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Large scale structure of the universe: Clusters and voids

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Abstract

In this work, we develop a new method for determination of the structure of a clastered distribution and apply this method studying the observed distribution of galaxies in the Universe.

Using the theory of probability, we carry out a quantative statistical analysis and find parameters which characterize differences in the distributions of galaxies in the sky and objects in an artificial uniform random field.

We are able to consider the phase space for objects in the field (for example, for stars), but in the case of galaxies, we consider the cut phase space taking into account observational data simultaneously - angular separations ρ between galaxies, and their radial velocities V.

We assume that the radial velocities of these objects indicate their Hubble distances and do not consider the peculiar velocities of galaxies.

We use the following classification of the objects:

1. members of clusters with different multiplicities;

2. uniform random distributed single objects in the galaxy field;

3. single objects inside voids.

We suggest a definition of the probability P_{nch} for objects to belong to these classes, and the critical values of P_{nch} for this classification. The relative uncertainties in the observations can be take into account.

In this paper, we consider only an area of the sky with coordinates approximately given by $0° < \alpha < 1°$ and $-90° < \delta < +90°$, and study a distribution of galaxies in the CfA-Catalogue. The number of galaxies with known redshifts inside this area is 116.

Our statistical analysis involves the number $N = 4.5 \times 10^7$ of objects in the celestial sphere with the radius corresponding to the maximum value $V_{max} = 45\ 000$ km/s of velocities of galaxies from CfA-Catalogue.

We take the mean number density of galaxies in the field

$\nu = 0.05 gal/Mpc^3$ and the Hubble constant $H = 75 km/s/Mpc$.

For every object under consideration , we calculate the probability P_{nch} and classify its. We identify the confident and probable members of galaxy groups and clusters.

As a result, we have found 19 new confident and probable galaxy groups with multiplicity $n < 7$ and one new 'supercluster' with $n = 31$ containing a few small clusters.

The members of the galaxy clusters are determined, on the average, with the probability P_{nch}=0.932 . For every cluster and different samples of galaxies we define the average values of basic characteristics - a angular separation ρ and a difference dV of radial velocities V of objects.

Between clusters and single galaxies often are confidently determined empty regions (voids) with the mean value of $P_{nch} = 0.995$.

Our calculations produce a highly skewed distributions of P_{nch} for galaxies which have above mean values of P_{nch}, and about 95% of galaxies have $P_{nch} = 0.85$ or more .

We conclude that 75.0% of the galaxies under study are members of clusters. Other galaxies in the field are either confident single ones (15 galaxies- 12.5%) distributed randomly in the field or belong (14 galaxies- also almost 12.5%) to voids.

We have here the cluster-void structure; sometimes in the field we observe few close clusters, connected with each other; between clusters and single galaxies we have often confident voids.

We have found also the number of chance pairs of galaxies in an artificial 'galaxy' field with an uniform random distribution of objects and the mean values of their characteristics ρ and dV: the number of confident chance pairs of 'galaxies' in this field with $V_{max} = 45$ 000 km/s and $N = 4.5 \times 10^7$ is 8.3×10^6; for such pairs with $V = V_{max}$ the mean values $< \rho_{ch} >= (3.0 \pm 1.2)'$ and $< dV_{ch} >= (1289 \pm 1087)$ km/s .

We also have found two confident real compact galaxy groups with large differences dV for their members: the values dV are equal to 2090 and 2076 km/s.

Keywords: **galaxy distribution functions - galaxy clusters - voids - chance and non-chance groups**

1 Introduction

In the extragalactic field, the basic subject of cosmology is a study of the large-scale structure of the Universe extending to large distances of giga-parsecs and a study of the dynamics and physical behaviour of a large number of galaxies.

The main problem of observational cosmology is the study of the distributions of the density and velocities of galaxies and quasars on small and large scales.

On very large scales, the Universe as a whole appears to be isotropic and homogeneous. But observationally, on small scales, on the scale of superclusters the distribution of

galaxies is inhomogeneous, there are filaments, clusters, superclusters, voids etc.

The detailed analysis of the observational data for galaxies and QSO's show that galaxies are not distributed uniformly on the sky. They are segregated into compact clusters of different multiplicity. Most clusters are spherically symmetric. Clusters of galaxies tend to be grouped into larger structures - superclusters. Superclusters have dimensions of the order of 50 Mpc, dimensions of typical galaxies are approximately 30-50 kpc. Most galaxies belong to large structures; about 90% of all galaxies belong to some form of hierarchical structure. Most clusters are spherical in shape but almost all superclusters are flat. Often, superclusters are called 'pancakes' (Zel'dovich 1970).

The observations of clusters and superclusters show that these objects are distributed randomly. But the detailed analysis of the distribution of galaxies indicates that this distribution is not uniform. We observe numerous clusters of galaxies as well as regions of the Universe which contain almost no galaxies. These regions are called 'voids' (de Lapparent et al.1986). Only 10superclusters.

Several theories of the structure of superclusters tried to explain the large-scale structure of the Universe,i.e the processes of formation of condensations of matter in spherical form with high density as well as of large scale filamentary and flat structures (cf. Peebles 1965, Zeldovich 1970, Rees 1985, Einasto et al. 1986, Contopoulos and Kotsakis 1987, Shandarin and Zel'dovich 1989, Alexander 1992).

Numerical simulations of the distribution of matter on large scales give structures of comparable size, shape and distribution to those observed (Aarseth et al. 1979, White et al. 1987, Doroshkevich et al. 1980, Kotok and Shandarin 1987, Shandarin and Zeldovich 1989, and the references therein). The well-known "Hot Big Bang" model of the Universe has been very successful on large scales where the Universe is almost homogeneous and isotropic. On smaller scales, there are two basic theories for the explanation of matter condensations which form via gravitational instability: isothermal and adiabatic.

In the theory of isothermal perturbations (Peebles 1965) any density perturbutions in the initial matter distribution of the Universe remains unchanged until gravity begins to dominate. At this time, any perturbation greater then 10^5 M_circ starts to accrete the surrounding matter and larger condensations of mass (clusters, superclusters) begin to form. By this theory, large structures require a long time for formation.

