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A NEW PARADIGM FOR QUARKS

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Abstract

It is shown that the algebra of the Lorentz group implies that a particle can consist of three sub-particles. The spin operator is represented as the sum of three commuting spin ½ operators instead of the usual two, leading to three boost operators that do not commute, that define vectors to these "particles" and also add up to the spin of the particle itself. Projection of the algebra onto axes fixed on the particle leads to operators that define its intrinsic properties, relating its isospin and strangeness to the spin in the way that has been observed for the basic spin ½ and spin 3/2 states. Some very accurate mass relations appear from the analysis.

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It has been known for many years[1] that the commutation relations between spin and boost operators s, σ :

$$(s_{\alpha\beta}, s_{\gamma\delta})\delta = -i\hbar(s_{\alpha\gamma}g_{\beta\delta} + s_{\beta\delta}g_{\alpha\gamma} - s_{\alpha\delta}g_{\beta\gamma} - s_{\beta\gamma}g_{\alpha\delta}) \tag{1}$$

can be written simply as

$$s = \hbar(N - N'); \quad \sigma = \hbar(N - N'); \quad \sigma_i = is_{oi} \tag{2a}$$

$$(N \times N) = iN; \quad (N' \times N') = iN'; \quad (N, N') = 0 \tag{2b}$$

so that

$$(s \times s) = i\hbar s = (\sigma \times \sigma); \quad (\sigma_i, s_j) = i\hbar\varepsilon_{ijk}\sigma_k \, (ijk = 123) \tag{3}$$

With the invariant Hamiltonian of the Dirac equation

$$H = -\gamma^\alpha P_\alpha (P_\alpha = p_\alpha - eA_\alpha)$$

and the relations

$$(s_{\alpha\beta}, \gamma_\gamma) = i\hbar(g_{\beta\gamma}\gamma_\alpha - g_{\alpha\gamma}\gamma_\beta) \tag{4}$$

$$s_{\alpha\beta} = i\hbar(\gamma_\alpha, \gamma_\beta)/4 \tag{5}$$

we also note that

$$\dot{x}^\alpha = \gamma^\alpha; \quad \dot{s}_{\alpha\beta} = P_\alpha\gamma_\beta - P_\beta\gamma_\alpha \tag{6}$$

We define $\rho = x - X$, where X denotes the position of the center of energy, so that ρ is the vector from the center to the particle, and $P_0\dot{X} = P\gamma_0$. Thus apart from an additive constant, and for a free particle

$$P_0\rho_i = s_{0i} = -i\sigma_i \tag{7}$$

consistent with Dirac's comment[2] that the electric moment of the electron is imaginary.

However, the representation of s and σ as combinations of commuting vector operators N, N' is not restricted to (2a) (2b), and here instead of (2a) we write

$$s = \hbar(N + N' + N'') = (\sigma + \sigma' + \sigma'') \equiv \Sigma$$

$$\sigma = \hbar(-N + N' + N'') \tag{8}$$

$$\sigma' = \hbar(N - N' + N'')$$

$$\sigma'' = \hbar(N + N' - N'')$$

and, in addition to (2b):

$$(N, N'') = 0 = (N', N''); \qquad (N'' \times N'') = i N'' \tag{9}$$

From (7), $\sigma, \sigma', \sigma''$ define the vectors ρ, ρ', ρ'' from their centers of energy to <u>three</u> pseudo-particles:

$$E\rho_i/c = s_{\sigma i} = -i\sigma_i \; ; \qquad E'\rho_i'/c = s_{\sigma i}' = -i\sigma_i' \; ; \qquad E''\rho_i''/c = s_{\sigma i}'' = -i\sigma_i'' \tag{10}$$

Equations (3) are still valid for each of the three σ's.

