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Toward Improving the Weisskopf-Wigner Approximation

G.V. Dass+

Department of Physics, Indian Institute of Technology, Powai, Mumbai, 400 076, India

Abstract

Hermiticity of the full Hamiltonian constrains the various structural properties (like $f \leftrightarrow \bar{f}$ oscillations) of the complex formed by a neutral flavoured meson f and its antiparticle $\bar{f}$. These "unitarity" constraints are written, following Kabir and Pilaftsis. Evaluation of the structural amptitudes at small values of the proper time brings out the inadequacy of the usual phenomenology based on the Weisskopf-Wigner approximation. Asymmetries which could experimentally check this are proposed.

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+ E-mail : guruv@niharika.phy.iitb.ernet.in

Fax: 091 22 578 3480

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We first define the "structural amplitudes" of an $(f, \bar{f})$ complex where f is a neutral flavoured meson (e.g., $f = K^o, D^o, B_d^0, B_s^o, ...$) and $\bar{f}$ is its antiparticle. Starting from the states $|f>$ and $|\bar{f}>$ initially (i.e. at proper time $t = 0$), let the time-development be given by

$$|f> \xrightarrow{t} a(t)|f> + b(t)|\bar{f}> + \Sigma_i\ c_i(t)|\rho_i>$$
$$|\bar{f}> \xrightarrow{t} \bar{b}(t)|f> + \bar{a}(t)|\bar{f}> + \Sigma_i\ \bar{c}_i(t)|\rho_i>, \tag{1}$$

where the $|\rho_i>$ (for i = 1,2, ...) are other states (orthogonal to $|f>$ and $|\bar{f}>$) produced with coefficients c_i and $\bar{c}_i$. By definition,

$$a(0) = \bar{a}(0) = 1\ ;$$
$$b(0) = \bar{b}(0) = 0\ ;$$
$$c_i(0) = \bar{c}_i(0) = 0\ , \quad \text{for all i} \quad . \tag{2}$$

We call the amplitudes $a, \bar{a}, b$ and $\bar{b}$ as "structural". While a and $\bar{a}$ respectively denote the diagonal transitions $|f> \rightarrow |f>$ and $|\bar{f}> \rightarrow |\bar{f}>$, the amplitudes b and $\bar{b}$ respectively denote the 'mixing' or nondiagonal transitions $|f> \rightarrow |\bar{f}>$ and $|\bar{f}> \rightarrow |f>$. In general, all the four structural amplitues are independent. Taking $|\bar{f}> = CP|f>$, structural CP-violation means $(a - \bar{a}) \neq 0$ or/and $(b - \bar{b}) \neq 0$. Let us state the meaning of "structure" in the language of the Weisskopf-Wigner approximation (called WWA henceforth) which is commonly used for a phenomenological description of the $(f, \bar{f})$ complex: The constant parameters describing the *structure* of the two independently propagating states (in terms of $|f>$ and $|\bar{f}>$), along with their propagation parameters, fully determine the four "structural amplitudes" $a, \bar{a}, b$ and $\bar{b}$. The WWA is used in this paper only for a comparison with the general unitary description.

The purpose of this paper is to write some constraints on $a, \bar{a}, b$ and $\bar{b}$ arising from unitarity (viz. hermiticity of the full Hamiltonian), on the lines of ref [1], and to use these constraints to deduce, at a purely phenomenological level, some general aspects of the t-dependence of these amplitudes. The small-t behaviour of various suitable combinations of these amplitudes is then explicitly derived, and contrasted with the corresponding behaviour obtained in the WWA. Future data on the experimental t-dependence of some CP-violating asymmetries (called $A_{1,2,3,4}$ below) can test this inadequacy of the WWA.

For deducing [1] the unitarity conditions, one utilizes the hermiticity of the full Hamiltonian H. The transition operator $\Omega(t) = exp(-iHt)$ is unitary:

$$\Omega^{+}(t) = [\Omega(t)]^{-1} = \Omega(-t) \quad .$$

Taking the lm-th element, we get the unitarity condition [1]

$$A^{*}_{ml}(-t) = A_{lm}(t) \equiv < l|\Omega(t)|m > , \tag{3}$$

for transition amplitudes. Restrictions on the even/odd nature (as a function of time) of the real and imaginary parts of the probability amplitudes A_{lm} follow from Eq.(3). For nondiagonal transitions, the unitarity condition [1]

$$\psi^{*}(-t)\psi(t) = 1 \tag{4}$$

for the ratio

$$\psi(t) = [A_{kj}(t)/A_{jk}(t)] \ , \quad j \neq k \tag{5}$$

can be satisfied by the form

$$\psi(t) = exp(g + ih) \tag{6}$$

where the unspecified real functions g and h are respectively odd and even functions of time.

