FPRINT-94-07

Fermilab Library
0 1160 0034257 0

MACROSCOPIC DESCRIPTION OF GRAVITATION*

Roustam M. Zalaletdinov† Tashkent, I/F

Department of Theoretical Physics, Institute of Nuclear Physics
Uzbek Academy of Sciences, Ulugbek, Tashkent 702132, Uzbekistan, CIS

August 22, 1993

1993/06/26

Abstract

The problem of macroscopic description of gravity and commonly used approaches to treat it are discussed. A new setting of the problem is proposed. A full geometric theory of macroscopic gravity is presented and described.

* *To appear in the Proceedings of the International Workshop on Experimental Gravitation, Nathiagali, Pakistan, June 26 - July 1, 1993*

† *E-mail address: zala@hq.iae.tashkent.su*

The idea of macroscopic gravity can be considered as an extension of Lorentz' idea (1895, 1916), formulated first for electrodynamics, about the existence of two levels, microscopic and macroscopic, of understanding classical physical phenomena. Lorentz showed that Maxwell's electrodynamics is a macroscopic theory of electromagnetism, and the Maxwell equations can be derived from a system of microscopic field equations called now the Maxwell-Lorentz ones, by averaging them out over space-time regions (see de Groot and Suttorp (1972), Novacu (1955) for details and references). Of great physical importance is the motivation for formulating such a macroscopic theory. Indeed, a space-time averaging means a measurement procedure by means of a measurement system having a defined finite space size and a measurement time and the averaging is therefore carried out over space regions and time intervals physically small (Lorentz 1895, 1916) compared with macroscopic distances and times characteristic of the process under consideration. Thus a macroscopic theory has a direct observational status and answers the questions of principle of which objects can be observed in classical measurements of fields and matter characteristics (recall, as an example, induction in electrodynamics).

The problem of macroscopic description of gravitation is a long-standing problem which has been posed by a number of authors (Shirokov and Fisher 1962, Sciama 1971, Ellis 1984; see Zalaletdinov (1993a) for a review). The motivation of a setting of the problem within general relativity came from cosmology where one is usually solving the Einstein equations with a continuously distributed stress tensor when looking for a cosmological solution. It implies that a space-time (or, ensemble) averaging of a real discrete matter distribution (stars, galaxies, etc.) has been carried out. A correct statement of the problem requires one to average out both a microscopic matter distribution and the Einstein field operator. It should be pointed out that taking Einstein's equations as the microscopic ones is well-grounded, for it is these equations that provide an exact solution for the gravitational field of an isolated point mass (Schwarzschild's solution).

Unlike electrodynamics, however, where the linear field operator of Maxwell's equations is known to be easily averaged out and the problem to be solved consists in constructing an electromagnetic medium model (Lorentz 1916), a realization of the programme within general relativity does require one to overcome severe troubles of both geometrical and statistical nature. Indeed, apart from the problem of a macroscopic gravitating matter description, a space-time (or, ensemble) averaging out of Einstein's field operator itself is a highly non-trivial problem arising due to a non-flat geometry underlying general relativity. This non-linear character of gravitational field raises, in particular, the question of how to treat consistently the field's correlations.

All attempts to formulate a macroscopic theory of gravity have been made mainly in the framework of linearized theory and no results have in fact been achived for a full theory (see Ellis (1984), Zalaletdinov (1992b, 1993a,c) for a review and references). All the papers

followed the generally adopted way, evoked by Lorentz's approach to electrodynamics, *of an averaging of Einstein's equations* to arrive at the macroscopic field equations and to construct thereby a macroscopic theory of gravity. Such an approach, however, failed to give and cannot in fact give a satisfactory solution to the problem since no proposal has been made about the correlation functions which should inevitably emerge in averaging a non-linear theory. The Einstein equations itself cannot provide us with a definition of the correlation functions and they together with the differential equations to find them must be defined in addition to the averaged field equations.

A new setting of the problem of formulating a full theory of macroscopic gravity has been proposed in Zalaletdinov (1992a,b,c, 1993a,b,c). Due to this statement one should study, first of all, the problem of how *to average out a (pseudo-)Riemannian space-time itself*, i.e Cartan's structure equations describing the structure of a Riemannian geometry. While doing this it is necessary to understand which averaged geometrical object - metric, connection, or curvature - can characterize an averaged space-time. Another important topic is the splitting of the the averages of products of the objects, being found in averaging out Cartan's structure equations. *This is the problem of introducing the correlation functions.* Upon deriving the structure equations for the averaged manifold, the Einstein equations which are Riemannian in their geometrical sense and are known to be additional conditions to Cartan's equations, can successfully be averaged out.