In the theory of adiabatic perturbations (Zel'dovich 1970), initial perturbations are large. They survive the dissipative processes of viscosity present in the early stages of the Universe's formation. Only concentrations more then 10^{13} M_o remain intact until the recombination epoch when they collapse via the Jean's instability. In this theory, the larger scale structures (superclusters) form first and elements of structures on smaller scales (clusters, groups, etc.) are formed later. In this case, the initial concentrations of matter are very flat in shape. Therefore, this theory is called the pancake theory. These 'pancakes' fragment to form galaxies.

Now it is difficult to say which of the two theories better explan the formation and evolution of large-scale structure of the Universe. But, in any case, one can say that, according these theories, the Universe is neither too young (in the case of the hierarchical theory, it would not have had enough time do develop) nor too old (in the case of the adiabatic theory, the observed structure of the Universe should not exist at present).

We also have observational evidence that dark matter is present on all distance scales of the Universe, from its effect on the motion of the Sun, the rotation of galaxies, in the dynamics of clusters and superclusters and also in the expansion of the Universe (Einasto et al. 1986, Trimble 1987).

2 Statistical analysis of large-scale structures

The main feature of the structures of the Universe is that they form from a random Gaussian field of density and velocity perturbations. Now from observational data, in principle, one cannot obtain the information about cosmological density perturbations at the initial linear stage of formation of the Universe. We only can obtain statistically a correlation between initial (linear) density fluctuations and the observed structures.

At present, there are many statistical approaches,describing the large-scale structures of the Universe to support or reject different theories of formation of these structures (e.g. Jones et al. 1988).

One of the methods is the correlation analysis which was developed by Peebles and co-workers (see Peebles 1980, and refernces therein).

The second is a statistical method based on cluster analysis, referred to as precolation analysi. First Zel'dovich and Shandarin (Zel'dovich 1982, Shandarin 1983, Shandarin and Zel'dovich 1983) applied this method for a study of the structure of the Universe.

In these statistical techniques usually authors consider independently the angular separations between the galaxies under study and their radial velocities. They use various critical values of these quantities to determine the members of clusters.

In this paper we suggest another method to study the large scale structure of the Universe. This technique was developed by J. Anosova and was earlier applied to groups of stars and galaxies (Anosova 1987, 1990a,b). In this method we consider the angular separations between galaxies, their velocities and the differences in the velocities. We do not consider the peculiar velocities of the galaxies. We assume that the radial velocities of these objects indicate their Hubble distances.

3 Method

This technique has now been extended to the study of the large scale structure of the Universe. We find a deviation of the actual distribution of the galaxies from a uniform random distribution in the general field.

Let's consider a phase space $\Sigma(R, U)$, which is a direct product of a sphere $\Sigma_1(R)$ of the coordinate space with radius R and a sphere $\Sigma_2(U)$ of the velocity space with radius U

$$\Sigma(R, U) = \Sigma_1(R) \otimes \Sigma_2(U). \tag{1}$$

Inside Σ_1 the number N of uniformly distributed objects of different kinds (stars, galaxies, quasars, their groups and clusters, superclusters etc.) is given by

$$N = 4\pi R^3 \nu, \tag{2}$$

where ν is the mean number density of objects in $\Sigma_1(R)$.

Assuming a homogeneous distribution of the N objects in the region $\Sigma(R,U)$ of phase space we can define

- The probability $P(n)$ that the n objects are inside the region $\sigma(n)$ by chance. Then the group of these n objects is a chance one.
- The expectation $EX(n)$ for the number of such chance groups in the full phase space $\Sigma(R,U)$.

The probability, $P(n)$, for the group of multiplicity n to be a chance one is given by (see Anosova 1987,1 990a,b)

$$P(n) = C_{N-1}^{n-1} B(n)^{n-1}[1 - B(n)]^{N-n} \tag{3}$$

where

$$B(n) \;=\; V(\sigma)/V(\Sigma), \tag{4}$$

is the probabilty that the n objects are inside the region $\sigma(n)$ by chance; $V(\sigma_n)$ and $V(\Sigma)$ are the volumes of region σ_n and the sphere $\Sigma \equiv \Sigma(R,U)$ in phase space; C_{N-1}^{n-1} is the number of combination of $n-1$ objects from $N-1$,

$$C_{N-1}^{n-1} = \frac{(N-1)!}{(N-n)!(n-1)!}; \tag{5}$$

the term

$$[1 - B(n)]^{N-n}, \tag{6}$$

is the probability that the other $N-n$ objects are outside the phase space σ_n occupied by the group of multiplicity n.

In order to obtain the expectation, $EX(n)$,for the number of such chance groups with the same multiplicity n and the same observational parameters in the full sphere Σ, one needs to do the following operations:

1. In expression (3) replace the indices $N-1$ and $n-1$ by N and n, since the main component A of the group may be any one of the N objects.
2. Multiply the new expression obtained by the ratio

$$V(\Sigma)/V(\sigma) = B(n)^{-1}, \tag{7}$$

   which is the number of regions σ inside the phase space Σ.

Then for the expectation $EX(n)$, we obtain the following expression

$$EX(n) = C_N^n B(n)^{n-1}[1 - B(n)]^{N-n}. \tag{8}$$

For galaxies we can consider only the angular separations between galaxies ρ and their radial velocities v. In this case, the expression for $B(n)$ is the following one:

$$B(n) = 0.25 \tan^2[\rho/2] \left(\frac{r_A}{R}\right)^3 \left|1 - \frac{r_n}{r_A}\right|^3 \tag{9}$$

and

$$r_n/r_A = v_A/v_n.$$

for two any members (A, n) of a galaxy group, the values of r_A,r_n and v_A and v_n are their three-dimensional distances and radial velocities. In expression (9) ρ is the angular separation between galaxies (in radians).

In order to define the maximal values of $P(n)$ and $EX(n)$ we should use

$$R = \max[r_A, r_n]. \tag{10}$$

We will use the expressions (3—9) for the probability $P(n)$ that the given group with multiplicity n is a chance one and the expectation $EX(n)$ for the number of such chance groups inside the phase space Σ.

