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Properties of Null-instanton Model from Magnetic Moments and Masses of Baryons and Nuclei

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The null-instanton model of baryons are studied based on the new experimental information and its important role in low-energy physics is clarified from the magnetic moments and masses of the nuclear states.

$ 1. Introduction

Recently, Bukina[1] has compared the magnetic moment results in the quark model with null instantons[2] and semibosonized Nambu-Jona-Lasinio (abbreviated as NJL hereafter) model[3] and pointed out that the difference between two models lies essentially in the evaluation of the magnetic moment of the Λ-hyperon. In the course of discussion she quoted the parameter values based on the old set of data in spite of the improvement of the experimental values[4], though it is not essential for her conclusion. There is an old question why the magnetic moments of odd-A nuclei deviate from the single-particle Schmidt lines.[5] Miyazawa[6] has proposed the quenching of the pion cloud for the bound nucleon, while Arima and Horie[7] (abbreviated as AH herefater) the configuration mixing of nucleons based on the nuclear shell model. Both models admit to explain the data appearing in between two Schmidt limits for both odd-Z and N nuclei, however, the magnetic moments which lie outside of the two limits are left unanswered.

In what follows if we admit the variation of two parameters of null-instanton unitary symmetric SU_6 model in the nuclear matter the difficulty pointed out above will be resolved by a simple picture. Accepting the validity of similar thought to the baryon masses in the nuclear matter the rapid variation of the binding energies of light nuclei[8] will be reduced to the same cause.

First of all we re-examine the parameters associated with the baryon magnetic moments under the new light of recent data in order to provide a standard in proceeding to test our proposal. An additional purpose is to examine the magnetic moment of the Λ-hyperon of our model numerically so as to give an indirect answer

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to the Bukina's proposal. In the course of this study we shall give additional sum rules among nine members of baryon magnetic moments and examine their physical meaning.

In nuclear physics one has to extract the single particle moment which will be used to find the corresponding parameter change of interest. This quantity of interest, which will contain solely the symmtry-breaking effect, will be obtained by combining suitably the experimental,[9] Schmidt and AH's results for individual nucleus. The magnetic moments of odd-odd nucleus are also used for the same purpose with some care without introducing the configuration mixing by extending the Schmidt method straightforwardly.

The standard variation of the broken symmetry in baryon masses becomes the ratio of mass term from instanton contribution and effective quark mass with appropriate numerical factor in the mass formula.[10] There is no definite standard of the variation of broken symmetry for the nucleon mass in nuclei. For this purpose we propose to take the modified packing fraction (mpf) as a numerator, where the binding energy contains the Coulomb correction. The denominator is chosen to be a coefficient of the volume term of Weizsäcker-Bethe mass formula,[11] by taking its proportionality to the mass number into account. Therefore the broken symmetry effect of the null-instanton model on bound nucleon mass is defined rather arbitrarily. However the satisfactory results found reliably from the studies, starting from the common theoretical basis, is of a great interest if it is confirmed further from an independent source.

$2 is devoted to the least-squares fit to the observed baryon magnetic moments with the null-instanton model and the test of sum rules. The magnetic moments of odd-A and odd-odd nuclei are studied in terms of null-instanton model in $3. A proposed study from the mass formula is presented in $4. The last section summarizes the result and suggests a probable test of the idea.

$ 2. Magnetic Moments of Baryons

Before entering into the analysis of the magnetic moments of baryons let us recapitulate the theoretical expression for them for the reader's sake. In the broken SU_6 symmetry with null-instanton picture we arrange them in the order of decreasing accuracy of presently available data:

$$\mu_p = (1+\frac{8}{9}z)\mu - \alpha , \qquad \mu_n = -\frac{2}{3}(1+\frac{1}{3}z)\mu ,$$

$$\mu_{\Xi^-} = \frac{1}{9}(-4\mu' + \mu) + \beta, \qquad \mu_{\Sigma^+} = \frac{1}{9}(8(1+z) + \mu') - \beta,$$

$$\mu_{\Lambda} = -\frac{1}{3}\mu' - \frac{1}{3}(\alpha - \beta), \qquad \mu_{\Xi^0} = \frac{1}{9}(-4\mu' - 2(1+z)\mu), \qquad (2.1)$$

$$\mu_{\Sigma^-} = \frac{1}{9}(-4\mu + \mu') + \beta, \qquad \mu_{\Omega^-} = -\mu',$$

$$\mu_{\Sigma^0\Lambda} = -\frac{1}{\sqrt{3}}(1 + \frac{2}{3}z)\mu + \frac{1}{6}(\alpha - \beta),$$

where z, α and β are the SU_6-breaking, the time averages for u-d and s-u (s-d) quark-exchange parameters in spin 0 state contained in an instanton and anti-instanton ball. The remaining two parameters μ and μ' are related to the u, d and s quark magnetic moments as

$$\mu_u = \frac{2}{3}(1+z)\mu, \quad \mu_d = -\frac{1}{3}\mu, \text{ and } \quad \mu_s = -\frac{1}{3}\mu'. \qquad (2.2)$$

In the least-squares fit we have repeated three fittings by (i) setting $\alpha = \beta = 0$, (ii) fixing three parameters μ, z and α determined from the first five states in Eq.(2.1) then released the remaining two parameters μ' and β free so as to minimize the chi-square and (iii) putting $\alpha = \beta$. The results (i) and (ii) are tabulated in Table I.

