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PLANE WAVE AS A CARRIER OF ACHRONAL SPIN

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ABSTRACT

The black body spectrum as well as its concomitant thermal fluctuation spectrum, which are predicted by quantum mechanics in a pair of uniformly accelerated frames, are traced to the existence and conservation of a new dynamical variable: achronal spin. It is found that a Klein-Gordon scalar charge has achronal spin $\frac{1}{2}$.

This spin precesses around a universal direction, the "Planckian" vector, whose components are the Planckian power spectrum and its concomitant thermal fluctuation spectrum. It is shown that if the scalar charge is in a plane wave state, then the achronal spin does not precess: it is aligned with its precession axis, the "Planckian" direction.

The invariance of the achronal spin precession axis and the invariance of the plane wave states express the same basic fact of Nature, the translation invariance of Minkowski spacetime. The difference is that pairs of accelerated frames express this translation invariance in terms of their invariant achronal spin precession axes, while inertial frames use the invariant plane wave momentum components to express the same thing.

I. INTRODUCTION

Classical mechanics was the starting point by which Newton and Einstein brought gravitation into our grasp. Their particle trajectories and world lines carry the imprint of the gravitational potential or the metric geometry from which one can deduce the properties of gravitation. It is by observing and examining classical orbits and world lines that we get to know the nature of the ambient spacetime and hence get to know gravitation and its properties.

However fundamentally Nature is quantum mechanical, and this means that the carrier of the imprints of gravitation should reflect this fact. If this is indeed the case, then our starting point should be purely quantum mechanical, and we must ask whether there exists a purely quantum mechanical carrier of the imprints of gravitation.

The purpose of this note is to put forth achronal spin as a candidate for such a carrier.

Achronal spin serves two roles. First, it is an attribute of a scalar particle. It is a dynamical variable which is conserved when the particle is free and not influenced by gravitation. Second, it indicates the reference frame in which the particle is observed. This is a familiar circumstance. For example, the constancy of all the components of a free particle's linear momentum indicates that the reference frame is inertial.

By contrast, the conserved achronal spin of a particle indicates that it is observed relative to a *pair* of achronal (non-timelike related, i.e. causally disjoint) uniformly accelerated frames. They were first considered by Rindler [1].

II. ACHRONAL SPIN: ITS EXISTENCE

In order to make evident the existence and the properties of achronal spin it is necessary that one's picture of spacetime be a union of pairs of accelerated frames. If one accepts this picture, then one finds that a single (scalar) charge is an entity with "achronal" spin $\frac{1}{2}$. Why? The existence of this spin $\frac{1}{2}$ is a direct consequence of the symmetry of the relativistic single particle Klein-Gordon quantum system.

A. Two Component Wave Equation

The wave equation for the system is defined on the two disconnected Rindler Sectors I and II,

$$\left.\begin{array}{l} t - t_0 = \pm\xi \sinh\tau \\ x - x_0 = \pm\xi\cosh\tau \end{array}\right\} \quad \begin{array}{l} +: \text{ Rindler Sector I} \\ -: \text{ Rindler Sector II} \end{array} , \tag{2.1}$$

and its explicit form is

$$\left[-\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} + \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - k^2\right]\psi = 0, \tag{2.2}$$

with $k^2 = k_y^2 + k_z^2 + \frac{m^2c^2}{\hbar^2}$. Here $\xi > 0$ and $-\infty < \tau < \infty$. Note that in spite of superficial appearances, this is actually two equations, one for each Rindler sector, Eq.(2.1). The wave function is defined on these two disconnected subdomains. Consequently, it must be expressed as a two-component wave function

$$\psi(\tau,\xi) = \begin{bmatrix} \psi_{\rm I}(\tau,\xi) \\ \psi_{\rm II}(\tau,\xi) \end{bmatrix} , \tag{2.3}$$

one component for each subdomain, Rindler I and Rindler II. The result is that relative to a pair of accelerated frames at the event, say, (t_0, x_0) the quantum mechanical behaviour of a single scalar charge is governed by the *two-component* wave equation

$$\left[-\frac{1}{\xi^2}\frac{\partial^2}{\partial\tau^2} + \frac{1}{\xi}\frac{\partial}{\partial\xi}\xi\frac{\partial}{\partial\xi} - k^2\right]\begin{bmatrix} \psi_{\rm I}(\tau,\xi) \\ \psi_{\rm II}(\tau,\xi) \end{bmatrix} = 0, \tag{2.4}$$