4 Analysis of the observed data

The basic general aims of our research are the following:

1. Definition of elements of structures of astronomical fields (observed and model) of different types, sizes and shapes (star and galaxy groups, chains, clusters, superclusters, filaments, voids etc.).

2. Selection of basic characteristic parameters of elements of the structure: mass and velocity distributions, distributions of magnitudes, morphological types for galaxies and spectral classes and classes of luminosities for stars, the geometrical parameters of structures under study. Determination of the average values of these basic characteristics.

3. Study of the dynamics of some samples of different elements of the field and the field as a whole within the framework of the gravitational N-body problem with various N. At any time we can define the structure of these elements and the full field and also follow the evolution of these structures.

4. Analysis and comparison of the obtained results for model and observed fields. Selection of the most realistic cosmological models.

In this work, we consider a small portion of the CFA catalogue of galaxies and examine a narrow strip of the sky with coordinates

$$0^\circ < \alpha < 1^\circ \qquad \text{and} \qquad -90^\circ < \delta < +90^\circ, \tag{11}$$

where α, δ are the equatorial coordinates of the galaxies. The number of galaxies in the CFA catalogue with known redshifts inside this area is 115. The full number of galaxies inside this area is 176. We omit one galaxy with negative radial velocity because we do not consider the peculiar velocities of galaxies. We assume that the radial velocities of these objects indicate their Hubble distances. In future, we plan to examine the full sky.

Initially, before our examination, we know nothing about the distribution of objects in the area under study; but after this investigation we will have full information about the

fine structure of the galaxy field in this region: every galaxy can be assigned to some element of structure of the general field.

We construct all possible pairs of objects. The number of pairs with different components is

$$C_n^2 = n(n-1)/2 = N_{pair} = 6555. \tag{12}$$

The full number of such pairs is $2 \cdot N_{pair} = 13110$. We determine angular separations ρ, the differences dV of radial velocities, and the 3-dimensional distances r_3 between the components of all pairs. We take the value of the Hubble constant to be

$$H = 75\,\mathrm{km/s/Mpc}. \tag{13}$$

Using our statistical criteria (3-9), we calculate the probability P that these pairs are chance ones and the expectation EX of such pairs in the full sky sphere. For our calculations we take the average number density ν of the galaxies in the field to be $0.05\,/\mathrm{Mpc}^3$. Comparing the calculated expectation EX with the corresponding number of observed pairs, we conclude whether these pairs are chance or non-chance ones.

Initially we used our earlier criterion (see Anosova 1987, 1990a,b):

1. The group with multiplicity n is a confident non-chance one if the expectation satisfies the inequality

$$EX(n) < 1, \tag{14}$$

   i.e., we do not expect any chance group with such observed data.

2. The multiple is a confident chance group if

$$EX(n) \approx N/n, \tag{15}$$

   where N is the total number of objects in the region Σ and N/n is the maximum possible number of groups with a given multiplicity n.

3. It is difficult to make any definitive statement about a group if the following inequality holds:

$$1 < EX(n) < N/n. \tag{16}$$

   In this case we can only identify the group as a probably non-chance or chance one.

Sometimes the value $EX > 1$ can be due to internal dispersions and uncertainties of radial velocities of the members of groups.

We classified the groups of objects (stars and galaxies) according to the following scheme:

I	Confident non-chance groups	
II	Probable non-chance groups	(17)
III	Probable chance groups	
IV	Confident chance groups	

If the redshifts indicate the distances of the galaxies from the Sun, close non-chance groups of galaxies are systems with physically connected components; the members of wide non-chance groups are confident single galaxies belonging to voids (between members of wide

non-chance groups there are - to a high degree of confidence - empty regions (voids)). Chance groups are fortuitous groupings of galaxies and may be called 'optical' ones. The members of such optical groups are single objects of the 'normal' galaxy field. If we consider pairs of objects and in the same region we have a few or many confident or probably non-chance pairs with physically connected components, then in this region we have a confident or probably non-chance group or cluster of these objects.

For all pairs of the galaxies under study only ≈ 1000 pairs (from $N_{pair} = 6555$) have $EX(n) > 1$. Therefore, the components of most pairs of galaxies with $EX(n) < 1$ are either members of clusters or single galaxies inside voids. Fig. 1 shows the distribution of all CfA galaxies in the area under consideration. At first sight, we can indeed identify a cluster with coordinates $0° < \alpha < 0.45°$ and $\delta \approx -36.4°$.

5 A model of the galaxy field: definition of the basic characteristics of chance and non-chance pairs

As seen above, the expressions (3), (8) contain two different terms $B(n)^{n-1}$ and $[1 - B(n)]^{N-n}$. The first is an indicator of clusters of objects inside the field, and the second is an indicator of voids. Therefore, we expect that with an increase in the volume $V(\sigma)$ occupied by the group, the values of $P(n)$ and $EX(n)$ also increase initially, reach their maxima, and then decrease till $P(n) \approx 0$, $EX(n) < 1$.

For every phase space volume $\Sigma(R)$ with radius R and the assumed average number density ν of the objects, we can find the maximal number

$$EX_{max}(n) = EX_{ch} \tag{18}$$

of chance groups with a given multiplicity n and the their typical basic characteristics.

We now introduce a new criterion for the classification of structures:

1. a galaxy pair is a chance one with the probability

$$P_{ch} = EX(n)/EX_{max}(n); \tag{19}$$

2. a group is a non-chance one with probability

$$P_{nch} = 1 - P_{ch}. \tag{20}$$

For confident chance pairs which satisfy condition (18) we have

$$P_{ch} = 1. \tag{21}$$

For a definition of $EX_{max}(n)$ we consider a model of the galaxy field which is the phase space sphere $\Sigma(R)$ of radius R corresponding to the largest radial velocity V of galaxies in this field.

We examine the galaxy fields with various V corresponding to the observed data for the CFA galaxies: $1000 \leq V \leq 45000$ km/s. Using the expression

$$N = \frac{4}{3}\nu R^3,$$

for every V we find the total number N of galaxies inside this field. We construct model pairs of galaxies with various values of the angular separation ρ and difference of radial velocities dV: we scan dV and ρ in the following intervals:

- we choose dV such that dV/V lies in the interval (0.0, 1.0).
- in order to obtain typical values of the parameters for galaxy pairs of all classes I-IV (see (17)) we examined a large range of angular separations between galaxies from 0° to 50°.