Such an approach to formulate a macroscopic theory of gravity is essentially non-perturbative and provides us with the geometry underlying the macroscopic gravitational phenomena. The geometry of the averaged (macroscopic) space-time is characterized by two curvature tensors, $M^{\alpha}{}_{\beta\gamma\delta}$ and $R^{\alpha}{}_{\beta\gamma\delta}$, being Riemannian and non-Riemannian, respectively, (the notation and averaging scheme hereafter follow these in Zalaletdinov (1992b); see also Zalaletdinov (1993c)). The curvature tensor $M^{\alpha}{}_{\beta\gamma\delta}$ is the induction tensor and corresponds to the average symmetric connection $\bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$. The second one is the average field tensor, $R^{\alpha}{}_{\beta\gamma\delta} = \bar{r}^{\alpha}{}_{\beta\gamma\delta}$, for another symmetric connection $\Pi^{\alpha}{}_{\beta\gamma}$. There is a remarkable relation between the two tensors, which results from averaging the second Cartan equation

$$M^{\alpha}{}_{\beta\rho\sigma} = R^{\alpha}{}_{\beta\rho\sigma} + Q^{\alpha}{}_{\beta\rho\sigma}.$$

The relation is of the form of a constitutive relation between the induction and average field and the origin of this geometric relation lies in the simple, but non-trivial, geometric fact of the non-linear definition of the affine curvature in terms of connection, which results in the curvature determined by the average connection not being equal to the average curvature. This difference

$$Q^{\alpha}{}_{\beta\rho\sigma} = 2 < \mathcal{F}^{\delta}{}_{\beta[\rho}\mathcal{F}^{\alpha}{}_{\underline{\delta}\sigma]} > -2\bar{\mathcal{F}}^{\delta}{}_{\beta[\rho}\bar{\mathcal{F}}^{\alpha}{}_{\underline{\delta}\sigma]}$$

is the field polarization tensor of the form of *a correlation tensor* with the polarization potential $A^{\alpha}{}_{\beta\gamma} = \bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma} - \Pi^{\alpha}{}_{\beta\gamma}$ that is an affine deformation tensor in its geometrical meaning. The equations for $A^{\alpha}{}_{\beta\gamma}$

$$A^{\alpha}{}_{\beta[\rho||\sigma]} - A^{\varepsilon}{}_{\beta[\rho}A^{\alpha}{}_{\underline{\varepsilon}\sigma]} = -\frac{1}{2}Q^{\alpha}{}_{\beta\rho\sigma}$$

are shown to be always integrable on an arbitrary averaged manifold and the tensor $A^{\alpha}{}_{\beta\gamma}$ does therefore exist.

A metric tensor $G_{\alpha\beta}$ compatible with the connection $\bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$, $G_{\alpha\beta||\gamma} = 0$, ($|$ and $||$ denote the covariant derivatives with respect to the connections $\Pi^{\alpha}{}_{\beta\gamma}$ and $\bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$, respectively) and

thus corresponding to the Riemannian curvature tensor $M^{\alpha}{}_{\beta\gamma\delta}$ is considered to be the metric tensor of the averaged space. The average field tensor $R^{\alpha}{}_{\beta\gamma\delta}$ is non-Riemannian due to the metric $G_{\alpha\beta}$ is incompatible with the connection $\Pi^{\alpha}{}_{\beta\gamma}$, $G_{\alpha\beta|\gamma} = N_{\alpha\beta\gamma}$ where $N_{\alpha\beta\gamma}$ is the non-metricity object. The averaged metric tensors $\bar{g}_{\alpha\beta} \neq G_{\alpha\beta}$ and $\bar{g}^{\alpha\beta} \neq G^{\alpha\beta}$ in general and they are not metric tensors any more, i.e. $\bar{g}^{\alpha\beta}\bar{g}_{\beta\gamma} \neq \delta^{\alpha}_{\gamma}$, although the averaging keeps them covariantly constant, $\bar{g}_{\alpha\beta||\gamma} = 0$, $\bar{g}^{\alpha\beta}_{||\gamma} = 0$. Nevertheless, one can always put (Zalaletdinov 1992c, 1993b,c)