Its domain is spanned by the coordinates τ and ξ. Physically we have a typical point (τ, ξ) refer to a *pair of events* (one in Rindler I, the other in RindlerII) which are equivalent under spacetime inversion

$$(t - t_0, x - x_0) \rightarrow (-t + t_0, -x + x_0). \tag{2.5}$$

Mathematically we have that the domain of our two-component wave equation is the union of the two Rindler Sectors I and II modulo the equivalence under spacetime inversion. If one wishes, one can in one's mind picture this domain as consisting of Rindler I or II, but in actuality it consists of the set of pairs of events.

[Parenthetical comment: Note that any (two-component) Cauchy surface of Rindler I and II (spacelike slice with intersection of their event horizons removed) has a domain of dependence [2] that *excludes* the future as well as the past ($|t - t_0| \geq |x - x_0|$) of the event, (t_0, x_0), where the two accelerated frames "touch". In other words, this future and this past are not part of the domain on which the wave equation ψ is defined. Consequently, these two spacetime regions are irrelevant for the two-component wave equation and thus play no role in the defining properties of achronal spin below.]

B. Complete set of observables

The measurable properties of the scalar charge are the eigenvalues of a complete set of operators. One of these operators is

$$\begin{aligned} P_\tau &\equiv i\frac{\partial}{\partial\tau} \\ &= i\left[(t - t_0)\frac{\partial}{\partial x} + (x - x_0)\frac{\partial}{\partial t}\right] \quad , \end{aligned} \tag{2.6}$$

the "boost" generator, whose eigenvalues are the scalar charge's moment of mass energy around the event (t_0, x_0). A typical eigensolution, a stationary state, satisfies $P_\tau\ \psi_\omega = \omega\ \psi_\omega$ and has the form

$$\psi_\omega = \begin{bmatrix} a_\omega \\ b_\omega^* \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} \quad . \tag{2.7}$$

Here $K_{i\omega}(k\xi)$ is the MacDonald function. We see that each eigenvalue ω is doubly degenerate. This implies we must find another operator, say $\not{h}_\omega$, (i) which may depend on the particular eigenvalue ω of P_τ, (ii)which commutes with P_τ,

$$[P_\tau, \not{h}_\omega] = 0 \quad ,$$

and (iii) whose eigenvalues can be used to distinguish the quantum states of a degenerate eigenvalue ω. If one can find such a $\not{h}_\omega$, then the eigenvalues of P_τ and $\not{h}_\omega$ will characterize uniquely each of their common eigenvectors.

To this end introduce one half the modified Pauli spin matrices

$$\vec{L} : \{L_1, L_2, L_3\} = \left\{\frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\right\}. \tag{2.8}$$

They obviously commute with the moment of mass-energy operator:

$$[P_\tau, L_i] = 0 \quad i = 1, 2, 3$$

Why pick the "modified" spin matrices? Answer: They are determined by the nature of the quantum mechanical inner product. That inner product is indispensible for the physical interpretation of a linear wave equation. For a scalar charge the inner product is that of Klein-Gordon

$$\begin{aligned} \langle\psi, \psi'\rangle &= \frac{i}{2}\int_\infty^0 \left(\psi_{II}^*\frac{\partial}{\partial\tau}\psi_{II}' - \frac{\partial}{\partial\tau}\psi_{II}^*\psi_{II}'\right)\frac{d\xi}{\xi} + \frac{i}{2}\int_0^\infty \left(\psi_I^*\frac{\partial}{\partial\tau}\psi_I' - \frac{\partial}{\partial\tau}\psi_I^*\psi_I\right)\frac{d\xi}{\xi} \\ &= \frac{i}{2}\int_0^\infty \left(\psi^\dagger\sigma_3\frac{\partial}{\partial\tau}\psi' - \frac{\partial}{\partial\tau}\psi^\dagger\sigma_3\psi'\right)\frac{d\xi}{\xi}. \end{aligned} \tag{2.9}$$