When we scan both values dV and ρ independently (ρ for fixed dV and vice versa), sometimes we obtain the different quantities

$$EX_{max}(n)(dV) \qquad \text{and} \qquad EX_{max}(n)(\rho). \tag{22}$$

In such cases, we calculate the average value

$$EX_{max}(n) =< EX_{max}(n) >= [EX_{max}(n)(dV) + EX(n)_{max}(\rho)]/2. \tag{23}$$

Further, we will use both these values, obtain two probabilities $P_{ch}(dV)$ and $P_{ch}(\rho)$ and calculate the average value

$$P_{ch} = [P_{ch}(dV) + P_{ch}(\rho)]/2. \tag{24}$$

Figure 2 shows the functions $P_{ch}(\rho)$ with argument ρ for $V = 1000\,\mathrm{km/s}$ for few different dV. We can see that for both values of V, for small and large values of ρ, $P_{ch}(\rho)$ is very small; for intermediate values of ρ, $P_{ch}(\rho)$ initially increases monotonically, attains a maximum and later decreases monotonically to zero. In fact, this behaviour is seen for every value of V. For $P_{ch}(dV)$ we have an almost similar behaviour, but for smaller V ($\approx 1000\,\mathrm{km/s}$) and larger dV this function cannot reach small values.

For every EX_{max} we fix the corresponding quantities dV and ρ. These values are the typical ones dV_{ch} and ρ_{ch} for confident chance pairs of galaxies satisfying the condition (21). For a given V and a range of values for one quantity (for example, dV) we can define the average value and r.m.s. deviations for the other quantity (ρ) and vice versa.

Table I shows these values for pairs of galaxies of the model fields: for a given V, this table contains the number N of galaxies in the phase space $\Sigma(R)$, the values $EX_{ch} = EX_{max}(2)(n = 2)$, as well as the average values dV_{ch} and ρ_{ch} and their r.m.s. deviations for confident chance pairs of galaxies. The last two quantities in Table I will be described below.

We can see that for small V, and correspondingly, small dV, for typical chance pairs the value of $< \rho_{ch} >$ is large. With increasing V and dV these values decrease quickly. For the largest V (45000 km/s) $< \rho_{ch} >= (3.0 \pm 1.2)'$. For this value of V, $< dV_{ch} >= (1289 \pm 1087)\,\mathrm{km/s}$. This is related to the problem of discordant redshifts of galaxies in compact groups. If the group of galaxies is very far from the Sun, the two dimensional projection may be compact, but velocity differences dV may be more then 1000 km/s for chance members. If dV is much less than this, then this group is a physically connected one; if dV is much more than 1000 km/s then it is a confident non-chance phenomenon.

It may be an effect of projection or the radial velocities of the galaxies are not indicative of their Hubble distances.

The statistical analysis involves the number N of objects inside the sphere $\Sigma(R)$. Note that for the largest V (45000 km/s) this number is very large viz., 4.5×10^7. The corresponding number of confident chance pairs EX_{ch} is also large: 8.3×10^6. It's interesting that for all V we obtain

$$EX_{ch} = 0.367N/2 = P(n = 2). \tag{25}$$

Now for all model and actual pairs of galaxies we will consider the ratios

$$k(\rho) = \rho/\rho_{ch} \qquad \text{and} \qquad k(dV) = dV/dV_{ch}. \tag{26}$$

We will call these values the normalized angular separations ρ between the galaxies and differences dV of their radial velocities.

For model galaxy fields with various V in the interval [1000,45000] km/s in steps of 1000 km/s, we define the following quantities:

1. for every $k(dV)$ in the interval (0.0,1.0) in steps of 0.1 the ratios $k(\rho)$ corresponding to values

   $$P_{ch} = 0.25,\ 0.50,\ 0.75,\ 1.00 \tag{27}$$

   (when the function $EX(n = 2)$ increases monotonically) and

   $$P_{ch} = 1.00,\ 0.75,\ 0.50,\ 0.25 \tag{28}$$

   (where this function decreases monotonically);

2. For every $k(\rho)$ from the same interval and with the same stepsize, the ratios $k(dV)$ for the same values of P_{ch}.

The main result of the foregoing analysis is that for various V and $k(dV)$ and $k(\rho)$ we have practically the same values of $k(\rho)$ in the first case and $k(dV)$ in the second case for every P_{ch} from (27) and (28).

Table II shows the average values $< k(\rho) >$ and $< k(dV) >$ and their r.m.s. deviations for various values of V for P_{ch} from (27,28). We can see that the r.m.s. deviations of these values are very small. Note that for the same value of P_{ch} to the left and right of $P_{ch} = 1$ we obtain very different values of $< k(\rho) >$ and $< k(dV) >$. For the P_{ch} in (27), these quantities are small for confident non-chance pairs with physically connected components; for the P_{ch} in (28), they are large for confident non-chance pairs with voids between the components.

6 Criteria for the classification of galaxy pairs

Now we propose to use P_{ch}, $< k(\rho) >$ and $< k(dV) >$ from Table III for a classification of the galaxy pairs into the following new classes I - VIII:

I. pairs with $0 \leq P_{ch} \leq 0.25$ are confident binaries with physically connected components

II. if $0.25 < P_{ch} \leq 0.50$, the pairs are probable binaries with physically connected components

III. if $0.50 < P_{ch} \leq 0.75$, the pairs are 'close' probable chance ones

IV. if $0.75 < P_{ch} \leq 1.00$, the pairs are 'close' confident chance ones

V. if $1.00 > P_{ch} \geq 0.75$, the pairs are 'wide' confident chance ones

VI. if $0.75 > P_{ch} \geq 0.50$, the pairs are 'wide' probable chance ones

VII. if $0.50 > P_{ch} \geq 0.25$, the pairs are probable non-chance ones. In the absence of any other galaxy between the components of a such pair, the components are separated by a void.

VIII. if $0.25 > Pch \geq 0$,the pairs are the confident non-chance ones with void between the components.

The ranges of $< k(\rho) >$ and $< k(dV) >$ corresponding to the various classes are determined from the critical values of these quantities given in Table III.