Table I. Magnetic Moments of baryons (n.m.)

baryon	SU_6	experiment	broken SU_6
p	2.79284739	2.79284739±0.00000006	2.79284739
n	-1.9130428	-1.9130428 ±0.0000005	-1.9130428
Ξ^-	-0.6108	-0.6507±0.0025	-0.6499
Σ^+	2.703	2.458 ±0.010	2.484
Λ	-0.701	-0.613 ±0.004	-0.598
Ξ^0	-1.552	-1.250 ±0.014	-1.417
Σ^-	-1.062	-1.160 ±0.025	-1.271
Ω^-	-2.10	-2.02 ±0.05	-1.98
$\Sigma^0\Lambda$	-1.63	-1.61 ±0.08	-1.56

The corresponding fitted parameters are summarized in Table II.

Table II. Parameters on fits in Table I

	(i)	(ii)
μ	2.915594286	3.095186725
μ'	2.1031	1.9765
z	-0.047362645	-0.21868392
α	---------	-0.299320732
β	---------	-0.1145
χ^2	1821	184

The third fit (iii) gives us

$$\mu = 3.016729592, \quad \mu' = 1.828, \quad z = -0.1463492709 \text{ and}$$
$$\alpha = \beta = -0.1685588436, \quad \text{with } \chi^2 = 558. \qquad (2.3)$$

This fit was repeated only as a memorial check of the best empirical rule found in the previous study. The reduced chi-square becomes 260, 36.7 and 92.9 for the fit (i), (ii) and (iii), respectively.

The estimated numerical value of the Λ magnetic moment in fit (ii) is not much different from the experimental value, so it seems too early to conclude on the validity of either the null-instanton model or the semibosonized NJL model unless we get a more refined information as discussed already by Bukina.

There are 9 independent experimental data. Each of them takes a different theoretical expression in our model with five parameters. In general one can construct 84 independent sum rules among them. It seems to be preferable to give some of them explicitly in order to decide which baryon should be chosen as the first object in the experimental studies.

For the moment 5 of them relatively accurately measured as pointed out at the beginning of this section. We shall give here 4 sum rules by joining the first five with the remaining four in succession. They are given by

$$8\mu_p + 3\mu_n + 4\mu_{\Xi^-} - 4\mu_{\Sigma^+} - 24\mu_\Lambda + 13\mu_{\Xi^0} = 0 \qquad (2.4a)$$

$$(37.05 - 34.42 = \pm(0.14 - 0.33)),$$

$$10\mu_p+20\mu_n+18\mu_{\Xi^-}-5\mu_{\Sigma^+}-30\mu_\Lambda-13\mu_{\Sigma^-}=0 \qquad (2.4b)$$

$$(61.40-62.26 = \pm(0.11-0.30)),$$

which is nothing but the one discussed by Bukina. She found one more sum rule in addition as an exception of the general combinatorial rule pointed out above. A new third one becomes

$$15\mu_p+4\mu_n+\mu_{\Xi^-}-14\mu_{\Sigma^+}-45\mu_\Lambda+13\mu_{\Omega^-}=0 \qquad (2.4c)$$

$$(69.48-68.97 = \pm(0.61-0.97)),$$

where the large error comes from that for Ω^-. Finally one finds

$$3(1+\sqrt{3})\mu_p+2(-5+\sqrt{3})\mu_n+(11-\sqrt{3})\mu_{\Xi^-}+4(2-\sqrt{3})\mu_{\Sigma^+}-12\mu_\Lambda+$$

$$6(3\sqrt{3}-1)\mu_{\Sigma^0\Lambda}=0 \qquad (2.4d)$$

$$(45.38-46.22 = \pm(1.99-2.11)).$$

Here the big error is caused, of course, from that involved in the last member of the sum rule. One learns from these analyses that the main bodies of the data for the last two states in (2.4c) and (2.4d), respectively, are rather accurately measured except for the error estimate, if one relies on the theory.

Before proceeding to next section it is convenient to define the standards of symmetry-breaking parameter z and α from baryon data suitable for nuclear physics. The z value of individual odd-N nucleus is un-ambiguously determined directly after the appropriate manipulation as will be shown in the next section. Thus we choose z found from baryon study in fit (ii) as the standard value.

$$z = -0.2187 \qquad (2.5a)$$

Assuming no additional symmetry-breaking occurs for the nucleons in the nuclear matter the Schmidt values of magnetic moments should be realized if we subtract the configuration mixing effect appropriately except for those staying outside of the Schmidt limits. In order to test the idea we have to standardize α to compare with that for odd-Z and odd-odd nuclei. There occurs three cases corresponding to three experimental situations: there exist (a) charge symmetric pairs, (b) no charge symmetric pairs and (c) odd-odd nuclei. We choose three standards α's:

the α found in fit (ii), the one extracted from the proton data alone by putting α = z and the estimated α value from the combined proton and neutron data by putting again α = z for case (a), (b) and (c), respectively. They are given by

$$\alpha = -0.2993, -0.1726 \text{ and } -0.1429 \text{ for (a), (b) and (c).} \quad (2.5b)$$

The last choice is most suitable for the charge symmetric odd-odd nuclei, although we take it as the standard for all odd-odd nuclei for convenience sake.