In addition we have

$$(\sigma_i', \sigma_j'') = -i\hbar\,\varepsilon_{ijk}\,\sigma_k; \quad (\sigma_i'', \sigma_j) = -i\hbar\,\varepsilon_{ijk}\,\sigma_k'; \quad (\sigma_i, \sigma_j') = -i\hbar\,\varepsilon_{ijk}\,\sigma_k'' \tag{11}$$

It follows from (8) that $\hbar N = \frac{1}{2}(\sigma' + \sigma'')$ etc. so that the vector to N, defined as in (10) by $\boldsymbol{R} = -2i\hbar c\, N/(E' + E'')$ is given by

$$\boldsymbol{R} = (E'\rho' + E''\rho'')/(E' + E'') \tag{12}$$

the weighted average of the vectors ρ' and ρ''. Thus we have two separate ways of resolving both $\boldsymbol{s}$ and Σ into the sum of three operators and of describing the coordinates of the corresponding "particles" relative to their centers of energy. From (11) these are seen to be inseparable and if any two of them have the same quantum numbers it would not be valid to invoke the Pauli exclusion principle any more than it is in the original formulation of equation (2). The N's are spin operators independent of each other, but the σ's are boost operators related to each other, so that they transform in a different way than the N's. However, from (8), the sum of the three σ operators is a spin operator that is equal to the spin of the particle.

Following the great Leonhard Euler, we could refer the operators to 'body axes' so that their components are characteristic of the particle or its sub-particles. The transformed components Σ_i^t of the vector Σ can then be only instantaneously equal to s_i, but they remain equal to s_i^t. Apart from a change of sign[3] in equation (3) the spin properties of $\boldsymbol{s}^t$ and Σ^t are the same as those of $\boldsymbol{s}$ and Σ.

For the case in which the three N operators are commuting spin ½ Pauli operators, s and Σ^t are spin ½ or spin 3/2 together. With the transforms of N, N', N'' to the body axes denoted by ν, ν', ν'', which are also commuting spin ½ operators, we write.

$$\Sigma^t = I + \mathbf{K} = \hbar(\nu + \nu' + \nu'') \tag{13}$$

where I is the isospin and the strangeness $S = -2$ K (for baryons). We find that this is equivalent to the quark model except that it provides relations between isospin, strangeness and the spin. For example, for K = 0, we have $I = 3/2$ for $\Sigma^t = 3/2$ and therefore spin $3/2$ (Δ) and $I = ½$ for $\Sigma^t = ½$ and therefore spin ½ (N). For $I = 0$ we have K $= 3/2$ ($S = -3$ for baryons) for $\Sigma^t = 3/2$, and therefore spin $3/2$ (Ω) and K $= ½$ $(S = -1)$ for $\Sigma^t = ½$ and therefore spin ½ (Λ). The states $I = 1$, K = ½ so that $S = -1$ (Σ) and $I = 1/2$, K $= 1$ $(S = -2)$ (Ξ) correspond to $\Sigma^t = ½$ or 3/2 and therefore to spins 1/2, 3/2. With $S = +2$ K, similar results hold for antibaryons.

The well-known data and SU (3) prediction for the basic spin 3/2 states yield a mean separation of $m_0 = 147$ *Mev*, the increase of $3\,m_0/2$ due to an increase in K by ½ and the decrease of $m_0/2$ due to the necessary decrease in I, i.e.[4]

$$\delta m = \delta(I + 3\mathrm{K})\, m_0 \qquad (\delta s = 0) \tag{14}$$

We may suppose that each ν-particle has approximately the same mass but that each may be raised independently to approximately the same mass level m_0 above its ground state. If the mass of ν is increased it can no longer be regarded as a component of isospin, and we denote it by **V**. The various possible combinations of ν and **V** then lead to the results quoted above and provide the usual results for the basic spin 3/2 states. Since spin ½ particles are included in the above, we may include the neutron-proton as a member with the family of spin 3/2 states if equation (14) is generalized to

$$\delta m = \delta(I + 3\mathrm{K} + \mathrm{s})\, m_0 \tag{14'}$$

since the Δ - N mass difference is $2\,m_0 = 294$ *Mev*. However, (14) and (14') are not true in general.

Double or triple excitation of a ν-particle leads to other quantum numbers and higher masses, and the extension to four N operators leaves room for baryon numbers and provides expressions for the electric charges.