The last two columns of Table I show, for different V, $\rho_{max}(I)$, the maximum value of ρ for class I, and $\rho_{min}(VIII)$, the minimum value of ρ for class VIII. Here $\rho_{max}(I)$ and $\rho_{min}(VIII)$ represent the actual angular separations, and are determined from the normalized values by using the average value of $k(dV)$ for each V. We can see that for smaller V, ρ is very large, and for the larger V it is very small.

7 Application to the CfA catalogue

Using the criteria described in the previous section, we search for and classify the actual pairs of galaxies from the region of the CfA Catalogue under study. This results in the determination of clusters, single galaxies and voids in the galaxy field.

We determine that indeed in the area with coordinates $-0.01° \leq \alpha \leq 0.45°$ and $\delta \approx -36.4°$ we have the largest cluster with $n = 31$. Moreover, this cluster consists of four smaller clusters. The cluster 1a consists of five subclusters. Hence we denote cluster 1 as 'supercluster-1'.

We also calculate the probabilities P_{nch} of confident and probable physical connection between the subclusters and the small clusters of supercluster-1. We show that subclusters of cluster 1a have confident physical connections; but, in most cases, the clusters 1a-1d have only probable physical connections with each other.

Fig. 3 presents the relative positions of the members of the supercluster-1 as well as clusters 2,3,6 and the single galaxies g4 and g89 which are visually close to the supercluster-1 but are separated when we apply our method.

We also determine single galaxies in the galaxy field. The nearest neighbour of such a galaxy forms, together with the given galaxy, a confident or probable chance pair (classes III - VI of our classification) or a pair with confident or probable voids between components (classes VII and VIII).

In the fact that P_{nch} for pairs of classes I,II and VII,VIII is often large (≈ 1); therefore,these pairs are identified very confidently.

8 The average confidence level of determination of galaxy groups and voids

At the beginning, for our classification (section 6) we used the critical confidence level $P_{ch}^{crit} = 0.75$ or $P_{nch}^{crit} = 0.75$. Now we can define the average values of the probabilities $< P_{nch} >$ and $< P_{ch} >$ for clusters and single galaxies in the galaxy field. These values $< P_{nch} >$ and $< P_{ch} >$ show the resulting average confidence level of our classification. We find that the real confidence level of this classification is high; for clusters we have

$$< P_{nch} >= 0.904 \pm 0.133 \tag{29}$$

and for voids

$$< P_{nch} >= 0.995 \pm 0.014. \tag{30}$$

Only for chance pairs, we obtain the average confidence level to be

$$< P_{ch} >= 0.752 \pm 0.112, \tag{31}$$

which is close to what we choose initially. If we exclude the 6 galaxies which are only probable members of the clusters with smallest P_{nch} (using the statistical criterion of a 2σ deviation), then we obtain the new average confidence level

$$< P_{nch} >= 0.928 \pm 0.099 \tag{32}$$

with a small r.m.s. standard deviation for the remaining 83 members of the clusters.

We also determine the average characteristics and their r.m.s. standard deviations for all the clusters: the three-dimensional distances $< r_3 >$, angular separations $< \rho >$ and the differences of radial velocities $< dV >$ between members of clusters or the single galaxies and their nearest neighbours; the quantities $< V >$, $< \alpha >$ and $< \delta >$ show the three-dimensional positions of the centers of the given cluster; the r.m.s. standard deviations of $< V >$, $< \alpha >$ and $< \delta >$ show the three average sizes of clusters in the line-of-sight and the equatorial coordinates.

The distributions of r_3, ρ and dV for the members of clusters and single galaxies of the field, as expected, are very different.

Tables III show the distribution of the multiplicity n for clusters and different samples. Here N_n is the number of clusters with multiplicity n, N_{gal} is the number of galaxies belonging to clusters and samples, and d_{n1}, d_{n2}, d_{n3} are the corresponding fractions. We can see that 75.0% of the galaxies are confident or probable members of clusters; only 25.0% are the single galaxies and half of these objects are inside voids.

9 The cluster-void structure of the Universe

Let's consider for every object n_1 (clusters and the single galaxies) the sequence of neighbours and find the number of nearest neighbours, n_2, with which the given object n_1 has

a confident or probable void-connection with probability $P_{nch} > 0.75$, $k(\rho) > k(\rho)^{crit} = 1.631$ and $k(dV) > k(dV)^{crit} = 2.767$ (classes VII and VIII of our classification). For members of clusters, we define the average values of the probabilities $< P_{nch} >$, three-dimensional distances $< r_3 >$, angular separations $< \rho >$ and differences $< dV >$ of radial velocities for pairs the components of which are the given object n_1 and the object n_2.

Because the subclusters 1a-5a of the cluster 1a and clusters 1a-d have confident or probable physical connections with each other, we consider together all 31 members of this supercluster-1. Clusters 11a,b and 12 have only confident or probable chance connections with each other; therefore, we consider these clusters separately. We can see that only these last clusters 11a,b and 12 have a few nearest neighbours (the same clusters) with chance connections. If we combine these clusters than we see that for all clusters and single galaxies the number n_2 is 1 or 2. In most cases (31 out of 49: 63.3%) $n_2 = 1$. Therefore, we can conclude that in the examined part of the Universe the normal galaxy field with random distributed galaxies does not exist. We find instead a cluster-void structure. The structure consists of single clusters, groups of a few connected clusters, single (isolated) galaxies and binaries. Between these we have voids with an average confidence level of

$$< P_{nch} >= 0.987 \pm 0.028.$$

10 Conclusion

Now we can formulate our basic results: 1.extention of the technique for the statistical study of the large-scale structure of the Universe; 2.a development of new objective criteria for classification of galaxies as confident or probable members of clusters or single galaxies inside the normal galaxy field and voids; 3. we apply our method to galaxies from the CFA-catalogue in narrow area of the sky and determine about the 20 confident and probable non-chance galaxy groups of different multiplicity as well as about 30 confident single galaxies in the field, 15 of them have nearest neighbours with confident or probable chance connection; 14 galaxies reside in confidently determined voids.