Notice that the idea of the symmetry-breaking effect on the nuclear magnetic moment can in principle be applied to any datum irrespective to its observed value.

$ 3. Odd-A and Odd-Odd Nuclei

Let us accept the variation of the symmetry-breaking parameters of free nucleons when they form the bound states.

In order to extract the suitable magnetic moments which should be compared with the Schmidt limits in the nuclear matter with the symmetry breaking we adopt the theoretical magnetic moments as

$$\mu_{th} = \mu_{Sch} - \mu_{\exp} + \mu_{AH}, \quad (3.1)$$

where μ_{Sch}, $\mu_{\exp}$ and μ_{AH} are the magnetic moments of Schmidt limit, experimental and AH value, respectively. The z and α for odd-N and Z nuclei may simply be estimated by substituting μ_{th} into the Schmidt formula following the classification suggested at the end of $ 2.

AH have estimated the effect of the configuration mixing in the first-order perturbation theory. They took into account the spin one unit excitation belonging to the same parity in both un-mixed and mixed configurations. All six contributions give the same sign so as to quench the magnetic moments in between the Schmidt limits common to odd-A nuclei. Owing to this fact and the observed anti-quenching found for the light nuclei the analysis was not carried out for nuclei up to A=16. The quenching mechanism was proposed by Miyazawa due to that of pion cloud for the bound nucleon irrespective to the nuclear structure. We are interested in the structure dependence of the effect so we confine ourselves to the former model. Owing to the perturbative nature of this model we have to retain two effective significant numbers in the numerical study. The all perturbative

effect vanishes for 1/2-($p_{1/2}$-) nucleon in the same approximation. We shall retain one more significant numbers in the estimates of z and α for $p_{1/2}$ nuclei. The average excitation energy appears as the denominator of the first-order matrix element in the estimate of expansion coefficient for the mixed configuration. The average excitation energy of the allowed configuration in light nuclei up to A=16 takes about 10 times or more relative to that for higher A, so we retain also effective three significant numbers without introducing the configuration mixing. In the case of odd-odd nuclei the additional configuration mixing appears than that in AH in the same approximation. For simplicity, we shall neglect all configuration mixing effect for odd-odd nuclei in our crude estimate of symmetry-breaking parameter by putting α = z for each nucleus. Here we have also chosen the single shell assignment for unpaired proton and neutron so as to keep the absolute magnitude of α as small as possible where the order of the shell structure alternates.

It is not difficult to estimate the configuration mixing effect once they are specified for odd A nuclei. We are interested in the shell structure dependent variation of the symmetry breaking parameters z and α. For our present purpose it will be sufficient to choose the odd-A nuclei with configuration mixing evaluated by AH. We have also chosen the calculated value for a single set of mixed configuration in ref.[7] when it gives the μ_{AH} close to the μ_{exp}, otherwise the average value of a few configurations instead of choosing $\mu_{th} = \mu_{Sch}$ in order to get a crude estimate of α for such a nucleus. To avoid a complication we shall not specify explicitly such cases in the tabulation, since the most of the estimates have been done under the assumed single configuration, so our choice does not affect to the general tendency found for the variation of z and α discussed in this paper.

In order to keep 1-1 correspondence with the Tables in the paper of AH, we took the same numbering of Tables except for the fact that we include additional low A data by the reason already explained.

(i) (1/2+) nuclei

Table III. Symmetry breaking parameters of (1/2+) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α
$^{3}_{2}He$	-2.128	0.0938	$^{3}_{1}H$	2.979	0.374

					α = z
			$^{19}_{9}F$	2.82	-0.16
$^{29}_{14}Si$	-1.89	-0.25	$^{31}_{15}P$	3.02	-0.043
$^{111}_{48}Cd$	-1.80	-0.38			
$^{113}_{48}Cd^{*}$	-2.06	0.79			
$^{115}_{50}Sn$	-1.72	-0.50			
$^{117}_{50}Sn$	-1.79	-0.40			
$^{119}_{50}Sn$	-1.81	-0.37			
$^{123}_{52}Te^{*}$	-1.92	-0.21			
$^{125}_{52}Te$	-1.64	-0.62	$^{203}_{81}Tl^{**}$	2.62	-0.27
$^{129}_{54}Xe$	-1.60	-0.60	$^{205}_{81}Tl$	2.61	-0.28

Here the first , second and third column correspond to the symbol of odd-Z nucleus, its magnetic moment corresponding to the Schmidt value after correcting configuration mixing and the corresponding z parameter, respectively, except for the light nuclei. The similar applies to the 4-th and 5-th column for odd-Z nucleus. In the last column the parameter α is shown. The symbol * (**) putted at the right superscript on the nucleus symbol means that it is the natural (artificial) radioactive nucleus. We put the two odd-Z data in the last two lines in order to save the space, although there is no correspondence among nuclear structures. The same convention is used up to Table XII unless stated otherwise.