Here σ_3 is the Pauli matrix

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad .$$

Each of the operators we are considering, P_τ, L_1, L_2, and L_3, is required to have the property that it is self-adjoint relative to this Klein-Gordon inner product,

$$\begin{aligned} \langle L_i\psi, \psi' \rangle &= \langle \psi, L_i\psi' \rangle \quad i = 1,2,3 \\ \langle P_\tau\psi, \psi' \rangle &= \langle \psi, P_\tau\psi' \rangle \quad . \end{aligned} \tag{2.10}$$

These conditions can be readily verified. For an eigenvector whose inner product with itself does not vanish, these self-adjointness conditions guarantee that its eigenvalues are real, as measurable quantities should be. They also guarantee that the finite symmetry transformations

$$e^{iL_1\alpha}, e^{iL_2\theta}, e^{iL_3\phi}, \text{ and } e^{iP_\tau\tau} \tag{2.11}$$

leave invariant the inner product

$$\begin{aligned} \langle e^{iL_1\alpha}\psi, e^{iL_1\alpha}\psi' \rangle &= \langle \psi, \psi' \rangle \\ &\vdots \\ &\text{etc} \end{aligned} \tag{2.12}$$

as all symmetry transformations must. The transformations form a group. In fact, the generators L_1, L_2, and L_3 obey the commutation relations

$$\begin{aligned} [L_1, L_2] &= -iL_3 \\ [L_2, L_3] &= iL_1 \\ [L_3, L_1] &= iL_2 \quad . \end{aligned} \tag{2.13}$$

of the group $SU(1,1)$. This group is however not a unitary group because the Klein-Gordon inner product, which is kept invariant, is not positive definite.

It is now straight forward to determine what that sought after operator $\hbar_\omega$ is. The commutatity of the boost generator P_τ with the L_i's implies that we are looking for some arbitrary linear combination of the L_i's:

$$\hbar_\omega = -n_1L_1 - n_2L_2 + n_3L_3 \quad . \tag{2.14}$$

Its square is

$$\begin{aligned} \hbar_\omega\ \hbar_\omega &= \frac{1}{4}(n_3^2 - n_2^2 - n_1^2)I \\ &\equiv \frac{1}{4}\vec{n}_\omega \cdot \vec{n}_\omega I \quad . \end{aligned} \tag{2.15}$$

Without loss of generality one may choose $\vec{n}_\omega$ to be normalized to one of the three possibilities

$$\begin{aligned} \text{"timelike"} &: \vec{n}_\omega \cdot \vec{n}_\omega = +1, \\ \text{"spacelike"} &: \vec{n}_\omega \cdot \vec{n}_\omega = -1, \\ \text{or "null"} &: \vec{n}_\omega \cdot \vec{n}_\omega = 0 \quad , \end{aligned} \tag{2.16}$$

Suppose ψ_ω is a simultaneous eigenstate of P_τ and $\hbar_\omega$:

$$P_\tau\psi_\omega = \omega\psi_\omega \tag{2.17a}$$
$$\hbar_\omega\psi_\omega = s\psi_\omega \tag{2.17b}$$

Apply $\not{h}_\omega$ to the second equation and obtain

$$\not{h}_\omega \not{h}_\omega \psi_\omega = s^2 \psi_\omega \tag{2.18}$$

With the help of Eq.(2.15) one concludes that for the case $\vec{n}_\omega \cdot \vec{n}_\omega = 1$ the allowed eigenvalues of $\not{h}_\omega$ are

$$s = \pm\frac{1}{2}.$$

What is their physical significance? To find out, note that the operator $\not{h}_\omega$ generates the rotation

$$e^{i\not{h}_\omega \alpha} = \cos\frac{\alpha}{2} + 2i \not{h}_\omega \sin\frac{\alpha}{2} \quad , \tag{2.19}$$

by the angle α around the vector $\vec{n}_\omega$ in a three dimensional Lorentz space. Consequently, the two eigenvalues $s = \pm\frac{1}{2}$ express a spin attribute of our quantum system, the scalar charge. If $s = \frac{1}{2}$ then it is in a quantum state corresponding to its *achronal* spin pointing "up", while $s = -\frac{1}{2}$ corresponds to achronal spin "down".

