign, and is larger in absolute value. Moreover, calculation shows that a physical solution requires that in the flat metric approximation, an electromagnetic wave energy tensor and the unknown tensor with an antigravity coupling carry, on the average, the same energy-momentum [12]. This is also expected for a photonic energy tensor since it must contain and, according to experiments, is essntially the energy-stress of the electromagnetic wave.

Thus, physics requires that the unknown tensor must be the energy tensor of photons, given that according to the bending of light, it is different from that of electromagnetism, but includes essentially the energy tensor

of the electromagnetic wave. In other words, ***there are two possibilities; either a distinct energy-tensor for photons is found or there are some more basic problems in general relativity.***

5. Anti-Gravity Coupling and the Photonic Energy-Stress Tensor.

To verify the conjecture as required by the bending of light, one must show that a valid modification can be obtained with a photonic energy-stress tensor, i.e. it would also lead to a physical solution and generate a geodesic equation for photons. From the arguments in previous section, the general form of the source is

$$T_{ab} = T(E)_{ab} + \alpha T(P)_{ab} \,, \tag{13a}$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are the energy-stress tensors for the electromagnetic wave and the related photons, and α is a constant to allow a possibly different coupling. Since both $T(E)_{ab}$ and T_{ab} (due to $\nabla^c G_{cb} \equiv 0$ and eq. [5a]) are divergence free and traceless, $T(P)_{ab}$ must also be divergence free and traceless.

Given that a photonic energy tensor should produce a geodesic equation, for a ***monochromatic*** wave, the tensor form should be similar to that of massive matter. Observationally, there is very little interaction, if any, among photons of the same ray. Theoretically, since photons travel in the velicity of light, there should not be any interaction among them. Therefore, the photonic energy tensor should be ***dust-like*** as follows:

$$T^{ab}(P) = \rho\, P^a P^b, \tag{13b}$$

where ρ is a scalar which, according to causality, is a function of u. The geodesic equation, $P^c \nabla_c P^b = 0$, is implied by $\nabla_c (\rho P^c) = 0$, and $\nabla_c T(P)^{cb} = 0$. $\rho(u)$ should be a non-zero function of the electromagnetic potentials and/or fields. This implies $\rho = \lambda A_m g^{mn} A_n$, where λ is a scalar constant to be determined.

Since light intensity is proportional to the square of the wave amplitude, ρ can be considered as the density function of photons if $\lambda = -1$. Due to $R = 0$ and eqs. (2) and (3) remain valid, $\rho(u)$ is Lorentz gauge invariant. Also, without any lost of generality, α can be selected first since λ can be adjusted accordingly.

In anticipation of an anti-gravity coupling, one may choose $\alpha = -1$ in eq. (13a), and obtains

$$T_{ab} = T(E)_{ab} - T(P)_{ab} = T(E)_{ab} - \lambda\, A_m g^{mn} A_n\, P_a\, P_b\,. \tag{13c}$$

Thus, a photonic energy tensor changes nothing in calculation, but gives another term for eq. (6) ***only***. The general form for a photonic energy–stress tensor, which involves different frequencies, will be given in §8.

To determine λ, let us consider a circularly polarized monochromatic wave (8). Then, we have,

$$T_{tt} = \frac{1}{2G}\omega^2 A_0{}^2\left[(1 + C)(1 + \lambda) - (1 - \lambda)B_\alpha \cos\alpha\right] \le 0 \tag{14}$$

since $P_t = \omega$ (in the units $c = \hbar = 1$) and eq. (11b) requires R_{tt} to be of ***second*** order and positive. Eq. (14) requires that $\lambda \le -1$ because the constants C and B_α are much smaller than 1. Causality requires that, in a flat metric approximation, the time average of T_{tt} is zero. This implies that, as expected,

$$\lambda = -1,$$

and

$$T_{tt} = -\frac{1}{G}\omega^2 A_0{}^2 B_\alpha \cos\alpha \le 0, \qquad \text{since} \qquad B_\alpha = \frac{K}{2}\, A_0{}^2 \cos\alpha. \tag{15}$$

where B_α is the amplitude of the gravitational wave and α is its phase difference to the electromagnetic wave. Note that, ***pure electromagnetic waves can exist*** since $\cos\alpha = 0$ is possible.

To confirm the general validity of $\lambda = -1$, consider a wave linearly polarized in the x–direction,

$$A_x = A_0 \cos\omega(t - z)\,. \tag{16a}$$

Then, one has

$$T_{tt} = \frac{g_{yy}}{2G}\, \omega^2 A_0{}^2 \{(-\lambda - 1) + (1 - \lambda)\cos\left[2\omega(t - z)\right]\}. \tag{16b}$$

Thus, the flat metric approximation again requires that $\lambda = -1$. Then,

$$T_{tt} = \frac{g_{yy}}{G}\, \omega^2 A_0{}^2 \cos\left[2\omega(t - z)\right]. \tag{16c}$$

Eq. (16c) implies $(g_{xx} + g_{yy})'$ to be of first order [12], and therefore its polarization has to be different.

Thus, $T_{ab}(P)$ has been derived completely from the electromagnetic wave, and general relativity also requires that the energy and momentum of a photon must be proportional to its frequency. Now, the need of a photonic energy tensor for geometric optics, is clearly not accidental. But, a photonic energy tensor, constructed by Misner et al. [14, §§ 22], is only an approximation of the time average of $T_{ab}(P)$.