Following the same [1] lines, we want to exploit Eq. (3) fully with the identifications

$$A_{kk} \equiv \bar{a} \ , \quad A_{jj} \equiv a \ , \quad A_{jk} \equiv \bar{b} \ , \quad A_{kj} \equiv b \ . \tag{7}$$

Defining the nondiagonal amplitudes

$$N_{\mp} = (b \mp \bar{b}) \ ,$$

Eq.(3) gives

$$N^{*}_{\mp}(-t) = \mp N_{\mp}(t) \ , \tag{8}$$

which contain Eq. (4) for the present case. For the diagonal amplitudes, the corresponding relations are

$$a^{*}(-t) = a(t) \ ; \tag{9}$$

$$\bar{a}^*(-t) = \bar{a}(t) \; ; \tag{10}$$

$$[a^*(-t) \pm \bar{a}^*(-t)] = [a(t) \pm \bar{a}(t)] \; . \tag{11}$$

One may note the comparison (contrast) of the unitarity relation (8) for $N_+(N_-)$ with the corresponding relations (9-11). Similarly, one gets

$$\eta^*(-t) = \eta(t) \; , \quad \eta \equiv \bar{a}/a \; ; \tag{12}$$

$$\eta(t) = exp(F + iG) \; , \tag{13}$$

where the unspecified real functions F and G are respectively even and odd functions of time; the general form (13) would hold for $\eta, a, \bar{a}, N_+$. The functions η and ψ are complementary in nature; Eqs. (6, 13). The constraint $F(0) = 0$ follows from $\eta(0) = 1$ implied by the first of Eqs. (2). If the ratio r of the two diagonal probabilities is constant, the first of Eqs. (2) shows r to be unity; see ref[1] for an analogous theorem for nondiagonal probabilities.

While data would be the final guide in deciding how the present phenomenology based on the WWA would be improved by using the above unitarity relations, we now concentrate on the behaviour of the structural amplitudes at small times. This is an important step in the direction of obtaining a unitary phenomenological scheme because the WWA is decidedly inadequate in that region of t, as the following paragraphs would show. The small-t behaviour of the various CP-violating ($\times$) and CP-conserving ($\surd$) combinations of the structural amplitudes, as following from the above unitarity relations, is given in table 1 which lists the real or imaginary nature of the coefficients of the constant (t^o) term, the even- power (t^{even}) terms, and the odd-power (t^{odd}) terms in the t- expansion of the amplitude combinations. The relation $|\psi(0)| = 1$ following [1] from Eq. (4) is implied by the purely imaginary nature of $[(b - \bar{b})/(b + \bar{b})]$ at $t = 0$, as seen in table 1.

The asymmetries

$$A_1 = \frac{|b|^2 - |\bar{b}|^2}{|b|^2 + |\bar{b}|^2} \; , \quad A_2 = \frac{|a|^2 - |\bar{a}|^2}{|a|^2 + |\bar{a}|^2} \tag{14}$$

can be seen to have the small-t behaviour

$$A_1 \sim t \; , \quad A_2 \sim t^2 \tag{15}$$

by using the results in table 1. A possible way to measure A_1 and A_2, for $f = K^o$, would be to look for ($K^- or \Lambda$) production arising from the final-state neutral kaon, at time t, if the primary reactions

$$\bar{p}p \rightarrow \pi^- K^+ \bar{K}^o \quad , \tag{16}$$

$$\bar{p}p \rightarrow \pi^+ K^- K^o \tag{17}$$

are taken to occur at t = 0. This would measure $|\bar{a}|$ and $|b|$ from the reactions (16) and (17) respectively; similarly, the production of K^+ from the final-state neutral kaon, at time t, would measure $|\bar{b}|$ and $|a|$ respectively from the reactions (16) and (17). For $f = B_d^o$, one may use decays of the $\Upsilon(4S)$

$$\Upsilon(4S) = |B^o \bar{B}^o - \bar{B}^o B^o > \tag{18}$$

by tagging beons at production and at decay, to get the diagonal ($|a|^2, |\bar{a}|^2$) and nondiagonal ($|b|^2, |\bar{b}|^2$) probabilities. Similarly, double-tagging of kaons could be used with the ϕ-meson

$$|\phi >= |K^o \bar{K}^o - \bar{K}^o K^o > \tag{19}$$

for $f = K^o$.