$$\bar{g}_{\alpha\beta} = G_{\alpha\beta}.$$

The covariantly constant symmetric tensor $\bar{g}^{\alpha\beta}$ is then an object of the theory, the presence of which makes the macroscopic space-time be reducible due to a theorem proved in Zalaletdinov (1992b, 1993c). The theorem gives a classification of all possible macroscopic space-times according to Petrov's types of the induction tensor and kinds of the macroscopic metric tensor reducibility.

In a 4-dimensional space-time there are three correlation tensors, the correlation 2-form

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]} = < \mathcal{F}^{\alpha}{}_{\beta[\gamma}\mathcal{F}^{\mu}{}_{\underline{\nu}\sigma]} > - \bar{\mathcal{F}}^{\alpha}{}_{\beta[\gamma}\bar{\mathcal{F}}^{\mu}{}_{\underline{\nu}\sigma]},$$

with $Q^{\alpha}{}_{\beta\gamma\lambda} = 2Z^{\delta}{}_{\beta[\gamma}{}^{\alpha}{}_{\underline{\delta}\lambda]}$, and the correlation 3-form $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}$ and 4-form $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\underline{\varphi}\psi]}$ which are defined similarily (Zalaletdinov 1992b,c, 1993b,c). It is a remarkable fact that the number of the tensors is finite due to a finite space-time dimension. These *tensors* are constructed from the connection coefficients and are the functions of a space-time point x (it means that the theory proposed is that of the equilibrium macroscopic gravitational processes; for a non-equilibrium theory (Zalaletdinov 1993e) it is necessary to introduce the many-points correlation functions, which become the correlation functions presented here in the case of all agruments coincidence). In their geometrical sense the correlation functions are non-trivial generalizations of the concept of the affine curvature tensor and there are the structure equation for the 2-form $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$

$$Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma||\lambda]} = Y^{\delta}{}_{\beta[\gamma}{}^{\alpha}{}_{\underline{\delta}\sigma}{}^{\mu}{}_{\underline{\nu}\lambda]} - Y^{\alpha}{}_{\beta[\gamma}{}^{\delta}{}_{\underline{\nu}\sigma}{}^{\mu}{}_{\underline{\delta}\lambda]}$$

$$+\frac{1}{2} < \mathbf{r}^{\alpha}{}_{\beta[\gamma\sigma}\mathcal{F}^{\mu}{}_{\underline{\nu}\lambda]} > -\frac{1}{2}R^{\alpha}{}_{\beta[\gamma\sigma}\bar{\mathcal{F}}^{\mu}{}_{\underline{\nu}\lambda]} - \frac{1}{2} < \mathbf{r}^{\mu}{}_{\nu[\gamma\sigma}\mathcal{F}^{\alpha}{}_{\underline{\beta}\lambda]} > +\frac{1}{2}R^{\mu}{}_{\nu[\gamma\sigma}\bar{\mathcal{F}}^{\alpha}{}_{\underline{\beta}\lambda]}$$

and the structure equation for the 3-form $Y^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi]}$ (Zalaletdinov 1992b,c, 1993b,c), both of them being integrable on an arbitary averaged manifold. It is the structure equation for $Z^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]}$ that is simultaneously a splitting rule for the average of the product $< \mathbf{r}^{\alpha}{}_{\beta[\gamma\sigma}\mathcal{F}^{\mu}{}_{\underline{\nu}\lambda]} >$, to enable one to average the differential Bianchi identities out. Given the correlation 4-form $X^{\alpha}{}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma}{}^{\theta}{}_{\underline{\kappa}\pi}{}^{\tau}{}_{\underline{\varphi}\psi]}$ and the correlators in the right-hand side of the structure equations (see Zalaletdinov (1992c, 1993b,c) for details), these are the differential equations to find them.

It should be pointed out that the geometry of averaged space-time with a metric, two curvature tensors, two connections, and three correlations tensors (for a 4-dimensional space-time) is a new geometry being a non-trivial generalization of the metric affine connection geometries and it is the geometry that underlies the macroscopic theory of gravity.