6. Unified Polarization of Plane-Wave Forms and Physical Solutions.

If a circularly polarized electromagnetic plane-wave results in a circularly polarized gravitational wave, one may expect that a linearly polarized electromagnetic plane-wave results in a linearly polarized gravitational wave. From the viewpoint of physics, the principle of causality would require that, for an x-directional polarization, gravitational components related to the y-direction, remains the same. In other words,

$$g_{xy} = 0 \ , \quad \text{and} \quad g_{yy} = -1 \ . \tag{17a}$$

Mathematically, condition (17a) is compatible with semi-unitary (i.e., g is a constant). Equation (17a) means that the gravitational wave is also linearly polarized. In the literature [10,14,24-26], there are other proposals. However, they all lead to unphysical solutions (see Appendix C & reference [27]).

It follows that equation (6) becomes

$$G'' = -2\,K\,G\,T_{tt}, \quad \text{and} \quad G = -g_{xx} \ . \tag{17b}$$

Then, the general solution for equation (17) is:

$$-g_{xx} = 1 + C_1 - (K/2)\,A_0{}^2\cos[2\omega(t-z)], \quad \text{and} \quad g_{tt} = -g_{zz} = \sqrt{g\,/\,g_{xx}} \ , \tag{18}$$

where C_1 is a constant. Note that the frequency ratio is the same as that of a circular polarization. For a polarization in the diagonal direction of the x-y plane, the solution is:

$$g_{xx} = g_{yy} = -1 - C_1/2 + (K/4)\, A_0{}^2 \cos[2\omega(t - z)] \ , \tag{19a}$$

$$g_{xy} = - C_1/2 + (K/4)\, A_0{}^2 \cos[2\omega(t - z)] \ , \tag{19b}$$

$$g_{tt} = - g_{zz} = \sqrt{-g}/\{1 + C_1 - (K/2)\, A_0{}^2 \cos[2\omega(t - z)]\}^{1/2} \ , \tag{19c}$$

Note that for a perpendicular polarization, the metric element g_{xy} changes sign. Solutions (18) and (19) imply that linear superposition of electromagnetic waves is only approximately valid. The time averages of their T_{tt} are also negative as required. If $g = -1$, relativistic causality requires $C_1 \geq K A_0{}^2/2$.

If the photonic energy tensor were absent (i.e., $\lambda=0$), the solution of equation (17) could have been

$$- g_{xx} = 1 + C_1 - (K/4)\, A_0{}^2 \{2\omega^2 (t - z)^2 + \cos[2\omega(t - z)]\} + C_2\, (t - z) \ , \tag{20}$$

where C_1 and C_2 are constants. Solution (20) is not physical because the term $(t - z)^2$ grows very large as time goes by. In disagreement with special relativity, $T(E)_{tt}$ has a time limit zero. Also, solution (20) illustrates that ***weak gravity is a physical requirement*** which may <u>not</u> be satisfied by an Einstein equation.

For a circularly polarized electromagnetic wave, the phase difference controls the amplitude of the gravitational wave (see eq. (17)), and the amplitude of the electromagnetic wave gives an upper bound. This is different from the case of linearly polarized waves for which the amplitude of gravity is fixed.

Now, in spite of the demanding physical requirements, a photonic energy tensor has been obtained. This is a very strong supporting evidence for general relativity. Physically, such a tensor should be unique. Besides, given all the severe physical requirements, it is difficult to imagine that a different tensor can be obtained.

Note that plane-waves (15), (18) and (19) are bounded ($|g_{ab}| <$ constant). But, the "plane-wave" of Bondi, Pirani, and Robinson [34] is not bounded. Therefore, it is not valid in physics [12,35]. One might blame all the problems to the non-linearity of the Einstein equation, and would wish a linear field equation for gravity. However, since the source can move with a light speed, such a linear equation, which was believed to be the first order approximation, violates causality and is therefore unacceptable [19].

7. A Unified Formalism for Plane Wave Polarizations and Semi-Unitary of the Metric.

In electrodynamics, because the Maxwell equation is linear, real waves are conveniently related to complex waves. However, quantum electrodynamics suggests that a complex wave is a necessity since it is required even by a hermitian operator. To prepare the discussion of complex waves (see Appendix D) and its polarizations, here we first show that there is a unified formalism for polarizations of real plane-waves.

Eq. (17a) can be interpreted as the transverse metric components are subjected to another constraint related to duality and polarization. Since both the momentum P^k and the conjugate momentum P_k are in the z-direction, one may conjecture that an electromagnetic plane-wave A_k and the contravariant potential A^k have the same plane of polarization. Thus, the ratios among their corresponding components are the same. Then one has the following additional equation:

$$(\alpha^2 - \beta^2)\, g_{xy} = \alpha\beta\, (g_{xx} - g_{yy}), \tag{21}$$

where

$$A_x = \alpha A, \quad A_y = \beta A, \quad \text{and} \quad |\alpha|^2 + |\beta|^2 = 1.$$

Equation (21) is equivalent to that, for an electromagnetic wave linearly polarized in the x-axis, $g_{xy} = 0$. For a circularly polarized wave, one could extend equation (21) to complex waves. Then, an electromagnetic and the accompanying gravitational wave have similar connection between real and complex waves.

A semi-unitary condition (i.e. g is a constant) simplifies eq. (6) considerably since eq. (7) implies that the last two terms on the left-hand side of eq. (6) cancel each other. Then eq. (6) is reduced into

$$C'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = 2K\left\{(F_{xt}^2\, g_{yy} + F_{yt}^2\, g_{xx} - 2F_{xt}F_{yt}\, g_{xy}) + GT(P)_{tt}\right\}. \tag{22}$$

Note that equation (22) includes only transverse metric elements, and C" can be of first order of deviations. The unitary condition lead to the same differential equation. But, g_{tt} and g_{zz} would be simpler. It should be noted also that the derivation of eq. (6) is also valid for complex functions.