From Table I one sees that the z and α are rather sensitive to the inputs. There are no resemblance among these for A=3 and standard ones, while α =-0.16 found for $^{19}_{9}F$ nucleus is close to the standard value -0.17. The most of z estimated for odd-N nuclei deviate largely from standard -0.22 except for a few cases. It is not clear whether we should take seriously the few exceptions at this stage.

(ii) (1/2-) nuclei

Table IV. Symmetry-breaking parameters of (1/2-) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α
$^{13}_{6}C$	0.762	0.0619	$^{13}_{7}N^{**}$	-0.322	0.299
$^{15}_{8}O^{**}$	0.719	0.136	$^{15}_{7}N$	-0.283	0.620
					$\alpha = z$
$^{71}_{32}Ge^{**}$	0.546	-0.619	$^{89}_{39}Y$	-0.137	-0.391
$^{77}_{34}Se$	0.534	-0.671	$^{103}_{45}Rh$	-0.888	0.135
$^{169}_{68}Er^{**}$	0.513	-0.762	$^{107}_{47}Ag$	-0.114	-0.528
$^{171}_{70}Yb$	0.493	-0.850	$^{109}_{47}Ag$	-0.131	-0.513
$^{195}_{80}Hg^{**}$	0.538	-0.653	$^{111}_{47}Ag^{**}$	-0.146	-0.500
$^{195}_{78}Pt$	0.606	-0.357	$^{113m}_{49}In$	-0.211	-0.445
$^{197}_{80}Hg^{**}$	0.524	-0.715	$^{115m}_{49}In$	-0.243	-0.417
$^{199}_{80}Hg$	0.503	-0.806			
$^{207}_{82}Pb$	0.589	-0.431			

The m after mass number on the left superscript means that it is a metastable nucleus. One sees that there is certain regularity in the estimated parameters between the light and heavy nuclei except $^{103}_{45}Rh$. The use of charge symmetry introduces rapid variation of z and α in the light two groups shown in the first two lines, while those for the heavy nuclei remain approximately constant. This indicate that the symmetry breaking effect are sensitive in light nuclei.

(iii) (3/2-) nuclei

Table V. Symmetry-breaking parameters of (3/2-) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z
			$^{7}_{3}Li$	3.256	-0.48
$^{9}_{4}Be$	-1.177	-1.3			
					α
$^{11}_{6}C^{**}$	-1.027	-1.5	$^{11}_{5}B$	2.689	0.98
					α = z
$^{53}_{24}Cr$	-1.93	-0.19	$^{63}_{29}Cu$	3.73	-0.21
			$^{65}_{29}Cu$	3.70	-0.23
$^{57m}_{26}Fe$	-1.36	-1.1	$^{69}_{31}Ga$	3.35	-0.43
			$^{71}_{31}Ga$	3.05	-0.60
$^{61}_{28}Ni$	-1.19	-1.3	$^{75}_{33}As$	3.97	-0.071
			$^{79}_{35}Br$	3.974	-0.20
			$^{81}_{35}Br$	3.74	-0.20
			$^{87}_{37}Rb$	3.83	-0.15

In Table V we tentatively arrange the pair of nuclei on the same line when the number N of odd-N coincide with Z of odd-Z nuclei. No correction from configuration mixing is done for the first four states. The large deviation of z and α from the standard values in these nuclei gives an additional evidence on the occurrence of the sensitivity to symmetry-breaking in light elements. The parameter value close to the standard one found for some of heavy elements indicate the success of the configuration-mixing approach from null-instanton model.

(iv) (3/2+) nuclei

Table VI. Symmetry-breaking parameters of (3/2+) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z
$^{33}_{16}S$	1.18	-0.14	$^{35}_{17}Cl$	-0.045	-0.011
			$^{37}_{17}Cl$	0.014	-0.068
$^{35}_{16}S$	1.12	-0.29	$^{39}_{18}K$	-0.15	0.085
			$^{41}_{18}K$	0.064	-0.12
$^{131}_{54}Xe$	1.16	-0.19	$^{191}_{77}Ir$	0.26	-0.31
			$^{193}_{77}Ir$	0.25	-0.30
$^{135}_{56}Ba$	1.25	0.029	$^{197}_{79}Au$	0.25	-0.30
$^{137}_{56}Ba$	1.16	-0.19			

One finds a more or less a similar behavior for z and α parameters to previous Tables, viz., they fluctuate below A=50 and stay nearly constant if we neglect a very few exceptions.