The average confidence level for determination of members of galaxy clusters is

$$P_{nch} = 0.932 \pm 0.067,$$

for voids it is

$$P_{nch} = 0.995 \pm 0.014.$$

We also shown that between all clusters and single galaxies there are confidently determined empty regions-voids.

Therefore, we conclude that in the examined part of the Universe the normal galaxy field with randomly distributed galaxies does not exist.

We plan to study in future distributions of galaxies and quasars in the full sky sphere.

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References

Aarseth,S.J.,Gott,J.R.,Turner,M.S.:1979,Astrophys.J.,228,664.
Alexander,D.:1992,Vistas in Astronomy,11.
Anosova,J.P.:1987,Astrofizika,27,535.
Anosova,J.P.:1990,Bull.Belgr.Astr.Obs.,142,1.
Anosova,J.P.:1990,Proc.IAU Coll.No.124,659.
Anosova,J.P.,Popovich G.:1990,Kinem. Phys.Celest.Bod.,6,93.
Contopoulos,G.,Kotsakis,D.:1987,Cosmology,sssspringer-Verlag,Berlin.
Einasto,J.,Klypin,A.,Saar,E.:1986,Mon.Not.R.astr.Soc.,219,457.
Jones,B.J.T.,Martinez,V.J.,Saar,E.,Einasto,J.:1988,Astrophys. J.Lett.332,L1.
Kotok,E.V.,Shandarin,S.F.:1987,Sov.Astron.31,600.
de Lapparent,V.,Geller,M.,Huchra,J.:1986,Astrophys.J.Lett.,302,L1.
Peebles,P.J.E.:1965,Astrophys.J.,142,1317.
Peebles,P.J.E.:1980.The Large-Scale structure of the Universe, Princeton University Press,Princeton.
Rees,M.:1985,Mon.Not.R.astr.Soc.,213,75.
Shandarin,S.F.:1983,Sov.Astron.Lett.9,104.
Shandarin,S.F.,Zel'dovich,Ya.B.:1983,Comments Astrophysics.Space Phys.10,33.
Shandarin,S.F.,Zel'dovich,Ya.,B.:1989,Reviews of Modern Physics, 61,2,185.
Trimble,V.:1987,Annu.Rev.Astron.Astrophys.25,425.
White,S.D.M.,David,M.,Efstathiou,G.,Frenk,C.S.:1987,Astrophys.J. 313,505.
Zel'dovich,Va.B.:1970,Astr.Astrophys.,5,84.
Zel'dovich,Ya.B.:1982,Sov.Astron.,59,636.
Zeldovich, Ya.B., Novikov, I.D.:1983, The Structure and Evolution of the Universe (University of Chicago, Chicago/London).

Table I: Characteristics of typical chance pairs in the galaxy field

V km/s	N	EX_{ch}	$< dV_{ch} >$ km/s	ρ_{ch} arcmin	$\rho_{max}(I)$ arcmin	$\rho_{min}(VIII)$ arcmin
1000	5.0×10^2	9.1×10^1	98±85	898±278	374	2250
2000	4.0×10^3	7.3×10^2	106±88	319±101	133	796
3000	1.3×10^4	2.5×10^3	162±138	174±55	72	433
5000	6.2×10^4	1.1×10^4	226±190	80±25	34	201
6000	1.1×10^5	2.0×10^4	262±222	61±19	25	152
8000	2.5×10^5	4.7×10^4	326±276	40±12	16	99
10000	5.0×10^5	9.1×10^4	387±327	29±9	12	72
20000	4.0×10^6	7.3×10^5	666±562	10±2	4.1	25
35000	2.1×10^7	3.9×10^6	1050±886	4.5±1.5	1.9	11
40000	3.2×10^7	5.8×10^6	1172±988	3.6±1.2	1.5	9.1
45000	4.5×10^7	8.3×10^6	1289±1087	3.0±1.2	1.3	8.0

Table II: The average critical values of the ratios $k(\rho)$ and $k(dV)$

P_{ch}	$< k(\rho)^{crit} >$	$< k(dv)^{crit} >$	Classes	
0.00	< 0.318	< 0.100		
0.25	0.318±0.012	0.100±0.002	I	confident clusters
0.50	0.480±0.019	0.229±0.004	II	probable clusters
0.75	0.646±0.025	0.416±0.008	III	'close' probably chance pairs
1.00	1.000±0.000	1.000±0.000	1V	'close' confident chance pairs
1.00	1.395±0.054	1.998±0.045	V	'wide' confident chance pairs
0.75	1.631±0.063	2.767±0.089	VI	'wide' probably chance pairs
0.25	1.914±0.074	3.902±0.229	VII	probable voids
0.00	> 1.914	> 3.902	VIII	confident voids

Table IIIa: The multiplicity and frequency of galaxy clusters

n	N_n	dn_1
31	1	0.267
6*	2	0.086
5	2	0.086
4	2	0.069
3	2	0.052
2	11	0.190
1	29	0.250

* Clusters 11b($n = 6$) and 12($n = 6$) have two common probable members.