8. General Relativity and Quantum Theory

Duality was considered exclusively only in quantum theory. But, the division of quantum and classical theories is man-made, and there may not be an insurmontable division between them in nature. Now, as observation requires, general relativity demands not only the notion of photon but also a distinct energy-stress tensor for photons. (In quantum electrodynamics gravity is neglected, and the electromagnetic wave and the related photons have the same energy-stress tensor.) However, since this tensor is also related to the electromagnetic wave, some kind of simple relationship toward the electromagnetic energy tensor can be expected. It turns out that the new source is related to a complex wave which is used in quantum theory.

To guard against accidental coincidence, two kind of polarizations are considered. For a linearly or circularly polarized plane-wave, their non-zero electromagnetic potentials are respectively:

$$A_x = A, \qquad \text{where} \qquad A = A_0 \exp\{-i\omega(t - z)\}; \quad \text{(linear)} \tag{23a}$$

and

$$A_x = \frac{1}{\sqrt{2}} A, \qquad \text{and} \qquad A_y = \pm \frac{i}{\sqrt{2}} A. \qquad \text{(circular)} \tag{23b}$$

Their energy-stress tensors are, for the linearly polarized wave (23a),

$$T(\tilde{E})_{tt} = - g^{mn}F_{mt}F_{nt} = - g_{yy}(-\omega^2)A^2/G = g_{yy}(\omega^2 A_0{}^2/G)(\cos 2\omega u - i \sin 2\omega u); \tag{24a}$$

and

$$T(\tilde{E})_{tt} = (\omega^2 A^2/2G)\left[g_{yy} - g_{xx} - (\pm)2ig_{xy}\right] = - (\omega^2 A_0{}^2/G)B_\alpha \exp(i\alpha), \tag{24b}$$

for the circularly polarized waves (23b). Note that $G = g_{xx}g_{yy} - g_{xy}{}^2$.

Now, in comparison with eq. (15) and eq. (16), one can see that

$$\mathrm{Re}\left[T(\tilde{E})_{ab}\right] = \{T(E)_{ab} - T(P)_{ab}\}, \tag{25}$$

i.e. the source is the real part of the energy tensor of the complex wave. Thus, duality is implicitly included

in the complex wave functions. Moreover, it seems, the utilization of complex waves has a deep meaning rooted in general relativity (see Appendix D). However, the meaning of an electromagnetic energy-stress tensor of a complex wave in electrodynamics remains to be further clarified by investigations.

The time average of $\mathrm{Re}[T(\tilde{E})_{ab}]$ for any plane-wave, in the flat metric approximation, is zero. Thus, eq. (25) may be extended as a general relation beyond the monochromatic plane-waves. Our interest in plane-waves derives from the presumption that at great distances from a finite source of waves, these waves must appear to be approximately plane [34]. Based on this presumption of spatial local idealization (see also § 9), it is conjectured that the validity of eq. (25) can be extended to any electromagnetic wave.

9. Discussions and Conclusions.

In general relativity, it is desirable to have criteria which enable one to obtain a physical solution and to determine the appropriateness of a source tensor without considering all the mathematically possible solutions. (Also, the belief that an unbounded solution can be valid [10,14,25,34,47] has made it even more difficult to decide.) To this end, related physical considerations (such as the bending of light, compatibility with the notion of weak gravity, relativistic causality, and polarization) and physical principles (such as the principle of causality, the equivalence principle, and the correspondence principle) would play a crucial role.

The bending of light manifests that there should be a distinct energy-stress tensor for the photons. This, would further clarify the meaning of duality, and supports that the quantum theory, in which gravity is neglected and an electromagnetic wave and related photons are considered as having the same energy-stress tensor, is not the final theory. Moreover, the gravitational effects, though very small, are the crucial factor in distinguishing the photonic energy tensor from the electromagnetic energy tensor. Thus, the perception that general relativity describes only the large structure of the universe whereas quantum mechanics deals with phenomena on extremely small scales, is proven to be problematic. This analysis supports Einstein's viewpoint that general relativity would encompass every fundamental aspects of physics. He [36] wrote, "The comparative smallness of what we know today as gravitational effects is not a conclusive reason for ignoring the principle of general relativity in theoretical investigations of fundamental character".

Since the photonic energy tensor is distinct, an electromagnetic energy-stress tensor may not be the only

source term. To examine the validity of a source tensor, one should choose a simple situation and then identify the physical requirements. Since physical influences can propagate with at most the light speed, the influence of an electromagnetic wave on its gravity must be spatially local. Therefore, an electromagnetic plane-wave, as a spatial local idealization, is a valid physical modeling to obtain the accompanying gravity. (Nevertheless, in disagreement with the light bending experiment, existing solutions are not bounded [12].)

For the case of an electromagnetic plane-wave, the principle of causality implies that the metric is a bounded plane-wave with transverse components. Then, duality and the Einstein tensor imply that, for a circularly polarized plane-wave, the metric is also circularly polarized (§ 4). This compatibility gives us added confidence to the Einstein tensor, and therefore the essential validity of the present form of Einstein equation.

However, since G_{tt} of a circularly polarized gravitational plane-wave is positive, validity of the Einstein equation is impossible unless there is another source tensor with an ***anti-gravity*** coupling. To obtain a physical solution, it turns out that the other tensor must be an energy-stress tensor for the photons (§ 4).

Thus, if the framework of general relativity is essentially valid, the remaining question becomes what is a photonic energy-stress tensor? For such a tensor, there are four physical requirements: i) It produces the ***null geodesic*** equation for the photons; ii) It makes, on the time average, the total source energy negative; iii) In the flat metric approximation, on the time average it should equal to the electromagnetic tensor as required by experiments; iv) The polarization of the resulting gravitational wave matches the polarization of the electromagnetic wave. Conditions i) and ii) require a photonic energy tensor distinct from an electromagnetic energy tensor. But, condition iii) requires them to be intimately related. In spite of these seemingly contradictory conditions, calculation shows that the resulting photonic energy tensor leads to the fulfillment of condition iv). It is interesting that the photonic energy tensor is also derived from the electromagnetic wave (§§ 4-6).