Another interesting asymmetry is

$$A_3 = \frac{N_{ff} - N_{\bar{f}\bar{f}}}{N_{ff} + N_{\bar{f}\bar{f}}} \tag{20}$$

between the numbers of $|ff >$ and $|\bar{f}\bar{f} >$ events produced by an initial $C = -ve$ state

$$|- >= |f\bar{f} - \bar{f}f > \tag{21}$$

like $\Upsilon(4S)$ and $|\phi >$. One gets, on dropping uninteresting overall constants,

$$N_{ff} = |e_1|^2 \ , \quad e_1 = [a(t_1)\bar{b}(t_2) + \epsilon \bar{b}(t_1) a(t_2)] \ ; \tag{22}$$

$$N_{\bar{f}\bar{f}} = |e_2|^2 \ , \quad e_2 = [b(t_1)\bar{a}(t_2) + \epsilon \bar{a}(t_1) b(t_2)] \ ; \tag{23}$$

where ϵ is the C-eigenvalue of the initial state; here, $\epsilon = -1$. If both the times (t_1 and t_2) of the production of the like-mesons (ff or $\bar{f}\bar{f}$) are small as measured from the creation of the $|- >$ state, results of table 1 imply, for $C = -ve$,

$$A_3 \sim (t_1 + t_2) \ . \tag{24}$$

On the other hand, if the initial state has $C = +ve$:

$$|+ >= |f\bar{f} + \bar{f}f >, \tag{25}$$

the corresponding time-dependence for the asymmetry A_3 would follow from Eqs. (22,23) with $\epsilon = +1$, and table 1, as

$$A_3 \sim (t_1^2 + t_2^2)/(t_1 + t_2) \ . \tag{26}$$

Another interesting asymmetry

$$A_4 = \frac{N_{f\bar{f}} - N_{\bar{f}f}}{N_{f\bar{f}} + N_{\bar{f}f}} \tag{27}$$

arises between $N_{f\bar{f}}$, the number of $|f\bar{f}>$ events with f appearing later than $\bar{f}$, and $N_{\bar{f}f}$ wherein $\bar{f}$ is later then f, starting with the states (21) and (25). One gets, in analogy to Eqs.(22,23), with $\epsilon = \pm 1$ for the states $|\pm>$ respectively, using $t_1 > t_2$,

$$N_{f\bar{f}} = |e_3|^2 \ , \quad e_3 = a(t_1)\bar{a}(t_2) + \epsilon\bar{b}(t_1)b(t_2) \ , \tag{28}$$

$$N_{\bar{f}f} = |e_4|^2 \ , \quad e_4 = b(t_1)\bar{b}(t_2) + \epsilon\bar{a}(t_1)a(t_2) \ . \tag{29}$$

Then, for small t_1 and t_2, the table gives, for both $\epsilon = \pm 1$,

$$A_4 \sim (t_1^2 - t_2^2) \quad . \tag{30}$$

How do the predictions (15), (24), (26) and (30) of the unitary theory compare with the corresponding results of the WWA? Two independently propagating states $|f_1>$ and $|f_2>$ characterise the WWA:

$$|f_1> = p_1|f> + q_1|\bar{f}>, \tag{31}$$

$$|f_2> = p_2|f> - q_2|\bar{f}>; \tag{32}$$

$$|f_1> \xrightarrow{t} \theta_1|f_1>, \tag{33}$$

$$|f_2> \xrightarrow{t} \theta_2|f_2>, \tag{34}$$

$$\theta_{1,2} = exp[-i\ t\ m_{1,2} - \frac{t}{2}\ \Gamma_{1,2}] = \ exp(-it\lambda_{1,2}) \ ; \tag{35}$$

$\theta_{1,2}$ (or, equivalently, $\lambda_{1,2}$) may be called the "propagation parameters" of the states $|f_{1,2}>$; here, $m_{1,2}$ and $\Gamma_{1,2}$ are the usual masses and widths for $|f_{1,2}>$; the complex constants $p_{1,2}$ and $q_{1,2}$ are used to express $|f_{1,2}>$ in terms of $|f>$ and $|\bar{f}>$. There are only two definitive time-dependences now: θ_1 and θ_2. The general constraints of Eqs. (2) imply that $(\bar{a} - a)$, b and $\bar{b}$ must have the time-dependence $(\theta_1 - \theta_2)$. Indeed, one finds (see ref [2] for a convenient review)