Table IIIb: The frequency of galaxies in the various samples

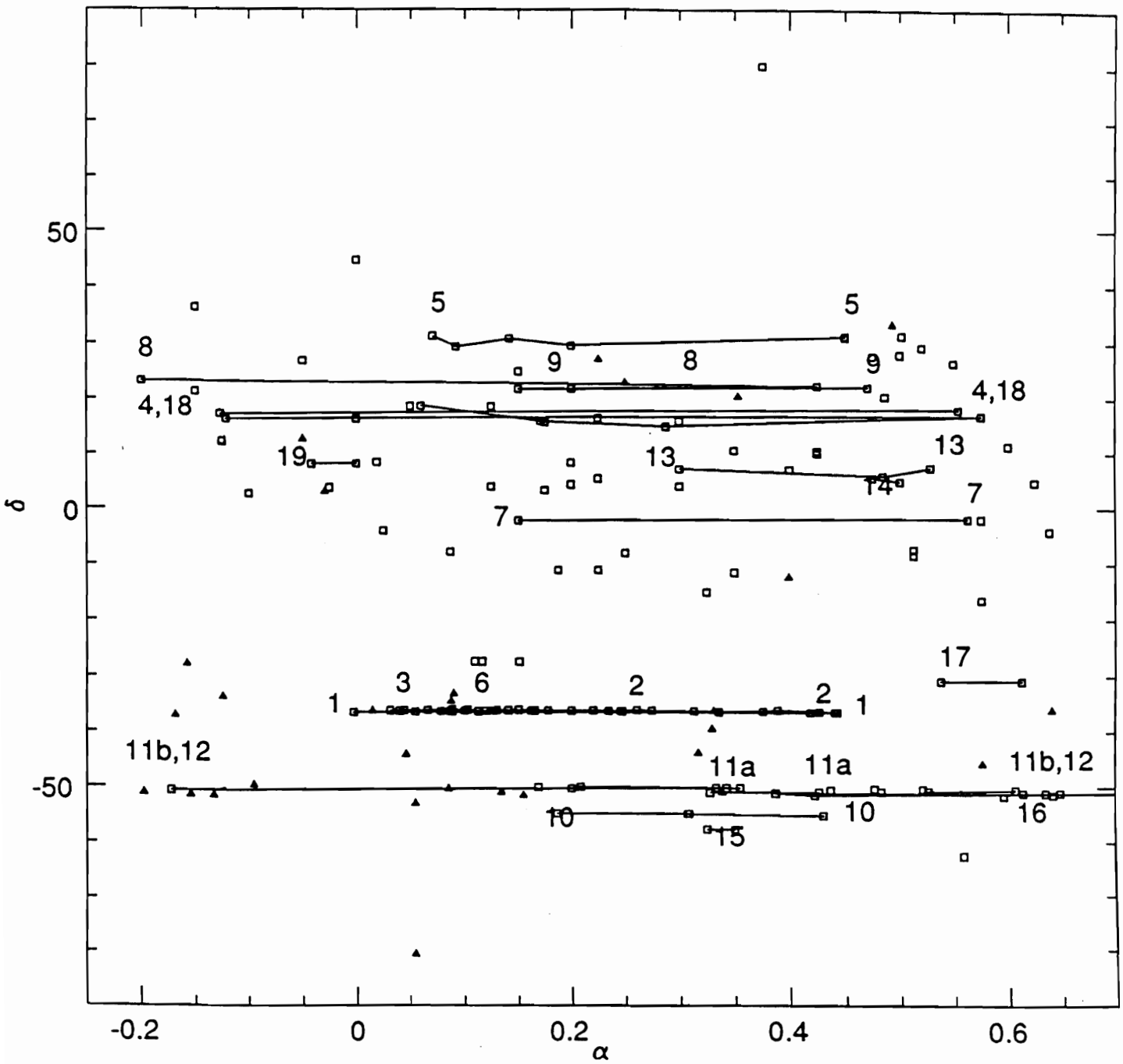
Sample	N_{gal}	dn_2	Sample	N_{gal}	dn_3
discords inside clusters	3	0.026	discords inside clusters	3	0.026
clusters with probable members	84	0.724	clusters with confident members	74	0.638
single galaxies	29	0.250	single galaxies	39	0.336
Total	116	1.000		116	1.000

Captions

Fig. 1. Distributions of galaxies on the sky: the galaxies with unknown radial velocities are indicated by empty squares; the members of clusters are connected by lines; the triangles indicate single galaxies in the field; the numbers correspond to the numbers of clusters.

Fig. 2. The dependence of P_{ch} on ρ for $V = 1000$ km/s and the various dV. The critical values of ρ^{crit}, P_{ch}^{crit} and the classes of the galaxy pairs are indicated.