The antigravity coupling implies that an energy-stress tensor of photons is the sum of the energy-stress tensor of the electromagnetic wave, and another part which provides for space-time curvatures of the gravitational wave components. Thus, it is clearer how electromagnetic waves and the photons are related. Although this would be a step to reconcile the difficulty between physical concepts and experiments, one should not consider this as suggesting that general relativity is now in a position to substitute quantum theory.

Moreover, because the photonic energy tensor $T(P)_{ab}$ must be included with an antigravity coupling, Ein-

stein's equation naturally produces an equation of motion also for the light. $\nabla^c T(P)_{cb}$ generates the geodesic for photons; but $\nabla^c T(E)_{cb}$ generates possibly only the Lorentz force and the equation of motion for a massive particle remains the same. Also, since the gravity due to light is verified to be very weak, its negligence in the calculation of light bending is justified within general relativity. Thus, the light bending as due to ***duality and the antigravity coupling of photons*** is another triumph of general relativity. Moreover, the bending of light would actually be the earliest ***experimental proof for the gravitational waves.***

Nevertheless, a skeptic might declare simply that the nonlinear Einstein–Maxwell equations do not allow a plane–wave. But, such a speculation is meaningless since it is not supported by physical considerations and certainly does not help solving the related problems. Experimentally, the bending of light requires, due to the accompanying gravitational wave, a distinct photonic energy tensor which generates a geodesic equation. The gravity due to a light beam must be negligible. The plane–waves are only examples to illustrate these needs. Although existing solutions for gravity of such a source [24–26] violate the equivalence principle as well as causality (see Appendix B), the electromagnetic plane–wave is a well–tested modeling in electrodynamics.

In general, the principle of causality implies that an electromagnetic wave is accompanied with a gravitational wave with a bounded amplitude. But, the gravitational waves are associated with the antigravity coupling [31], whereas the coupling to electromagnetism has the same sign of that of massive matter. Hence, the photonic energy tensor is necessary. Also, without a photonic energy tensor, the energy conservation would not allow to have a gravitational wave. ***Thus, within the theoretical framework of general relativity, there is no alternative other than obtaining a distinct photonic energy-stress tensor.***

For a linearly polarized electromagnetic waves, gravity is generated by a term of second order in time–average, only a very small portion of the total energy. For a circularly polarized wave, the gravitational wave component can even be zero. This establishes explicitly the dependence of gravity on the form of energy. Such a dependence would be crucial to resolve the problem of the so–called inevitable gravitational collapse. Also, the accompanying wave manifests that a gravitational wave propagates in a same manner as the light [37].

If the gravitational radiation is associated with an antigravity coupling, the inadequacy of the source is not limited to the case of electromagnetic waves. Indeed, due to theoretical consistency and agreement with experiments [31], the Einstein equation for massive matter has to be modified as follows:

$$G_{ab} \equiv R_{ab} - \frac{1}{2} g_{ab}R = -K\,[T(m)_{ab} - t(g)_{ab}], \tag{26}$$

where $T(m)_{ab}$ and $t(g)_{ab}$ are respectively the energy-stress tensors for massive matter and gravity. Then,

$$\nabla^c\, T(m)_{cb} = 0, \qquad \text{and} \qquad \nabla^c\, t(g)_{cb} = 0, \tag{27}$$

because of the coupling signs. (Note that, in eq. (13), $\nabla^c\, T(E)_{cb} = \nabla^c\, T(P)_{cb} = 0$.) If gravity is generated by massive matter, then $Kt(g)_{ab}$ is expected to be of second order. This tensor verifies that the antigravity coupling is a ***physical*** feature. Thus, eq. (26) confirms also the necessity of a photonic energy tensor.

It has been a long way to arrive eq. (26), the Einstein equation of 1995 version necessitated by experiments. The 1915 version is supported only for the cases of weak and static gravity [31,38]. In 1953, Hogarth [39] conjectured that, for the two-particle problem, the energy of gravitational radiation must be represented in the source tensor. In accord with this, Lo [12] proved in 1992 that there is no bounded plane-wave solution. Before that, theorists [40] were unaware of that the "plane-waves" (satisfying $R_{ab} = 0$) are unphysical. Also, theorists such as Bonnor, Griffiths and MacCallum [41] have questioned the physical validity of known time-dependent solutions because of the nature of their singularities.

The antigravity coupling, which is necessitated by the gravitational waves, is the most profound difference between general relativity and Newtonian gravity which allows neither the gravitational wave nor an antigravity coupling. This coupling in eq. (26) explains why there is no dynamic solution for the 1915 version, and implies that the energy conditions in the singularity theorems [29,30] are not valid in physics. Detailed discussions of the antigravity coupling and its implications will be provided in a separate paper [32].

The necessity of a photonic energy-stress tensor illustrates that there is a connection between general relativity and quantum theory. This is further manifested by the fact that a general form of the photonic energy tensor can be derived from the electromagnetic energy-stress tensor of the related complex wave in quantum electrodynamics. (Note that Einstein had hoped [42] that a modified Maxwell equation would have light-quantum solutions.) It is interesting to note also that both general relativity and the concept of photon were proposed by Einstein. As shown, these two seemingly independent theories are actually inextricably related.

Einstein maintained the hope for a total solution on the lines of a classical field theory until the end of his life. Pauli [18] considered this would be the great open problem of the relation of relativity theory to quantum theory. But, he pointed out that a clear connection between the general theory of relativity and quantum mechanics is not yet in sight. (This viewpoint could be challenged, however, since Bohr has to use general relativity to justify his uncertainty principle [42,43].) Now a connection between general relativity and quantum theory is clearly there. Would Einstein's dream occupy physicists for a long while to come?