What is the meaning of this "achronal" spin pointing "up" or "down"? To this end let $\vec{n}_\omega = (1,0,0)$, so that $\not{h}_\omega = L_3$. In this special case the two eigenvectors corresponding to $s = \pm\frac{1}{2}$ are

$$s = \frac{1}{2}: \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau}; \quad s = -\frac{1}{2}: \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} \quad . \tag{2.20}$$

Consequently, achronal spin "up" expresses the circumstance where the charge has non-zero amplitude only in Rindler Sector I and none in Sector II. For spin "down" the amplitude is non-zero only in Rindler Sector II. The spin is "achronal" because events in Rindler I and II are related achronally [2] to each other.

To summarize: Achronal spin is an additional dynamical degree of freedom with discrete values, $s = \pm\frac{1}{2}$. These values, together with the moment of mass-energy ($-\infty < \omega < \infty$), specify completely the quantum state of a charge relative to a pair of accelerated frames. The wave function for such a state is that solution to the two-component wave equation (2.4) which satisfies the two eigenvalue equations (2.17). Their eigenvalues s and ω are the labels for a unique quantum state.

III. ACHRONAL SPIN: ITS PRECESSION

The achronal spin operator $\not{h}_\omega$ imparts geometrically and physically appealing properties to the quantum states of the scalar charge. These properties become obvious when one looks at plane wave solutions to the wave equation

A. Scalar plane waves via spin modes

Consider a plane wave state

$$e^{\mp i(t-t_0)k\cosh\theta + i(x-x_0)k\sinh\theta} \tag{3.1}$$

of either positive (upper sign) or negative (lower sign) Minkowski frequency. The angle θ, we recall, is the angular parameter on the mass hyperboloid $\omega_k^2 - k_x^2 = k_y^2 + k_z^2 + \frac{m^2}{\hbar^2} \equiv k^2$,

$$\begin{aligned} \omega_k &= k\cosh\theta > 0 \\ k_x &= k\sinh\theta \end{aligned} \tag{3.2}$$

A plane wave state is spacetime translation invariant. Consider its expansion in terms of the boost invariant eigenfunctions, Eq.(2.20), in each of the two Rindler frames I and II, Eq.(2.1). The result is [3]

$$\begin{aligned} \int_{-\infty}^{\infty} \begin{bmatrix} e^{\pi\omega/2} \\ e^{-\pi\omega/2} \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} e^{i\omega\theta} d\omega &= \begin{bmatrix} e^{-i(t-t_0)k\cosh\theta + i(x-x_0)k\sinh\theta}\big|_I \\ e^{-i(t-t_0)k\cosh\theta + i(x-x_0)k\sinh\theta}\big|_{II} \end{bmatrix} \\ &= \begin{pmatrix} \text{positive frequency} \\ \text{plane wave state} \end{pmatrix} \end{aligned} \tag{3.3a}$$

and

$$\int_{-\infty}^{\infty}\begin{bmatrix} e^{-\pi\omega/2} \\ e^{\pi\omega/2}\end{bmatrix}\frac{K_{i\omega}(k\xi)}{\pi}e^{-i\omega\tau}e^{i\omega\theta}d\omega=\begin{bmatrix} e^{i(t-t_0)k\cosh\theta+i(x-x_0)k\sinh\theta}|_{I} \\ e^{i(t-t_0)k\cosh\theta+i(x-x_0)k\sinh\theta}|_{II}\end{bmatrix}$$

$$=\begin{pmatrix}\text{negative frequency}\\ \text{plane wave state}\end{pmatrix} \qquad (3.3b)$$

These are expansions of the form

$$\begin{bmatrix}\psi_{\rm I}(\tau,\xi)\\ \psi_{\rm II}(\tau,\xi)\end{bmatrix}=\int_{-\infty}^{\infty}\psi_\omega d\omega \qquad (3.4)$$

where ψ_ω are the two-component spin eigenstates, Eq.(1.7), of a to-be-determined spin operator. The natural question is: What is the direction of the concomitant achronal spin carried by each boost eigenstate?