As in the quark model, mesons are described by $\nu\mu$, νW, VW etc. where μ^+, W^+ are the anti-particles to ν^-, V^- respectively. As one example we consider the reaction between I^- states with zero kinetic energy (very difficult to execute experimentally!).

$$\Phi\,(1019.412) + \rho^-\,(768.3) \rightarrow \overline{\mathrm{K}}^0\,(896.10) + \mathrm{K}^+\,(891.83)$$

All quantum numbers are conserved and the masses on the left side add up to 1787.71 *Mev/c*, and on the right to 1787.93 *Mev/c*. The relation may be written as

$$V^- W^+ + \nu^+ \mu^+ \rightarrow \mu^+ V^- + \nu^+ W^+ \tag{15}$$

or

$$s\bar{s} + u\bar{d} \rightarrow s\bar{d} + u\bar{s}$$

since Φ decays into $S = \pm 1$ states. The signs denote charges $\pm 1/2e$ and, for VW, the sign of S. This is a simple change of partners as in a chemical reaction that is neither exo- nor endo- thermic. Another example involves $\Phi(1^-)$ and $K(1^-)$ with spin ½ baryons (masses in Mev given in brackets).

$$\Sigma^0 + \Phi(2212.26) \rightarrow \Xi^0 + K^0(2211.0) \rightarrow \Xi^- + K^+(2213.15)$$

i.e., with charges adding up to $Q - \frac{1}{2}B$,

$$\nu^+ \nu^- V^- + V^- W^+ \rightarrow \nu^+ V^- V^- + \nu^- W^+ \rightarrow \nu^- V^- V^- + \nu^+ W^+ \tag{16}$$

or, adding to Q, $uds + s\bar{s} \rightarrow uss + d\bar{s} \rightarrow dss + u\bar{s}$

Similar relations apply to positively and negatively charged states, with $\nu^- - \nu^+ = K^0 - K^\pm = 4.27$ Mev. The mass values of ν and V determine the masses of two observable states and during the transformation (16), $2\nu + 3V$ is unchanged, but $\delta\nu = 130$ Mev $\delta V = -87$ Mev approximately ± 43.5 Mev per particle. For the pair Σ^+, K^+, data require that $V^+ = 2\nu^+$, so that $\Sigma^+ = 4K^-/3$, and for Σ^+, Φ, we have $V^+ = 3\nu^+/2$, so that $\Sigma^+ = 7\Phi/6$, both accurately confirmed. The changes from the first pair to the second are $\delta\nu = 42.5$ Mev, $\delta V = 85$ Mev, the sum of course being $\Phi - K^\pm$.

In summary, it is an overlooked but very natural conclusion from the algebra of the Lorentz group that a particle may consist of three sub-particles that resemble quarks, with specified position vectors relative to their energy centers. Although no forces are introduced these interior particles cannot be, and never have been, separated because they are part of the structure of the Lorentz group itself. The isospin and strangeness are shown to be related to the magnitude of the spin in the way that has been observed.

References and Notes

1. E. Wigner Ann. of Math. 40 149 (1939). V. Bargmann and E. Wigner Proc. Nat. Acad. Sci. 34 211 (1946). V. Bargmann Ann. of Math 48 568 (1947), Harish-Chandra Phys. Rev. 71 793 (1947). L. M. Gel'fand and A. M. Iaglom Zhur. Eksp. i. Teoret. Fiz. 18 (8) 703 (1948). H. J. Bhabha Rev. Mod. Phys. 21 451 (1949).

2. P. A. M. Dirac. Quantum Mechanics (Oxford 1935) 2nd. ed. p. 264.

3. H. C. Corben Classical and Quantum Theories of Spinning Particles (Holden Day 1968) pp. 57- 60.

4. Note that for zero momentum the ρ, ρ', ρ'', execute S.H.M. naturally with frequency $m_0 c^2 h$. Their 3-components commute, but no pairs of different components do, and independently each ρ_3 contributes a mass $\frac{1}{2} m_0$ for the ground states ν, and $3m_0/2$ for the first excited "strange" states $\mathbf{V}$. For a given mass the number of oscillators that contribute to the mass is $2I$ (contributing $\frac{1}{2} m_0$ each) and $2K$ $(3m_0/2$ *each)*. This also shows why neither I nor $\frac{1}{2}|S|$ can exceed 3/2.