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Appendix A: Dynamics of Particles, Electromagnetism, and Theoretical Consistency in General Relativity

Some theorists believe that only accurate predictions of a theory is important, but not consistency, since a current theory does not deal with all aspects of physics. For instance, the Maxwell's theory cannot explain why a charged particle can exist. Obviously, this incorrect view is not shared by Einstein since a motivation of general relativity is theoretical consistency. Moreover, one should observe that while the question of charge is beyond Maxwell's theory, gravity of electromagnetism is necessarily an integral part of general relativity.

In general relativity, there are two central problems, namely, 1) the source T_{ab} of the field equation $G_{ab} = -KT_{ab}$ and 2) the equation of motion for a particle. Also, there is a problem of consistency since

$$\nabla^c T_{cb} = 0 \tag{A1}$$

is implied by $\nabla^c G_{cb} \equiv 0$, and eq. (A1) must be compatible with the equation of motion. Thus, in principle, <u>all</u> the information should be derivable from the source T_{ab}. For massive matter, usually $T(m)_{ab} = \rho u_a u_b +$

$p(u_a u_b - g_{ab})$, where ρ is the mass density, p is the pressure, and $u^a = dx^a/d\tau$ is the velocity of ρ [13].

In general relativity, for a neutral particle, the equation of motion is the geodesic equation [21],

$$u^c \nabla_c u^b = d^2x^b/d\tau^2 + \Gamma^b{}_{ca}\, dx^c/d\tau\, dx^a/d\tau = 0 , \quad \text{where } d\tau^2 = g_{mn}\, dx^m\, dx^n , \qquad (A2)$$

and $\Gamma^b{}_{ca} = (\partial_c g_{ka} + \partial_a g_{kc} - \partial_k g_{ca})\, g^{bk}/2$. Observation confirms the validity of eq. (A2).

However, the equation of motion is <u>insufficient</u> to determine the source since an additional divergence free tensor can always be added. On the other hand, an appropriate source tensor must lead to a physical solution. Einstein [22] was aware of this difficulty, and in fact, encouraged a modification of the source term. Thus, an appropriate source tensor is a difficult problem related to the <u>validity</u> of the theory [13].

Consider a particle with charge e and mass m. The accepted equation of motion for a particle is

$$m\, u^c \nabla_c u^b - e\, F^b{}_c\, u^c = 0 , \qquad (A3)$$

where $F^b{}_a$ is the electromagnetic field tensor. In equation (A3), the radiation reaction force has been omitted. For a continuous charge distribution, the second term in eq. (A3) should have the form $\sigma\, F^b{}_c\, u^c$, where σ is the charge density. The Lorentz force $-\sigma\, F^b{}_c\, u^c = \nabla_c\, T(E)^{cb}$, the divergence of the electromagnetic energy-stress tensor. Thus, ***electromagnetism must be involved as a source in Einstein's equation.***

An accelerated charge and its field do not simply move together since an electromagnetic wave would be emitted. If the equation of motion for the electric energy were related to a geodesic equation, the electric energy ***would be*** attracted by a massive particle with a force $\approx -\theta c^2 \Gamma^a{}_{tt} \approx -\theta \frac{c^2}{2} \partial g_{tt}/\partial x^a$, where θ is an electric energy density. But a charge creates a repulsive force (of different r-dependence according to the Reissner-Nordstrom metric) on another massive particle. Thus, the interaction between these particles would be viewpoint-dependent. In other words, a theoretical inconsistency would be created.

Experiment requires that an energy-stress tensor of light $T(L)_{ab}$ must be distinct from $T(E)_{ab}$ since

$$\nabla_c T(E)^{cb} = 0 \qquad (A4)$$

cannot generate a geodesic equation. On the other hand, if the light includes other than electromagnetic radiation, its energy-stress tensor must be different from $T(E)_{ab}$. Moreover, in current theory of general relativity, a photonic geodesic equation cannot be derived from the Maxwell-Einstein equations [14].

One might disregard this failure as unimportant; and argue that the geodesic equation and/or the Huyghen's principle can simply be added to the theory. However, the validity of these methods actually depends on an unproven assumption that general relativity is self-consistent in physics. Thus, whether it is possible to derive the experimentally supported geodesic equation for photons within the theoretical framework of general relativity, would be a test of the theory's viability. In other words, on the contrary, this difficiency is important because it manifests that the current theory is ***not yet self-consistent for dynamic problems*** (see § 9).

For dynamic problems, the theoretical self-consistency of general relativity has not been adequately addressed. The three well-known tests examine only the static gravity. In the light bending calculation, the gravity due to the presence of light is neglected. However, it was not clear whether the theoretical gravity generated by the light is indeed weak as observed (see § 1), although "the equivalence principle appears as the best bridge between the theories of gravitation and of elementary particles [6]." The existing solutions violate the equivalence principle which is critical for a valid application of $ds^2 = 0$ [22,27].

For massive sources, the inadequacy of the Einstein equation for dynamic problems was first raised in 1921 by Gullstrand [44,45] in his report to the Nobel Committee. (On the perihelion effect, he expressed the opinion that other, long-known deviations from the pure two-body Newtonian law should be re-valuated with general relativistic methods before there could be even an attempt to identify the residual effect to be explained [42].) The non-existence of dynamic solution was recently proved by Lo [31] in 1995. It has also been shown the impossibility of having a gravitational plane-wave for $G_{ab} = 0$ [12] in 1997.