(v) (5/2+) nuclei

Table VII. Symmetry-breaking parameters of (5/2+) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z
$^{17}_{8}O$	-1.93	-0.19			
$^{25}_{12}Mg$	-1.70	-0.53	$^{27}_{13}Al$	4.53	-0.32
$^{91}_{40}Zr$	-1.41	-0.95	$^{121}_{53}Sb$	4.92	-0.10
$^{95}_{42}Mo$	-1.72	-0.50	$^{127}_{53}I$	5.02	-0.043
$^{97}_{42}Mo$	-1.63	-0.63	$^{131}_{55}Cs^{**}$	4.83	-0.15

$^{105}_{46}Pd$	-1.74	-0.47	$^{141}_{59}Pr$	4.65	-0.25
$^{111}_{48}Cd$	-2.01	-0.078	$^{151}_{63}Eu$	4.02	-0.61
			$^{185}_{75}Re$	4.81	-0.16
			$^{187}_{75}Re^{*}$	4.76	-0.19

In terms of μ_{th} the data is not much irregular but much in z and α for odd-N and odd-Z, respectively, owing to the sensitivity of the latter. Notice that z = -0.19 for $^{17}_{8}O$ is close to the standard value z = -0.219.

(vi) (5/2-) nuclei

Table VIII. Symmetry-breaking parameters of (5/2-) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z
$^{67}_{30}Zn$	1.44	-0.069	$^{85}_{37}Rb$	0.89	-0.19

The α found for $^{85}_{37}Rb$ shows again supports the sufficiency of configuration mixing theory in high mass region, viz., the null-instanton force does not affect on the bound nucleon at such high mass number provided that the configuration mixing is good enough.

(vii) (7/2-) nuclei

Table IX. Symmetry-breaking parameters of (7/2-) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z
			$^{45}_{21}Sc$	5.77	-0.19
$^{43}_{20}Ca$	-2.03	-0.049	$^{51}_{23}V$	5.66	-0.25
$^{49}_{22}Ti$	-1.39	-0.98	$^{57}_{27}Co^{**}$	5.19	-0.52

			$^{59}_{27}Co$	5.27	-0.47
$^{143}_{60}Nd$	-1.69	-0.54			
$^{145}_{60}Nd$	-1.90	-0.24			
$^{147}_{62}Sm^{*}$	-1.91	-0.22			
$^{149}_{62}Sm$	-2.05	-0.020			

Looking at α values in Table IX one gets a similar conclusion found from Table VIII.

(viii) (7/2+) nuclei

Table X. Symmetry-breaking parameters of (7/2+) nuclei

odd-Z nucleus	μ_{th}	α = z
$^{123}_{51}Sb$	1.63	-0.11
$^{133}_{55}Cs$	1.56	-0.058
$^{135}_{55}Cs^{**}$	1.67	-0.14
$^{137}_{55}Cs^{**}$	1.55	-0.050
$^{139}_{57}La$	1.13	0.26
$^{175}_{71}Lu$	1.89	-0.30
$^{181}_{73}Ta$	1.67	-0.14

(ix) (9/2+) nuclei

Table XI. Symmetry-breaking parameters of (9/2+) nuclei

odd-N nucleus	μ_{th}	z	odd-Z nucleus	μ_{th}	α = z

			$^{93}_{41}Nb$	6.78	-0.18
			$^{99}_{43}Tc^{**}$	6.72	-0.21
$^{83}_{36}Kr$	-1.77	-0.43			
$^{87}_{38}Sr$	-1.50	-0.82	$^{113}_{49}In$	6.89	-0.12
			$^{115}_{49}In^{*}$	6.84	-0.15

(x) (9/2-) nuclei

Table XII. Symmetry-breaking parameters of (9/2-) nuclei

odd-Z nucleus	μ_{th}	α = z
$^{209}_{83}Bi$	1.97	0.28

Tables X, XI and XII give an additional information on the subject already discussed.

(xi) Odd-Odd nuclei

The magnetic moment formula for the odd-odd nuclei corresponding to the Schmidt model may be derived as follows. Owing to its simplicity of derivation we shall explain shortly in words and then give the result explicitly. Let us specify the spin operators $\vec{j}_p$ and $\vec{j}_n$ for unpaired single proton and neutron in the nucleus, respectively. The total magnetic moment operator becomes the sum of Schmidt operators by replacing the total spin of the corresponding single particle operators by these spin operators for the proton and neutron part, respectively. Firstly one takes scalar products of this operator with the dyads $\vec{j}_p\,\vec{j}_p/\vec{j}_p^{\,2}$ and $\vec{j}_n\,\vec{j}_n/\vec{j}_n^{\,2}$ for proton and neutron part, respectively, and then take that operator with the dyad $\vec{J}\,\vec{J}/\vec{J}^{\,2}$ made from the total spin $\vec{J}$. The coefficient of the total spin $\vec{J}$ thus calculated gives the desired magnetic moment. Specifying the orbital angular momentum of the unpaired proton and neutron by $\vec{l}_p$ and $\vec{l}_n$, respectively, the results are summarized as follows for the sake of reader:

For simplicity let us define first the four quantities by

$$x(l_p-) = \frac{\mu_p + j_p - \frac{1}{2}}{j_p}(J(J+1) + j_p(j_p+1) - j_n(j_n+1)), \quad (3.2a)$$

$$X(l_n-) = \frac{\mu_n}{j_n}(J(J+1) + j_n(j_n+1) - j_p(j_p+1)), \quad (3.2b)$$

$$X(l_p+) = \frac{-\mu_p + j_p + \frac{3}{2}}{j_p+1}(J(J+1) + j_n(j_n+1) - j_p(j_p+1)), \quad (3.2c)$$

$$X(l_n+) = \frac{\mu_n}{j_n+1}(J(J+1) + j_n(j_n+1) - j_p(j_p+1)), \quad (3.2d)$$

where μ_p and μ_n are the magnetic moments of free proton and neutron, respectively, $l_p- = j_p-1/2$, $l_n- = j_n -1/2$, $l_p + = j_p+1/2$ and $l_n+ = j_n +1/2$. Making use of these notations one finds the four possible magnetic moments of interest as

$$\mu(l_p-, l_n-) = \frac{1}{2(J+1)}[X(l_p-) + X(l_n-)], \quad (3.3)$$

$$\mu(l_p-, l_n+) = \frac{1}{2(J+1)}[X(l_p-) - X(l_n+)], \quad (3.4)$$

$$\mu(l_p+, l_n-) = \frac{1}{2(J+1)}[X(l_p+) + X(l_n-)], \quad (3.5)$$

$$\mu(l_p+, l_n+) = \frac{1}{2(J+1)}[X(l_p+) - X(l_n+)]. \quad (3.6)$$

The proton and neutron part of these expressions coincide with the Schmidt formula by making an appropriate substitution as will be checked easily. The numerical results are summarized by putting $\mu_{Sch} = \mu_{exp}$ in our approximation in Table XIII.

Table XIII. Symmetry-breaking parameter of odd-odd nuclei

nucleus	shell assignment	J^π	μ_{exp}	$\alpha = z$

${}^{2}_{1}H$	$(s_{1/2}, s_{1/2})$	1^+	0.857	-0.164
${}^{10}_{5}B$	$(p_{3/2}, p_{3/2})$	3^+	1.801	-0.22
${}^{14}_{7}N$	$(p_{1/2}, p_{1/2})$	1^+	0.4036	-0.228
${}^{38}_{19}K^{**}$	$(d_{3/2}, d_{3/2})$	3^+	0.5399	-0.30
${}^{60}_{27}Co^{**}$	$(f_{7/2}, p_{3/2})$	5^+	1.3735	-0.23
${}^{6}_{3}Li$	$(p_{3/2}, p_{3/2})$	1^+	0.8220	0.41
${}^{8}_{3}Li$	$(p_{3/2}, p_{3/2})$	2^+	1.6532	0.42
${}^{12}_{5}B^{**}$	$(p_{3/2}, p_{1/2})$	1^+	1.0029	-0.92
${}^{20}_{9}F^{**}$	$(d_{5/2}, d_{5/2})$	2^+	2.094	0.10
${}^{22}_{11}Na^{**}$	$(d_{5/2}, d_{5/2})$	3^+	1.746	-0.11
${}^{24}_{11}Na^{**}$	$(d_{5/2}, d_{5/2})$	4^+	1.6902	-0.12
${}^{32}_{15}P^{**}$	$(s_{1/2}, d_{3/2})$	1^+	-0.2523	-0.50
${}^{36}_{17}Cl^{**}$	$(d_{3/2}, d_{3/2})$	2^+	1.2854	-1.17
${}^{40}_{19}K^{*}$	$(d_{3/2}, f_{7/2})$	4^-	-1.2981	-0.37
${}^{42}_{19}K^{**}$	$(d_{3/2}, f_{7/2})$	2^-	-1.1410	0.76
${}^{44}_{21}Sc^{**}$	$(f_{7/2}, p_{3/2})$	2^+	2.56	0.12
${}^{44m}_{21}Sc$	$(f_{7/2}, f_{7/2})$	$6^{(+)}$	3.88	0.47
${}^{46}_{21}Sc^{**}$	$(f_{7/2}, f_{7/2})$	4^+	3.04	1.21
${}^{50}_{23}V^{*}$	$(f_{7/2}, f_{7/2})$	6^+	3.347	-0.12

${}^{52}_{25}Mn^{*}$	$(f_{7/2}, f_{7/2})$	6^{+}	3.075	-0.42
${}^{54}_{25}Mn$	$(f_{7/2}, p_{3/2})$	3^{+}	3.302	-1.28
${}^{56}_{27}Co^{**}$	$(f_{7/2}, p_{3/2})$	4^{+}	3.803	-0.54
${}^{58}_{27}Co$	$(f_{7/2}, p_{3/2})$	2^{+}	4.035	-1.31

The first, second, third, fourth and fifth column of Table XIII correspond to the symbol of odd-odd nuclei, the assumed orbits for the unpaired proton and neutron, the total spin and parity, the observed magnetic moments and the estimated α by putting α = z, respectively. In the Table we have arranged the data for parallel spins of proton and neutron shell and non-parallel one: the first five states and the remaining eighteen states, respectively. If there is an ambiguity in shell assignment we choose a unique one so as to make the absolute value of α less than 1 as much as possible. Except for nuclei with mass number A up to16 the estimated numbers should not be taken too seriously, because we did not take into account the effect of configuration mixing.