To average out the metric structure equations and the Einstein equations it is necessary (Zalaletdinov 1992b,c, 1993b,c) to assume the following splitting rule for the average of a

covariantly constant tensor $C^{\mu\ldots}_{\nu\ldots}$ times the connection coefficients:

$$< \mathcal{F}^{\alpha}{}_{\beta\gamma}\mathbf{C}^{\mu\ldots}_{\nu\ldots} >= \bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}\bar{C}^{\mu\ldots}_{\nu\ldots}$$

which is always true due to the error done being less than that of averaging. Then the rule for splitting *the average of metric times curvature can be derived* (Zalaletdinov 1992b,c, 1993b,c)

$$< \mathbf{r}^{\alpha}{}_{\beta\gamma\lambda}\mathbf{g}^{\varepsilon\rho} > -R^{\alpha}{}_{\beta\gamma\lambda}\bar{g}^{\varepsilon\rho} = -2Z^{\alpha}{}_{\beta[\gamma}{}^{\varepsilon}\underline{\delta}_{\lambda]}\bar{g}^{\delta\rho} - 2Z^{\alpha}{}_{\beta[\gamma}{}^{\rho}\underline{\delta}_{\lambda]}\bar{g}^{\varepsilon\delta}$$

(and there is similar one of course for the covariant metric tensor). This rule is of great physical importance, for *it is the only rule needed for the Einstein equations to be averaged out.* The result is the macroscopic field equations

$$G^{\alpha\varepsilon}M_{\varepsilon\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}G^{\mu\nu}M_{\mu\nu} = -\kappa T^{\alpha(macro)}_{\beta}$$

where the macroscopic stress tensor $T^{\alpha(macro)}_{\beta}$ is of the form

$$\kappa T^{\alpha(macro)}_{\beta} = \kappa < \mathbf{t}^{\alpha(micro)}_{\beta} > -(Z^{\alpha}{}_{\mu\nu\beta} + \frac{1}{2}\delta^{\alpha}_{\beta}Q_{\mu\nu})\bar{g}^{\mu\nu} + U^{\alpha\varepsilon}M_{\varepsilon\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}U^{\mu\nu}M_{\mu\nu},$$

the stress tensor being conserved with respect to the connection $\bar{\mathcal{F}}^{\alpha}{}_{\beta\gamma}$

$$T^{\alpha(macro)}_{\beta||\alpha} = 0.$$

Here $Z^{\alpha}{}_{\mu\nu\beta} = 2Z^{\alpha}{}_{\mu[\varepsilon}{}^{\varepsilon}\underline{\nu}_{\beta]}$ is a Ricci-tensor like object for the correlation tensor, $Q_{\mu\nu} = Q^{\varepsilon}{}_{\mu\nu\varepsilon}$, and $< \mathbf{t}^{\alpha(micro)}_{\beta} >$ is the averaged energy-momentum tensor. The covariantly constant tensor $U^{\alpha\beta} = \bar{g}^{\alpha\beta} - G^{\alpha\beta}$ is determined due to a theorem (Zalaletdinov 1993c). Though somewhat unexpected, the result is natural: a space-time averaging out of the Einstein equations brings *the Einstein equations again for the induction Ricci tensor defined through the macroscopic metric*, and a macroscopic stress tensor includes, in addition to the averaged matter, the correlation tensor terms for the geometric correction of the averaged matter. It is remarkable that the macroscopic Einstein equations can be put (Zalaletdinov 1993d) in the form of Hooke' law

$$(M^{\alpha}{}_{\mu\nu\beta} - Z^{\alpha}{}_{\mu\nu\beta})\bar{g}^{\mu\nu} = -\kappa(< \mathbf{t}^{\alpha(micro)}_{\beta} > -\frac{1}{2}\delta^{\alpha}_{\beta} < \mathbf{t}^{\varepsilon(micro)}_{\varepsilon} >)$$

with $M^{\alpha}{}_{\mu\nu\beta} - Z^{\alpha}{}_{\mu\nu\beta}$ and $\bar{g}^{\alpha\beta}$ being as the tensor of elastic moduli and the deformation tensor of the macroscopic space-time, respectively.