Meanwhile, in 1936 the existence of wave solution was questioned by Einstein [7] himself. In 1953 the non-existence of dynamic solution was conjectured by Hogarth [39]. However, in 1993 Christodoulou and Klainerman [46] claim that "dynamic solutions" in vacuum can be constructed from the strong asymptotically flat "initial data sets". But, they have not proved that their "initial data set" is related to a dynamic source as physics requires. Thus, they actually have not proven the existence of a dynamic solution. Moreover, their initial data sets are incompatible with the field equation for weak gravity and Einstein's radiation formula.

Appendix B: The Principle of Causality, Symmetry, and Validity of a Field Equation

The concept of causality describes the ideas of cause and effect (which can be probabilistic). There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacua. This requirement restricts also the choice of space-time coordinates [12,23, 27]. The time-tested assumption that ***phenomena can be explained in terms of identifiable causes*** will be called the principle of causality. Hence, any parameter in a solution for physics must be related to some physical causes. Here, this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution, and consequently in the validity of a field equation in physics.

In practice, based on observation when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. Then, any deviation from the normal state must have physically identifiable causes. Since the principle of causality implies that symmetry breaking must have causes, a symmetry must be preserved if no cause breaks it. For example, the electromagnetic field is zero in a normal state. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law.

In general relativity, the ***normal state*** of a metric is the flat metric in special relativity. The constant flat metric possesses all the symmetry allowed by special relativity. Thus, if a non-constant metric does not possess a certain symmetry, then there must be physical cause(s) which has broken such a symmetry. In other words, the metric should have at least the same symmetry as its physical cause(s). For example, in the Schwarzschild solution, causality requires that the metric is spherically symmetric and asymptotically flat. Note that, in a spherical coordinate system, the angular dependence of this metric is related to the infinitesimal solid angle $d\Omega$ because such a dependence exists inherently for the normal state [37]. Also, without the principle of causality, the flat metric would not be the only constant solution for an empty space.

However, ***the physical cause(s) should not be confused with the mathematical source term in the field equation.*** Such a confusion would be possible because, for some situations, such a distinction does not seem to be meaningful. For instance, in electrodynamics, the physical cause of an electromagnetic field and the source term in Maxwell's equation, are the same charge currents. In general relativity, the cause of gravity is the physical matter, but not the source term in Einstein's field equation. The energy-stress tens-

ors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. In the Schwarzschild case, the cause is the mass distribution. Thus, one should not infer the symmetries of the metric based on the source term (instead of its causes) although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. Sometimes, the source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave is not rotationally invariant with respect to the direction of propagation. But the related electromagnetic energy-stress tensor can be rotationally invariant and even be a constant [12]. In the literature (see §13 & §21 of [10], eq. (35.40) in [14], and also [24-26]), the metric is incorrectly assumed to be rotationally invariant. This assumption violates causality and results in theoretical difficulties (see [12,23]).

Classical electrodynamics implies that the flat metric is an accurate approximation of a bounded metric, caused by the presence of a weak electromagnetic plane-wave. This physical requirement is supported by the principle of causality which implies such a metric to be a bounded periodic function. However, this boundedness, though also required by experiments, is not satisfied by solutions in the literature [24-26]. These solutions, in fact, violate causality directly since they involve parameters without any physical cause (see [23]). Moreover, although these solutions have the metric signature of a Minkowski space, in contrast to the incorrect belief on a Lorentz manifold [5,9,14,47], they do not satisfy the equivalence principle [12,23,27,28].

Theoretically if any physical principle is violated, then the equivalence principle should not be satisfied; otherwise its validity as a physical principle would be questionable. Also, although the principle of causality and relativistic causality are distinct physical requirements, in practice they are often either satisfied or violate simultaneously [23,27]. Apparently, there are invisible connections among physical principles.

The compatibility of symmetry due to the principle of causality is a physical requirement. Thus, symmetry consideration can be used as a criterion for the validity of a solution and even the field equation.

Nevertheless, for some equations, the symmetries of a solution can be very different from that of the source term (which may or may not be the physical cause). For example, consider the following equation,

$$\eta^{mn}\partial_m\partial_n F(t,x,y,z) = [4\partial_u\partial_v - \partial_x\partial_x - \partial_y\partial_y] F(t,x,y,z) = f(u), \tag{B1}$$

where η_{ab} is the flat metric $(+,-,-,-)$, $u \equiv (t - z)$, and $v \equiv (t + z)$. If F is a function of only t and z, then the inhomogeneous solution of eq. (B1)) depends not only on u but also v, and is

$$F(t,z) = \frac{v}{4} \int^{u} f(t)\, dt\ . \tag{B2}$$

It should be noted that, due to the factor v, F(t,z) is not bounded even for a *very small* source term f(t).

Then, one may examine a field equation after the related physical cause is identified. The left-hand side of eq. (B1) can be considered as a Maxwell's equation or an equation in linearized gravity. For the case of Maxwell's equation, the principle of causality implies that the source term may not be in the form of plane-waves. This restriction is satisfied physically because, in nature, a charged particle is invariably massive.

For linearized gravity, the function F(t,x,y,z) in eq. (B1) relates to the deviations from a flat metric. (An implicit assumption of weak gravity is that an empty space has a flat metric. This assumption is identical to the requirement of a normal state.) If the physical cause is an electromagnetic plane-wave propagating in the z-direction, then the related source energy-stress tensor can be a function of u [10,14,24-26], and its lowest order approximation is a function of u, and thus the source term in linearized gravity would have the form f(u). Then, according to solution (B2), the metric would have a factor v and is unbounded. (Theorists were not aware of that such an unbounded solution is also in disagreement with experiments [25].)

On the other hand, the principle of causality implies that the metric is a function of u only [10,12,14]. This contradiction suggests that, for gravitational waves, eq. (B1) is not an appropriate form. Thus, causality implies that ***there are weak gravity exact solutions, which cannot be approximated with linearized gravity.*** One might argue that a solution could be a function of only u through a gauge transformation. This is not possible physically nor mathematically since a flat space-time metric has to remain flat.