Fig. 3. Positions of the members of supercluster 1: subclusters a1-a5 of cluster 1a, and clusters 1b-d. The members of clusters 2, 3, and 6 as well as two single galaxies g4, g89 in the same region of the sky are indicated.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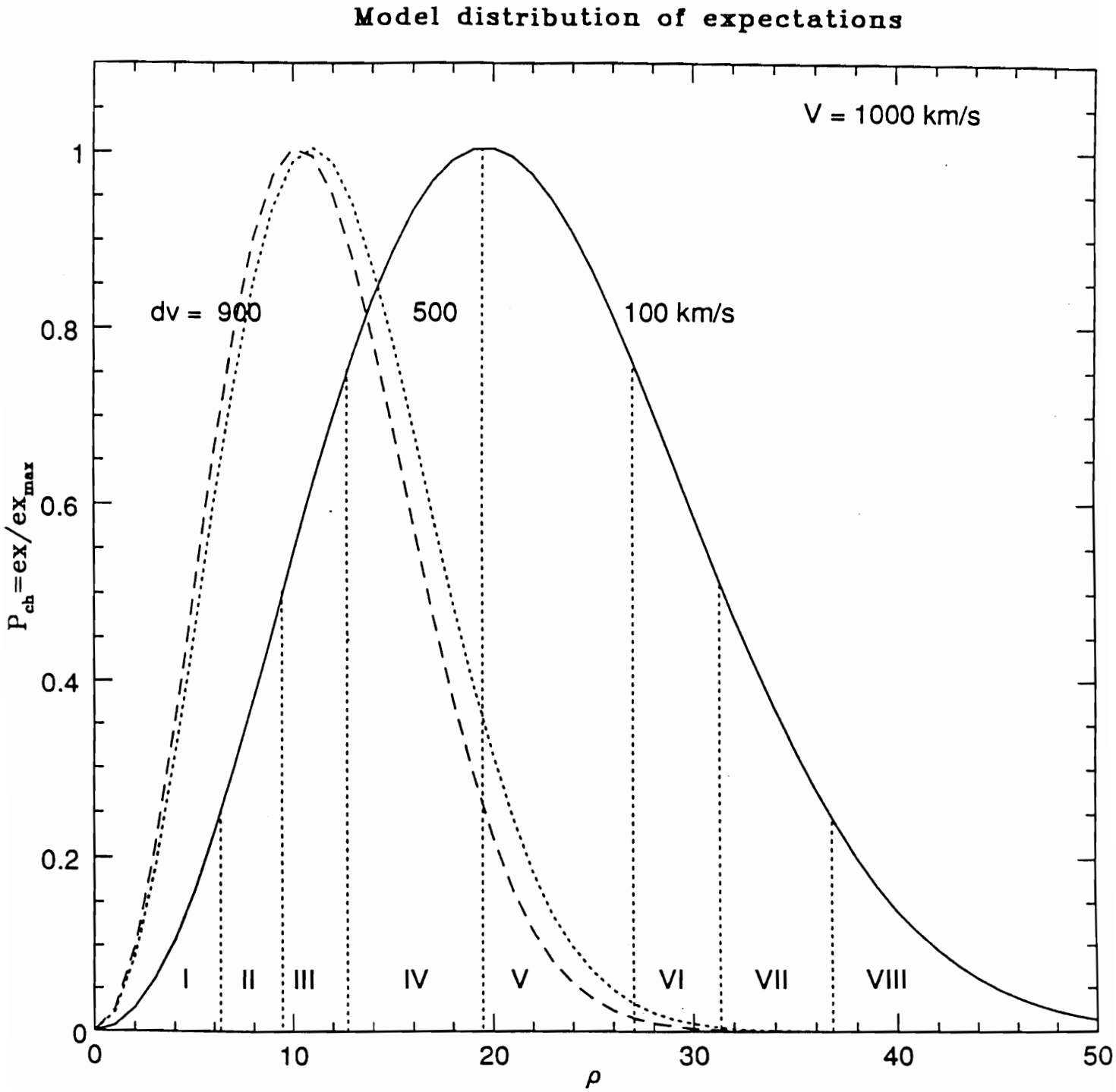
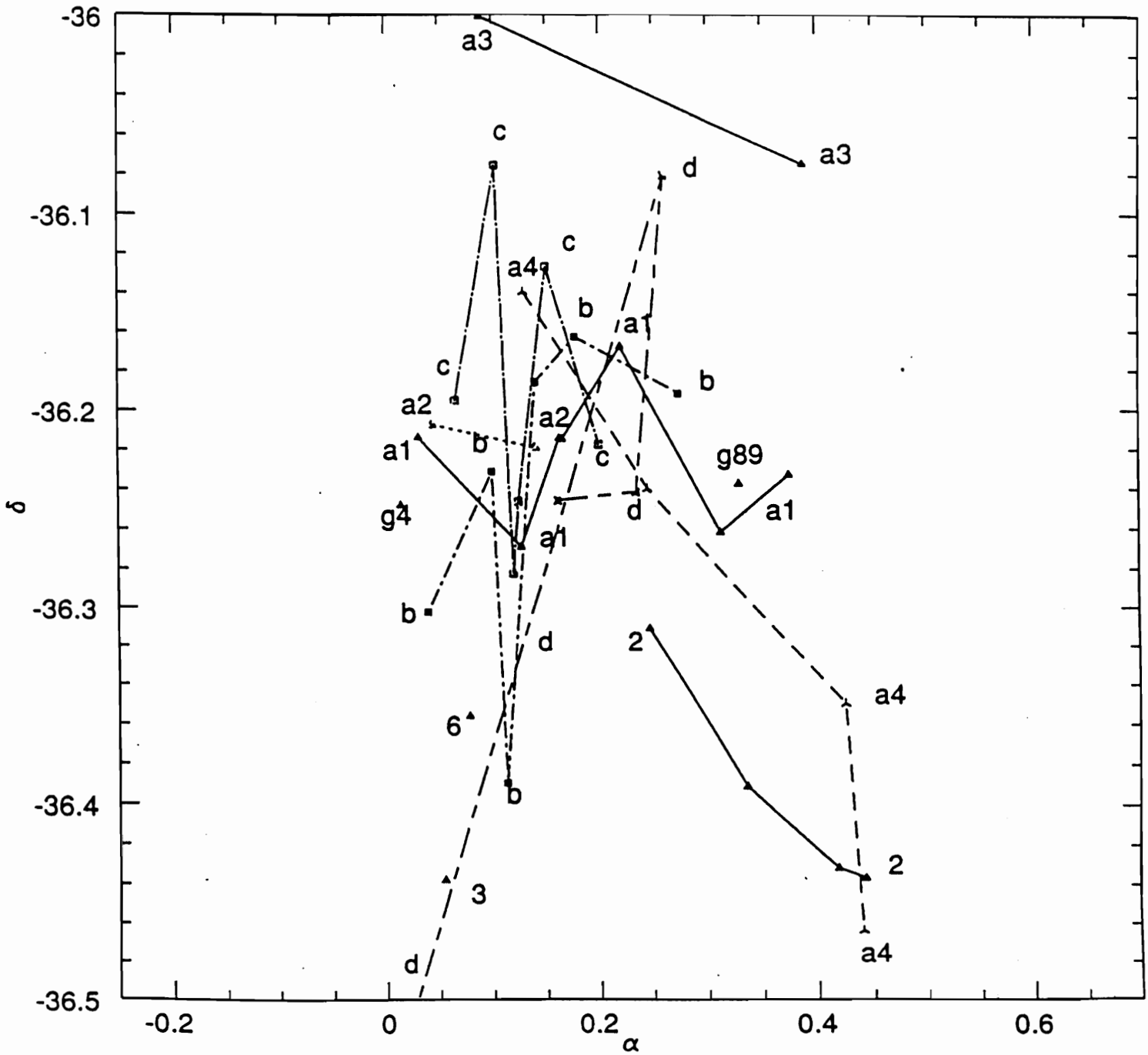
0
-50
δ
-0.2
0
0.2
0.4
0.6
α
5
5
8
9
8
9
4,18
4,18
19
13
13
14
7
7
17
3
6
2
2
1
1
11b,12
11a
11a
11b,12
10
10
16
15


Model distribution of expectations
V = 1000 km/s
dv = 900
500
100 km/s
I
II
III
IV
V
VI
VII
VIII
$P_{ch}=ex/ex_{max}$
ρ
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50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-36.1
-36.2
-36.3
-36.4
-36.5
δ
-0.2
0
0.2
0.4
0.6
α
a3
a3
c
d
c
a4
b
a1
c
b
a2
a2
b
c
a1
g89
g4
d
a1
a1
b
d
2
a4
6
b
2
3
a4
d