$$(a + \bar{a}) = \theta_1 + \theta_2 \ ; \tag{36}$$

$$(a - \bar{a}) = [(p_1/q_1) - (p_2/q_2)]\ b, \tag{37}$$

$$\bar{b} = [p_1 p_2/(q_1 q_2)]\ b, \tag{38}$$

$$b = q_1 q_2 (\theta_1 - \theta_2)/(q_1 p_2 + q_2 p_1)\ . \tag{39}$$

The contrast between the WWA and the general theory now comes out. Eqs. (36-39) show that, unlike the results in table 1, one gets, in the WWA, the last 3 amplitude-combinations of the table $\{(b-\bar{b})/(b+\bar{b}), (b-\bar{b})/(\bar{a}-a), (\bar{a}-a)/(b+\bar{b})\}$ to have constant values determined by $p_{1,2}$ and $q_{1,2}$. For the remaining 8 amplitude-combinations, in the WWA, coefficients of the various powers of t are in general complex and determined by integral powers of the complex messes $\lambda_{1,2}$. That contrasts with the general theory wherein the coefficients of the successive powers are relatively imaginary for all the 11 amplitude-combinations of the table. In fact, in the WWA, the time - dependence of $(a+\bar{a})$ is $(\theta_1+\theta_2)$ and it is $(\theta_1-\theta_2)$ for $(\bar{a}-a)$, $(b-\bar{b})$ and $(b+\bar{b})$, leading to the time-dependence $[(\theta_1-\theta_2)/(\theta_1+\theta_2)]$ for the three amplitude-combinations numbered 6,7 and 8 in table 1.

For the asymmetries considered above, the WWA gives

$$A_1 \equiv \text{constant, for all times}\ , \tag{40}$$

and

$$A_2 \sim t\ , \quad \text{for small times}\ . \tag{41}$$

Also, for the $C = -ve$ case (see, for example, ref. [3]), one gets, in the WWA,

$$A_3 \equiv constant\ ; \tag{42}$$

here, the relation $e_1 = -[p_1 p_2/(q_1 q_2)]\ e_2, \epsilon = -1$ holds between the amplitudes defined in Eqs.(22-23). For the $C = +ve$ case, one gets, for small t_1 and t_2,

$$A_3 = constant \quad ; \tag{43}$$

here, $e_1 = [p_1 p_2/(q_1 q_2)]e_2$, with $\epsilon = +1$ holds, apart from terms which are relatively negligible for small t_1 and t_2. Coming to A_4, the WWA gives, by using Eqs. (~~31-38~~ 36-39), for both $\epsilon = \pm 1$, viz. $C = +ve$ and $-ve$,

$$A_4 \sim (t_1 - t_2) \quad , \tag{44}$$

for small t_1 and t_2.

One may note that the behaviour of A_1 and A_3 at small times depends crucially on $Re[(b-\bar{b})/(b+\bar{b})]$, and that of A_2 and A_4 on $Re[(\bar{a}-a)/(\bar{a}+a)]$. For all the four asymmetries, the WWA gives a variation slower by one power of a suitable time-variable, as seen by comparing the WWA results (40-44) with respectively, (15,24,26,30) of the general theory.

As pointed out already [1], the WWA cannot exactly satisfy unitary. Future experiments should prove the general results (15,24,26,30) in contrast to the corresponding results (40-44) of the WWA. Unfortunately, the tests are difficult. Also, there is no unique way of defining "small times"; one can try our tests at smaller and smaller times for eventually discovering departures from the WWA. There are, at present, no measurements of the *time − dependences* of $A_{1,2,3,4}$, though *some* data on $A_{1,2}$ for $f = K^0$ [4] and A_3 for $f = B_d^o$ [5] do exist.

In summary, we have deduced the small-time behaviour of the structural amplitudes of the $(f, \bar{f})$ complex by utilizing the hermiticity of the full Hamiltonian. This behaviour is not satisfied by the commonly used Weisskopf-Wigner approximation. We have proposed test for this.

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Serial No.	Amplitude Combination	CP-property	Coefficient of t^o	Coefficient of t^{even}	Coefficient of t^{odd}
1	a or $\bar{a}$	Not Applicable	1	Real	Imaginary
2	$a+\bar{a}$	$\surd$	2	Real	Imaginary
3	$\bar{a}-a$	$\times$	0	Real	Imaginary
4	$b+\bar{b}$	$\surd$	0	Real	Imaginary
5	$b-\bar{b}$	$\times$	0	Imaginary	Real
6	$(\bar{a}-a)/(\bar{a}+a)$	$\times$	0	Real	Imaginary
7	$(b+\bar{b})/(\bar{a}+a)$	$\surd$	0	Real	Imaginary
8	$(b-\bar{b})/(\bar{a}+a)$	$\times$	0	Imaginary	Real
9	$(b-\bar{b})/(b+\bar{b})$	$\times$	Imaginary	Imaginary	Real
10	$(b-\bar{b})/(\bar{a}-a)$	$\surd$	Imaginary	Imaginary	Real
11	$(\bar{a}-a)/(b+\bar{b})$	$\times$	Real	Real	Imaginary

Table: 1. Real or Imaginary nature of the coefficients of the constant (t^0) term, the even-power (t^{even}) terms, and the odd-power (t^{odd}) terms in the t-expansion of the various amplitude combinations (listed in second column), due to the unitarity constraint of Eq. (3), using the identifications of Eq.(7). The symbols ($\surd$) and ($\times$) in the third column mean, respectively, CP-conserving and CP-violating.