In general, the principle of causality implies that, in agreement with the correspondence principle, a weak (non-singular) source should produce a weak field and that a physical solution is bounded in magnitude (see also [35]). Thus, the basis of Einstein's notion of weak gravity is also the principle of causality.

Appendix C: Compatibility of the Polarizations of Plane-Wave Components

Einstein's field equation shall be examined with different polarizations. Then, the relations between the polarizations of electromagnetic and gravitational wave components are established. It should be noted also that an inappropiate polarization would result in an unphysical solution.

It has been established that for a circularly polarized electromagnetic wave of frequency ω the solution is also a circularly polarized gravitational wave, but of frequency 2ω. Then, its curvature tensor is

$$R_{tt} = 2\omega^2 B_\alpha{}^2/G \ , \quad \text{where} \quad G = (1 + C_1)^2 - B_\alpha{}^2 , \tag{C1}$$

and

$$T(E)_{tt} = \omega^2 A_0{}^2 (1 + C_1 - B_\alpha \cos\alpha)/2G \tag{C2}$$

where B_α and A_0 are the gravitational and the electromagnetic wave amplitudes. C_1, the determinant g and g_t are constants. The frequency ratio suggests that $-K A_0{}^2$ is of first order of deviations. Thus, $\omega^2 A_0{}^2/2G$ in (C2) must be canceled by the photonic energy-stress tensor. (This means that, in the flat metric approximation, an electromagnetic wave and its photons carry, on the average, the same energy-momentum.)

Now, we consider also a linearly polarized electromagnetic wave, whose only non-zero component is

$$A_x = A(t - z) = A_0 \cos[\omega(t - z)] \ . \tag{C3}$$

From equation (6), the equation of the lowest order terms is

$$f''/2 = - K (A')^2 , \quad \text{where} \quad -f(u) \equiv (g_{xx} + g_{yy}) + 2 \tag{C4}$$

Indeed, there is a constant $-K\omega^2 A_0{}^2/2$ to be canceled. The modified equation of the lowest order should be

$$f'' = - 2K T(u), \tag{C5}$$

and the time average of T(u) is zero. Now, equation (6a) becomes

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 - G'(g'/2g) = 2K\, g_{yy}T(u). \tag{C6}$$

There are four unknowns in eq. (C6). The previous case suggests that eq. (C6) can be reduced.

From eqs. (7) and (C4), only one of g and g_t can be a constant. (The Schwartzchild solution suggests that g would likely be a constant.) Nevertheless, it is still possible to simplify (C6). To this end, define

$$F_1'' \equiv -2K\,T(u)\ , \quad \text{and} \quad f \equiv f_1 + F_1(u) + F_2(u)\ , \tag{C7}$$

where $F_2(u)$ is of second order and f_1 is the time average of f. Then, one obtains

$$G'' - g_{xx}'g_{yy}' + (g_{xy}')^2 = -(fg_{yy}' + F_2'g_{yy})' - F_1''g_{yy} + F_2'g_{yy}'$$

$$- [2\gamma_{yy}\gamma_{yy}'' + (\gamma_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2]. \tag{C8}$$

where the deviation $\gamma_{ab} = g_{ab} - \eta_{ab}$. Then (C6) is reduced to

$$G'(g'/2g) + (fg_{yy}' + F_2'g_{yy})' - F_2'g_{yy}' =$$

$$- [2\gamma_{yy}\gamma_{yy}'' + (\gamma_{yy}')^2 + 2g_{xy}g_{xy}'' + (g_{xy}')^2]. \tag{C9}$$

where

$$G' = -(fg_{yy})' - 2[\gamma_{yy}\gamma_{yy}' + g_{xy}g_{xy}'].$$

For a physical solution, it requires that the time average of the term $(fg_{yy}' + F_2'g_{yy})'$ is zero.

If g is a constant, then eq. (C9) implies $g_{yy}' = g_{xy}' = 0$, since the time average of the right hand-side of eq. (C9) cannot be positive. These equations in turn imply $F_2'' = 0$, and consequently $F_2 = 0$. Thus, both

g_{yy} and g_{xy} are constants, and equation (C6) is reduced to

$$g_{xx}'' = 2K\,T(u). \tag{C10}$$

If the constants are independent of the wave amplitude, then one has

$$g_{xy} = 0\ , \text{ and } \ g_{yy} = -1\ . \tag{C11}$$

On the other hand, in general, (C11) implies

$$G'(g'/2g) - F_2'' = 0\ . \tag{C12}$$

It follows that g_t cannot be a constant in a linear polarization. To illustrate this, let us assume

$$g_{tt} = 1\ ,\ g_{xy} = 0\ , \text{ and } \ g = -1\ . \tag{C13a}$$

as suggested by linearized gravity. Then equation (6), with the photonic energy tensor $T(P)_{ab}$, becomes

$$(g_{xx}')^2 = 2K\,g_{xx}\left[(A')^2 + g_{xx}T(P)_{tt}\right]\ . \tag{C13b}$$

Without a photonic energy-stress tensor (i.e., $\lambda = 0$), the solution for eq. (C13) is:

$$g_{xx} = -\,(1 + C \pm i\sqrt{K/2}\,A)^2\ , \quad \text{and} \quad g_{yy} = g_{xx}^{-1}\ , \tag{C14}$$

where C is a constant which is zero when $A = 0$. The imaginary sign comes from the fact that special relativity is an accurate approximation. Solution (C14) is not physical because it is essentially imaginary for a real electromagnetic wave. Moreover, the frequency ratio between gravitational and electromagnetic wave

components should be two. Also, it is easy to see that no value of λ can make g_{xx} a real wave function.