It is remarkable to point out that the estimated α for the deuteron is close to the standard value α =-0.143 estimated from the combined free proton and neutron data, where the proton and neutron are loosely bound. The α values in next four states, which however does not contain the effect of configuration mixing, are not much different from the standard value.

Throughout these studies we conclude within an approximation made the range of instanton induced long-range force in nuclear matter may be about nuclear size of A=50 or less. If the higher order effect of configuration mixing explains completely the magnetic moments of odd-A nuclei, viz., the discrepancy between each datum which appears in between two Schmidt limits for each of odd-N and odd-Z group, the apparent force range under discussion may become close to the nuclear size of A=16, or less. The force range under discussion is the instanton-anti-instanton induced inter nucleon one.

In next section we shall investigate this problem from nuclear binding energies by manipulating suitably the data so as to find the associated symmetry-breaking parameter α which will be

compared with that estimated for individual nuclear magnetic moment in this section in a suitable manner.

$ 4. Nuclear Mass

In the mass formula of baryons with null-instanton model we have neglected the proton and neutron mass difference and find

$$\frac{1}{2}(m_p + m_n) = 3\,\mathrm{U} - \frac{3}{2}\alpha_{mass}\,, \tag{4.1}$$

where the subscript mass is attached to α which represents the symmetry-breaking effect in the baryon mass formula. By its way of definition its treatment is different from that in the magenetic moment. Let us define the standard value of symmetry-breaking parameter for the baryon mass by

$$\alpha' = \frac{\alpha_{mass}}{2U} = 0.2468, \tag{4.2}$$

by making use of the numbers found from baryon mass study.[10] This standard is not used explicitly in the study of this paper. It should be taken as a memory that the symmetry breaking in the null-instanton model is occurring already in the baryon-mass level.

There is no definite criterion to extract the α' for the nuclear mass corresponding to the α for the magnetic moment. We shall introduce a new symmetry-braking parameter for the bound nucleon mass independently to the above standard by the same notation.

$$\alpha' = \frac{mpf}{u_V}\,, \tag{4.3}$$

where mpf is the modified packing fraction after making the Coulomb correction to the observed binding energy B, viz.,

$$\mathrm{mpf} = (\mathrm{B+Coulomb})/\mathrm{A}, \tag{4.4}$$

u_V is the coefficient of the volume term in the Weizsäcker-Bethe mass formula. According to this definition the smaller the α' becomes the more closer the free nucleon state is reached. The numerical value u_V has been estimated for three choices of the range of mass numbers[11] : (i) A=13-257, (ii) 40-257 and (iii) 100-267. They are 15.36, 15.98 and 17.41 MeV for (i), (ii) and (iii),

respectively. There is not much meaning to discriminate three choices of u_V in the present work except for a minor reason. We tabulate the estimated results in Table XIV.

Table XIV. Symmetry-breaking parameters in nuclear masses

nucleus	mpf (MeV)	$\alpha'(i)$	$\alpha'(ii)$	$\alpha'(iii)$
D	1.11226	0.0724	0.0696	0.0639
T	2.827	0.1841	0.1769	0.1624
${}^{4}_{1}H$	0.820	0.0534	0.0513	0.0471
${}^{4}_{2}He$	7.0740	0.4607	0.4427	0.4064
${}^{5}_{1}H$	5.8867	0.2100	0.2018	0.4064
${}^{5}_{2}He,{}^{5}_{3}Li$	5.5683	0.3626	0.3485	0.3199
${}^{6}_{2}He,{}^{6}_{3}Be$	4.9554	0.3227	0.3101	0.2847
${}^{6}_{3}Li$	5.5655	0.3624	0.3483	0.3197
${}^{7}_{3}Li,{}^{7}_{4}Be$	5.8413	0.3804	0.3656	0.3356
${}^{8}_{2}He$	3.5708	0.2325	0.2235	0.2051
${}^{8}_{3}Li,{}^{8}_{5}B$	5.3495	0.3484	0.3348	0.3073
${}^{8}_{4}Be$	7.4417	0.4846	0.4657	0.4275
${}^{9}_{3}Li$	5.1723	0.3368	0.3237	0.2971
${}^{9}_{4}Be,{}^{9}_{5}B$	6.7710	0.4409	0.4237	0.3890
${}^{10}_{4}Be,{}^{10}_{6}C$	6.8055	0.4432	0.4257	0.3909
${}^{10}_{5}B$	6.9880	0.4551	0.4373	0.4014
${}^{11}_{4}Be$	6.2537	0.4073	0.3914	0.3592