B. The Stokes map

To this end consider the Stokes map

$$\vec{n}_\omega \longleftrightarrow \psi_\omega \quad . \qquad (3.5)$$

It is a one-to-one correspondence between the direction $\vec{n}_\omega=(n_1,n_2,n_3)$ of the operator, $\not{h}_\omega=-n_1L_1-n_2L_2+n_3L_3$, and the two quantum states in Eq.(2.17b). This correspondence is based on the inner product

$$<\psi_\omega,\psi_\omega>=\psi_\omega^\dagger\sigma_3\psi_\omega, \qquad (3.6)$$

and is given by the three "hyperbolic" Stokes parameters [4],

$$n_i=2\frac{<\psi_\omega,L_i\psi_\omega>}{<\psi_\omega,\psi_\omega>}\quad i=1,2,3. \qquad (3.7)$$

which establishes the one-to-one correspondence. One verifies that $\vec{n}_\omega=(n_1,n_2,n_3)$ is a point that lies on the "Stokes hyperboloid" [4] of two sheets, $\vec{n}_\omega\cdot\vec{n}_\omega\equiv -n_1^2-n_2^2+n_3^2=1$. Conversely, given $\vec{n}_\omega$ on one of these two sheets, one constructs $\not{h}_\omega$ and then finds one of the two Klein-Gordon solutions ψ_ω which satisfies the eigenvalue Eq.(2.17). The other eigensolution corresponds to $-\vec{n}_\omega$ on the other sheet of the hyperboloid.

C. Planckian spectrum: Precession of achronal spin

The achronal spin-induced one-to-one correspondence of Stokes, Eq.(3.7), implies a striking group theoretical confrontation between Planck's thermal spectrum and the translation invariant distinction between particles and antiparticles in the absence of gravitation.

First consider the "Planckian spectral" vector

$$\begin{aligned}\vec{n}_\omega&=(n_1,n_2,n_3) &(3.8)\\ &=2\{\left[\frac{1}{\exp\frac{\hbar\omega}{kT}-1}+\frac{1}{(\exp\frac{\hbar\omega}{kT}-1)^2}\right]^{\frac{1}{2}},0,\frac{1}{2}+\frac{1}{\exp\frac{\hbar\omega}{kT}-1}\} \quad .\end{aligned}$$

Its components are the "zero point energy" plus "Planckian thermal power" spectrum for n_3 and the "r.m.s. thermal fluctuation" spectrum for n_1. Here

$$kT=\frac{\hbar}{c}\frac{g}{2\pi}.$$

is the familiar Davies-Unruh temperature. We are using dimensionless units for the frequency ω. Consequently, the D-U temperature is merely $kT = \hbar/2\pi$. Next, consider the two quantum states

$$\begin{bmatrix} e^{\pm\pi\omega/2} \\ e^{\mp\pi\omega/2} \end{bmatrix} \frac{K_{i\omega}(k\xi)}{\pi} e^{-i\omega\tau} = \begin{cases} \text{spin "up" (upper sign)} \\ \text{spin "down" (lower sign)} \end{cases}, \tag{3.9}$$

with $-\infty < \omega < \infty$. The Stokes map, Eq.(3.7), implies that they are orthogonal (relative to Eq.(2.6)) eigenvectors corresponding to the eigenvalues $s = \pm\frac{1}{2}$ of that achronal spin operator $\not{h}_\omega$, Eq.(2.14), whose direction $\vec{n}_\omega$ is parallel and anti-parallel to the "Planckian spectral" vector, Eq.(3.8).

At last, compare these spin eigenstates with those occurring in the plane wave expansion Eqs.(3.3).