If one assumes, in violation of causality as Misner et al. [14, eq. (35.40)] did, that

$$g_{xx} = g_{yy} , \quad g_{xy} = 0 , \quad \text{and} \quad g_{tt} = -g_{zz} = 1 , \tag{C15a}$$

then, with the photonic energy tensor $T(P)_{ab}$ and its coupling constant $K\lambda$, the resulting equation is

$$2LL'' = -K\left[(A')^2 + \lambda L^2 T(P)_{tt}\right] , \quad \text{where} \quad g_{xx} = -L^2 . \tag{C15b}$$

For $\lambda = 0$, equation (C15b) implies that L is not bounded. For $\lambda = -1$, equation (C15b) becomes

$$2LL'' = K\omega^2 A_0^2 \cos(2\omega u) . \tag{C16}$$

However, equation (C16) implies that L is not a periodic function of u since L is approximately 1 for weak gravity. In fact, there is no λ which can make L a periodic function. This example demonstrates again that Misner et al. do not fully understand the physical meaning of plane-waves [12].

In the above calculations, the sign of R_{tt} and an appropriate polarization are crucial for obtaining a physical solution. Also, the unverified assumption that there are bounded gravitational plane-wave solutions for $R_{ab} = 0$ [14,47] is incorrect [12]. Moreover, through exact solutions such as gravitational waves (11), (18), and (19), it can be shown that a solution of the linearized equation can be totally unrelated to an approximation of the exact solution [23].

Appendix D: Real-Complex "Wave-Duality" and Particle-Wave Duality.

In § 8, it is not clear how the imaginary part of the energy tensor $\mathrm{Im}[T(\tilde{E})_{ab}]$ is related to the metric g_{ab}. Here, we consider this problem further from the viewpoint of duality. In classical electrodynamics, a real wave can be considered as the real part of a complex wave. These waves satisfy related Maxwell's equations in which the source term of the real equation is the real (or imaginary) part of the complex source. It seems

that such a "wave duality" is only a mathematical convenience and that complex waves are mathematical auxiliaries. In quantum electrodynamics (QED), however, complex wave functions must be used although the field operator is a hermitian operator [48] and it is impossible to have QED based on real functions alone. This suggests that ***the wave-duality should have a physical origin, particle-wave duality.***

If "wave-duality" indeed has a physical origin, then wave-duality should also be valid in general relativity eventhough the equation is nonlinear. This conjecture is strengthened by the relation that the real part of the energy-stress tensor $T(\tilde{E})_{ab}$ of a complex wave is the modified source tensor (13), i.e.

$$\mathrm{Re}\left[T(\tilde{E})_{ab}\right] = T_{ab}(E) - T_{ab}(P). \tag{D1}$$

Eq. (D1) implies that, for a static electromagnetic field, $T(\tilde{E})_{ab}$ is real. Since its imaginary part may not be zero, from the viewpoint of completeness, complex $\tilde{T}(E)_{ab}$ should satisfy a complex Einstein equation, i.e.,

$$\tilde{G}_{ab} = -K\,\tilde{T}(E)_{ab}; \tag{D2}$$

and the real part of $T(\tilde{E})_{ab}$ of a complex gravitational wave satisfies a modified real Einstein equation. This would be compatible with that, ***whereas particle-wave duality is explicitly manifested in a real Einstein equation; duality is implicitly included in a complex Einstein equation.***

Eq. (D2) is supported by the facts that eqs. (2) and (3) are valid for complex functions and that the geodesic equation and the generalized Maxwell's equation can be extended to a complex metric. Since eq. (20) has only transverse metric elements, one may expect that wave-duality is valid for those elements. However, since the metric is semi-unitary, one may expect wave-duality to be only approximately valid for g_{tt} and g_{zz}. In short, wave-duality is valid at least for weak gravity.

To see that wave-duality is valid for directionally polarized electromagnetic waves, let us consider the complex wave (23a) related to the wave (16a), the complex gravitational metric elements are:

$$-g_{xx} = 1 + C - KA^2/2, \qquad \text{and} \qquad g_{tt} = -g_{zz} = \sqrt{g\,/\,g_{xx}}\ , \tag{D3}$$

where C is a complex constant. These metric elements satisfy equations (2), (3), (7), and (22). For wave (D3), eq. (22) is simplified to the following differential equation

$$G'' = -2K(A')^2, \qquad \text{where} \quad G = -g_{xx}. \tag{D4}$$

To further support wave-duality, one can calculate the case of circularly polarized electromagnetic waves. A circularly polarized electromagnetic complex wave would be wave (23b). Then, the gravitational complex wave should also be circularly polarized. Since equation (21) implies $\tilde{T}(E)_{tt}$ in (D2) is zero, eq. (21) and eq. (D2) imply that the gravitational complex wave is circularly polarized as follows:

$$g_{xx} = -1 - C + B, \quad g_{yy} = -1 - C - B, \quad \text{and} \quad g_{xy} = \pm iB, \tag{D5a}$$

where

$$B = B_\alpha \exp\{-i[2\omega(t - z) + \alpha]\}, \qquad C = C_1 \pm iB_\alpha,$$

and

$$G = (1 + C)^2 = (1 + C_1)^2 - B_\alpha^2 \pm i2(1 + C_1)B_\alpha. \tag{D5b}$$

Formula (D5) indeed further confirms wave-duality.

The above calculations confirm that the complex Einstein eq. (D2) and Einstein eq. (13) are compatible. While (D4) for the linear polarization is a linear equation, the circular polarization involves a non-linear equation. Thus these confirmations strongly support the conjecture that wave-duality has its origin from particle-wave duality. Note also that eq. (D1) can be used to calculate beyond the plane-waves.

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