${}^{11}_{5}B,{}^{11}_{6}C$	7.4302	0.4839	0.4650	0.4268
${}^{12}_{5}B,{}^{12}_{7}N$	7.0523	0.4593	0.4413	0.4051
${}^{12}_{6}C$	8.3118	0.5413	0.5202	0.4775
${}^{13}_{5}B$	6.8815	0.4481	0.4306	0.3953
${}^{13}_{6}C,{}^{13}_{7}N$	8.0475	0.5241	0.5036	0.4623
A= 50		0.6867	0.6679	0.6276
A=100		0.7360	0.7190	0.6809
A=150		0.7539	0.7388	0.7039
A=200		0.7590	0.7456	0.7135

The first , second, third, fourth and fifth column of the Table give the symbol of nucleus, the estimated mpf, the corresponding α' for choices (i), (ii) and (iii), respectively, except for the last 4 lines. In these four cases the Coulomb correction is done by making use of the corresponding parameter u_c in the mass formula by averaging over B's and Z dependencies from available data at A. Owing to the difference of numerical coefficient u_c in in three chosen range A, we get three slightly different mpf in 4 different A's. More precisely they become

mpf= 10.54, 10.67, 10.93 MeV for (i), (ii), (iii) at A=50, (4.5a)

mpf= 11.30, 11.49, 11.85 MeV for (i), (ii), (iii) at A=100, (4.5b)

mpf= 11.58, 11.81, 12.25 MeV for (i), (ii), (iii) at A=150, (4.5c)

and

mpf= 11.65, 11.91, 12.42 MeV for (i), (ii), (iii) at A=200. (4.5d)

These as well as the tabulated result show that there is the saturation of instanton induced force in the nuclear matter roughly at about A=16. The α'(i) (i=1,2,3) in the last four lines of the Table correspond to those in Eq.(4.5a-d).

The following summary of this study will give us the definite support to the variation of symmtetry-breaking parameters of null-intanton model in the nuclear matter:

(1) The smallness of α' for D supports the closeness of α found from D magnetic moment to that of free proton-neutron data.

(2) The rapid variation of α' at light nuclei and α at light odd-A and odd-odd nuclei indicate that the instanton-induced force is relatively short range.

The α' stays nearly constant say above A=16, while z and α cannot be fitted exactly to the standard values in the individual state by the common choice of energy gap between the relevant shell structures, but high A states show roughly the saturation of them with some exceptions.

(3) If we assume that the quenching of the magnetic moment at high mass number is solved completely by the configuration mixing, the longest range of null-instanton induced force in nuclear matter is at most the nuclear radius of A=16. This estimate will be somewhat reduced if a many-body effect is taken into account.

$ 5. Discussion and Conclusion

In this paper we have examined the properties of null-instanton force from magnetic moments of baryons and those from odd-A and odd-odd nuclei. For many years we cannot understand why some of odd-A magnetic moments stay outside of the two Schmidt limits. It is shown that this long-standing problem can be reduced to the change of symmetry-breaking parameters of nucleons in nuclear matter. The study done on the magnetic moments of odd-odd nuclei gives us an additional clue to understand this problem. The range of null-instanton force in nuclear matter is at most the radius of A=16 nucleus. One learns that this result is supported by the variation of α' in low and high mass states.

We have pushed our idea based on the analysis of the nuclear magnetic moments made by AH in earlier days. At the same time the null-instanton model explains the reason why they have succeeded to explain the observed magnetic moments in their assumed mass range.

Let me digress shortly to comment on Bukina's work. Her broken SU_3 electromagnetic current contains 6 parameters so she find a single sum rule from 7 data, from a constraint on 6 parameters her relation splits into two sum rules: one of them is

Eq.(2.4b), the other holds also accidentally for the null-instanton model. Both relations are not completely in success as far as we rely on the presently available data, especially for the latter. In the semibosonaized NJL model there appears 7 parameters with 7 inputs so that she finds a discrepancy between the null-instanton and that model through the magnetic moment of the Λ-hyperon.

In this paper we are concerned the symmetry-breaking of null-instanton force between nucleons in nuclear matter. The present accuracy of the sum rule (2.4b) and numerical results in Table I and II will not affect the conclusion found in this paper even if the future study favors the semibosonized NJL model.

Recently the null-instanton model has paid much attention at high energy interaction with a certain success[12] where the short-range property of the instanton-induced effect can be studied by the perturbative approach.

In this paper we have suggested the rapid variation of the bound nucleon mass in light nuclei by the presence of the instanton induced force. It will be worthwhile to measure simultaneously the mini-max of the momentum distributions of proton and the residual nucleus in (e,p) collision at a fixed angle for various target nuclei. The instantaneous nature of the force and the presence of the uncertainty principle make obscure the direct check. Even a negative answer teaches us how much instantaneous the propagation velocity of the force is.

We hope that a further study on the nature associated with the instanton will be made in order to clarify the physics involved in QCD.

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