It is now evident that a Minkowski particle (resp. antiparticle) plane wave state is composed entirely of an orchestra of boost eigenstates, each one of equal intensity but with different phase and each one carrying achronal spin $\frac{1}{2}$ parallel (resp. antiparallel) to the "Planckian spectral" vector $\vec{n}_\omega$. The spectral decompositions, Eqs.(3.3), express the fact that ***a plane wave is a carrier of achronal spin.*** Indeed, the plane wave, Eq.(3.3a) (resp. Eq.(3.3b)) carries the amount $s = \frac{1}{2}$ (resp. $s = -\frac{1}{2}$) in each boost spectral component. (The subscripts I and II designate, as usual, evaluation in the two respective Rindler sectors.)

Recalling that plane wave states are invariant under translations in Minkowski spacetime, one sees that achronal spin leads to the following conclusion: *The translation invariance of Minkowski spacetime is characterized by the achronal spin of the system pointing into the "Planckian spectral" direction, Eq.(3.8), at each event of spacetime. Moreover, for particle states the achronal spin points parallel to this direction, while for antiparticles it points into the antiparallel direction.*

The "Planckian spectral" direction is the same at all events of Minkowski spacetime. Consequently, under translation from one event to another the invariance of each of the plane wave states is equivalent to the invariance of the achronal spin axis: it keeps pointing along the direction of the "Planckian spectral" vector, Eq.(3.8). In brief

$$\begin{pmatrix} \text{invariance of} \\ \text{each plane} \\ \text{wave state} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \text{invariance of} \\ \text{each chromatic} \\ \text{achronal spin axis} \end{pmatrix}$$

By contrast, a general non-invariant quantum state, e.g.. some superposition of plane waves, has a (boost) spectrum of achronal spins each one of which *precesses* around an *invariant axis*, which is the "Planckian spectral" direction [5].

IV. ACHRONAL SPIN AND PAIRS OF ACCELERATED REFERENCE FRAMES

The path breaking calculations of Fulling [6], Davies [7], and Unruh [8] have been with us for about twenty years. Their calculations imply the geometrical picture necessary for bringing the key feature of Einstein's accelerated local box [9,10] into agreement with the global nature of quantum mechanics: Spacetime is to be viewed as a union ("fiber bundle") of pairs of oppositely accelerated frames.

This implicit framework allowed Fulling, Davies, and Unruh to consider the quantum mechanical analogue of Einstein's seminal gedanken experiment: free particles observed relative to a linearly accelerated frame. Their version of this experiment led inexorably to a new and astonishing result: a particle independent (Davies-Unruh) temperature. Its physical nature was made plausible by Unruh, who, motivated by Fulling, found that empty ("no particles") flat spcetime manifests itself by measurable thermal black body radiation ("many particles") relative to an accelerated frame. This is a far-reaching result because it simultaneously touches every major aspect lying at the heart of what revolutionized twentieth century physics.

A repeatedly asked question that arises in this context is this: Why does Nature, in the context of pairs of accelerated frames, favor one particular function, the Planckian spectrum, over all other possible functions?

Indeed, from the viewpoint of quantum mechanics, it is rather striking, if not mysterious, that the Planckian spectrum appears under seemingly diverse circumstances. Even though there are many very different ways [11,12,13,14,15,16] of establishing the existence of the thermal ambience in accelerated frames, at the end one always obtains the same mathematical "coincidence": Relative to each member of a pair of accelerated frames, globally empty spacetime is characterized (for bosons) by the Planckian spectral function! Why?

The existence of achronal spin, and its "Planckian" orientation encoded into any plane wave gives a direct answer to this question:

The Planckian is Nature's way of using spin to express the parallel transport symmetry of Minkowski spacetime.

V. CONCLUSION

Although the implied connection between thermal physics and group theory ($SU(1,1)$) is intriguing, we emphasize that this conclusion is not based on any thermodynamic or multiparticle hypothesis. It is simply a statement about the quantum mechanical properties of a single charge in the absence of gravitation.

