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ADDING LONG WAVELENGTH MODES TO AN N–BODY SIMULATION

G. TORMEN[1,2], E. BERTSCHINGER[2]

[1] *Dipartimento di Astronomia dell'Università di Padova, Vicolo dell'Osservatorio, 5 I–35122 Padova, ITALY*

[2] *Department of Physics, MIT, Cambridge MA 02139, USA*

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Abstract

We have devised and applied a new method to add long wavelength power to an evolved N–body simulation. We briefly describe how our method works and discuss its possible application in the study of the large scale structure.

1 Why The Issue?

Computer simulations of large scale structure play today a fundamental role in cosmology by providing a better understanding of the many issues related to structure formation. Simulations are usually performed in a cubic volume L^3 on which periodic boundary conditions are imposed. For such reason, the density fluctuation power spectrum $P(k)$, given as initial condition to the simulation, is sampled discretely in the Fourier space. The sampling is made on a regular cubic lattice centered on $\vec{k} = \vec{0}$, ($|\vec{k}| \equiv 2\pi/\lambda$), with intergrid size $\Delta k = k_{\min} = 2\pi/L$ and extension in every direction determined by the number N^3 of particles used, via the Nyquist relation: $k_{\max} = \pi N/L$.

Ideally one would like to have a continuous sampling of $P(k)$, corresponding to having an infinite number of particles in an infinite volume. This would allow a detailed analysis of the model over a wide dynamical range, would overcome the problem of statistical fluctuations arising from different realizations at the lowest $\vec{k}$, and would assure that all the important contribution from large scales are included in the evolution. We have developed an approximation method that meets partially these goals.

2 The MAP Idea In A Nutshell

Our goal is to improve, at small $\vec{k}$, the sampling resolution in Fourier space. The basic idea is to take a simulation originally in a periodic cube of length L, replicate it m^3 times in a larger cube, and add

to it the power missing from the longer wavelengths not sampled in the original cube, up to $\lambda = mL$. Hence the acronym MAP (Mode Adding Procedure).

We must start with a simulation whose long waves ($\lambda \approx L$) are still linear and evolving independently from the short ones and we must devise a way to add the long waves power correctly. For this we chose to use the Zel'dovich approximation, following these steps:

1. We take the original simulation, and from the particles' final positions compute the comoving displacement field (final minus initial positions) on a regular Eulerian grid in position space.

2. We then subtract from such field the contribution of the lowest modes (longest waves) contained in a cube centered on $\vec{k} = \vec{0}$ and move the particles accordingly; we also change each particle's velocity, taking advantage of the linear relation of the velocity to the displacement. We make m^3 copies of the particles on a cube of length mL.

3. We generate a set of new random long-wavelength displacements in the $(mL)^3$ cube by randomly sampling the power spectrum. These displacements (and the corresponding velocities) are added to the particles at their Eulerian positions.

The final result is a set of particle positions and velocities which now include the effect of waves as long as mL and is more suitable than the original sample for, e.g., peculiar velocity studies.

3 Fourier Space Manipulation

The Fourier space of a periodic simulation can be thought as a cubic lattice. Each cube of side $\Delta k = 2\pi/L$ is associated with a discrete Fourier component $\hat{\delta}_{\vec{k}}$ of the density fluctuation field $\delta(\vec{x})$, with mean power per mode $\langle|\hat{\delta}_{\vec{k}}|^2\rangle \approx P(k)(\Delta k)^3$. $P(k)$ is the power spectral density corresponding to the volume Δk^3.

The procedure that we described corresponds to removing some cubes with side $\Delta k = 2\pi/L$ around $\vec{k} = \vec{0}$ (step 2 in previous section) and replace them with a larger number of smaller cubes of side $\Delta k = 2\pi/mL$ (step 3). The total power (spectral density times volume) of the replaced cubes shall be equal, in the root-mean-square sense, to the total power of the removed ones. In the region of $\vec{k}$-space where the substitution is performed the small cubes sample the power near $\vec{k} = \vec{0}$ more accurately than the big ones; they are associated with a power per mode nearly m^3 times smaller. The net effect of this procedure is that we gain a higher resolution of the fluctuations on large scales (small $\vec{k}$) — i.e., we have created a larger sample.

4 MAP Simulation

We started with a P^{3}M simulation of standard cold dark matter (Harrison–Zel'dovich initial power spectrum, $\Omega = 1$, $H_0 = 50$ km s^{-1} Mpc^{-1}, normalized to have $\sigma_8 = 0.7$ for a top–hat filter) in a cube of length $L = 100$ Mpc, with 144^3 collisionless particles each of mass $2.3 \times 10^{10}\ M_\odot$. We made 8^3 copies of the simulation, keeping a random subsample of 32,000 particles from each copy, and added to the cube of side 800 Mpc so obtained the long wavelength modes. The resulting MAP simulation has enough mass and force resolution to study both the small–scale (e.g. velocity dispersion) and large–scale (bulk flows) peculiar motions as well as the small–scale dynamics of dark matter halos.

We do not claim here that a MAP simulation is equivalent to a real, full size N–body simulation. The sampling of the fields in Fourier space is different, and some small scale periodicity, albeit modulated by long waves, usually appears in the final product. Even so, the MAP simulation behaves in general very well from the statistical point of view, and substantially fulfills what it is supposed to do. These facts, and especially the insights that such method has been giving on the gravitational clustering issue, insights only partially presented here, make it an interesting tool for opening new perspectives in the dynamics of large gravitating systems.