The correspondence principle for the theory has been formulated in Zalaletdinov (1993a,b,c). It reads that the macrovacuum equations

$$M_{\alpha\beta} = Q_{\alpha\beta} \qquad \bar{g}^{\alpha\varepsilon}Q_{\beta\varepsilon} = Z^{\alpha}{}_{\mu\nu\beta}\bar{g}^{\mu\nu}$$

become Isaacson's equations (Isaacson 1968)

$$M_{\alpha\beta} = -\kappa T^{(GW)}_{\alpha\beta}$$

in the high-frequency limit. As a result, the correlation tensor $Q_{\mu\nu} = 2Z^{\delta}{}_{\mu[\nu}{}^{\varepsilon}{}_{\underline{\delta\varepsilon}]}$, which serves as the macrovacuum source and is equal to $-\kappa T^{(GW)}_{\alpha\beta}$ in the high-frequency limit, has been established to describe the energy momentum of the macrovacuum gravitational field. Thus, in the macroscopic theory of gravity there is *the tensor object for the energy momentum of macroscopic gravitation*

$$-\kappa T^{\alpha(grav)}_{\beta} = (Z^{\alpha}{}_{\mu\nu\beta} + \frac{1}{2}\delta^{\alpha}_{\beta} Q_{\mu\nu})\bar{g}^{\mu\nu},$$

due to the conservation of the macroscopic stress tensor $T^{\alpha(macro)}_{\beta}$ the following equation of motion for the averaged matter holding

$$\kappa < \mathbf{t}^{\alpha(micro)}_{\beta} >_{||\alpha} = (Z^{\varepsilon}{}_{\mu\nu\beta||\varepsilon} + \frac{1}{2} Q_{\mu\nu||\beta})\bar{g}^{\mu\nu}.$$

Indeed, a simple consideration shows that after being summed up in averaging out over a space-time region the gravitational field energy becomes localizable and can be treated, as a consequence, by a tensor object. As a matter of principle, this fact has first been established by Isaacson (1968) within the high frequency approximation for general relativity. The full macroscopic theory proposed has provided a general solution for the problem.

Thus, a solution for the problem of the macroscopic description of gravitation has been found only along the geometric line of consideration. A rich geometric structure turned out to govern the field correlations and a new geometry which generalizes the Riemannian geometry of the space-times in general relativity has been established to underlie the macroscopic gravitational phenomena and describe thereby the structure of macroscopic space-times. The role of the correlation tensors in the evolution of Universe and in the large-scale structure formation is under study.

Acknowledgment

This work was supported, in part, by a Meyer Foundation Grant awarded by the American Physical Society.

References

de Groot S R and Suttorp L G 1972 *Foundations of Electrodynamics* (Amsterdam: North-Holland)

Ellis G F R 1984 *General Relativity and Gravitation* ed B Bertotti, F de Felici and A Pascolini (Dordrecht: Reidel)

Isaacson R A 1968 *Phys. Rev.* **166** 1272

Lorentz H A 1895 *Versuch einer Theorie der Elektrischen und Optischen Erscheinungen in Bewegten Körpern* (Leiden)

—1916 *The Theory of Electrons* (Leipzig: Teubner)

Novacu V 1955 *Introducere in Electrodinamica* (Bucharest: Editura Academiei)

Sciama D W 1971 *Modern Cosmology* (Cambridge: Cambridge University Press)

Shirokov M F and Fisher I Z 1962 *Astron. Zh.* **39** 899 (in Russian)

Zalaletdinov R M 1992a *GR13 Conference* (Cordoba: Universidad Nacional de Cordoba) Abstracts

—1992b *Gen. Rel. Grav.* **24** 1015

—1992c *Proc. 5th Reg. Conf. on Mathematical Physics (Edirne, Turkey)* ed M Koca (Singapore: World Scientific)

—1993a Macroscopic gravity: A new setting of the problem *Preprint* Institute of Nuclear Physics, Uzbek Acad. Sci. (November 1992/May 1993) submitted for publication

—1993b Macroscopic gravity: The geometry and field equations *Preprint* Institute of Nuclear Physics, Uzbek Acad. Sci. (November 1992/May 1993) submitted for publication

—1993c Towards a theory of macroscopic gravity, *Gen. Rel. Grav.* **25** 673

—1993d The macroscopic Einstein equations as Hooke's law *Preprint* Institute of Nuclear Physics, Uzbek Acad. Sci. (June 1993) to be submitted for publication

—1993e in progress