The reason for emphasizing this last fact is this: *deviations of these q.m. properties away from their Minkowskian values serve as a purely quantum mechanical characterization of gravitation.* This is the principle guiding our investigations. *Achronal spin is to serve as a purely quantum mechanical probe of gravitation, analogous to the one served by geodesics in Einstein's endeavor.*

The significance of Eqs.(3.3) goes beyond the mathematical fact that the set of spin "up" and spin "down" states is unitarily equivalent to the set of positive and negative Minkowski frequency (particle and antiparticle) plane wave states.

Indeed, Eqs.(3.3) relate two different philosophies. The plane wave states express the familiar picture of spacetime as a union of instantaneous Lorentz frames. By contrast, the two-component states on the left hand side embody the picture of spacetime as a union of pairs of accelerated frames, one pair at each event.

In the presence of gravitation, or relative to a curvilinear coordinate system, an instantaneous Lorentz frame (tangent space) at an event yields only the lowest order approximation to phase of the wave function of a quantum mechanical system. In other words, a Lorentz frame yields only the first two terms of a Taylor series expansion of the phase of a wave function. This limitation vitiates a particle being described in terms of quantum mechanics relative to the instantaneous Lorentz frame. Quantum mechanics demands the globally defined wave function.

From the viewpoint of classical physics, restricting one's attention to an instantaneous Lorentz frame is no problem. By resorting to the high frequency approximation and by using the principle of constructive interference, one recovers the classical world line of a particle. In an instantaneous Lorentz frame this amounts to recovering an infinitesimal line segment, i.e. the event and the four-velocity, and hence the classical state of the particle. The whole subsequent history of the particle is determined by this classical and pointwise determined state.

Quantum mechanics does not permit such a classically deterministic description of Nature.

Nevertheless this is what Newton and Einstein used to characterize gravitation, each in his own way. The classical geodesic world lines carry the imprint of gravitation by means of geodesic deviation. This leads to metric geometry and curvature, i.e. to Einstein's classical way of characterizing gravitation.

Consistency with quantum mechanics demands that gravitation instead be formulated in such a way, that its foundation, and hence its characterization, be purely quantum mechanical. The proposal of this note is to assign this task to achronal spin. In other words, achronal spin is to take over the role which particle geodesics used to play in Einstein's classical formulation of gravitation.

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[14] U. H. Gerlach, Found. Phys. 16, 667 (1986) arrives at the Planckian and its rms fluctuation spectrum by examining zeropoint fluctuations. See also T. Boyer, Phys. Rev. D 21, 2137 (1980); U. H. Gerlach, Phys. Rev. D 27, 2310, (1983).

[15] D.W. Sciama in *QUANTUM GRAVITY 2, A Second Oxford Symposium* edited by C.J. Isham, R. Penrose, and D.W. Sciama (Clarendon Press, Oxford, 1981) p210-222, has considered the reason for the specifically thermal distribution of energy among the normal modes viewed relative to an accelerated frame. His reasoning, based on statistical thermodynamics, traces the answer to a *preexisting equilibrium* among these modes when they are in their vacuum state: the introduction of a coupling among them (e.g. by a speck of dust) does not change their relative zeropoint energy.

[16] P.C.W. Davies and S.A. Fulling in Proc. R. Soc. London, A348, p393 (1976), and in Proc. R. Soc. London, A356, p237 (1977), and summarized in N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, (Cambridge University Press, New York, 1984) Chapter 4.4 arrived at

the Planckian by having an inertial detector view Minkowski zero-point radiation bouncing off a mirror accelerating to the "left". The two (spacetime) dimensional nature of this thought experiment makes it isomorphic to the one in reference [7] where an observer accelerates to the "right" and views Minkowski zero-point radiation bouncing off an inertial mirror. In other words, the detection of reflected zero-point radiation in a frame relative to which $ds^2 = -dt^2 + dx^2$ (mirror is accelerated to the left, worldline of inertial observer: x =fixed) is experimentally indistinguishable from the detection of that same reflected zero-point radiation in a frame relative to which $ds^2 = -\xi^2 d\tau^2 + d\xi^2$ (mirror is inertial, worldline of observer accelerated to the right: ξ =fixed).